Optimizing Algorithms and Code for Data Locality and Parallelism Targeting Multicore Architectures Using Cilk++

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What is this tutorial about?

Optimizing algorithms and code

- Improving code performance is hard and complex.
- Requires a good understanding of the underlying algorithm and implementation environment (hardware, OS, compiler, etc.).

Optimizing for data locality

Computer cache memories have led to introduce a new complexity measure for algorithms and new performance counters for code. Optimizing for data locality brings large speedup factors.

Optimizing for parallelism

All recent home and office desktops/laptops are parallel machines; moreover "GPU cards bring supercomputing to the masses." Optimizing for parallelism improves the use of computing resources. And optimizing for data locality is often a first step!
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- Optimizing for parallelism improves the use of computing resources.
- And optimizing for data locality is often a first step!
Once upon a time, everything was slow in a computer.
The second space race . . .
What are the objectives of this tutorial?

1. Understand why data locality can have a huge impact on code performances.
2. Acquire some ideas on how data locality can be analyzed and improved.
3. Understand the concepts of work, span, parallelism, burdened parallelism in multithreaded programming.
4. Acquire some ideas on how parallelism can be analyzed and improved in multithreaded programming.
Acknowledgments and references

Acknowledgments.

- Charles E. Leiserson (MIT), Matteo Frigo (Axis Semiconductor) Saman P. Amarasinghe (MIT) and Cyril Zeller (NVIDIA) for sharing with me the sources of their course notes and other documents.
- Yuzhen Xie (Maplesoft) and Anisul Sardar Haque (UWO) for their great help in the preparation of this tutorial.

References.

- [http://www.csd.uwo.ca/~moreno/HPC-Resources.html](http://www.csd.uwo.ca/~moreno/HPC-Resources.html)
Plan

1. Data locality and cache misses
   - Hierarchical memories
   - Impact on our programs

2. Multicore programming
   - Multicore architectures
   - Cilk / Cilk++ / Cilk Plus
   - The fork-join multithreaded programming model
   - Practical issues and optimization tricks
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A CPU cache is an auxiliary memory which is smaller, faster memory than the main memory and which stores copies of the main memory locations that are expectedly frequently used.

Most modern desktop and server CPUs have at least three independent caches: the data cache, the instruction cache and the translation look-aside buffer.
Each location in each memory (main or cache) has
- a datum (cache line) which ranges between 8 and 512 bytes in size, while a datum requested by a CPU instruction ranges between 1 and 16.
- a unique index (called address in the case of the main memory)

In the cache, each location has also a tag (storing the address of the corresponding cached datum).
When the CPU needs to read or write a location, it checks the cache:

- if it finds it there, we have a cache hit
- if not, we have a cache miss and (in most cases) the processor needs to create a new entry in the cache.

Making room for a new entry requires a replacement policy: the Least Recently Used (LRU) discards the least recently used items first; this requires to use age bits.
The replacement policy decides where in the cache a copy of a particular entry of main memory will go:

- **fully associative**: any entry in the cache can hold it
- **direct mapped**: only one possible entry in the cache can hold it
- **$N$-way set associative**: $N$ possible entries can hold it
Data locality and cache misses

Hierarchical memories

---

**Cache Performance for SPEC CPU2000** by J.F. Cantin and M.D. Hill.

The SPEC CPU2000 suite is a collection of 26 compute-intensive, non-trivial programs used to evaluate the performance of a computer’s CPU, memory system, and compilers (http://www.spec.org/osg/cpu2000).
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A typical matrix multiplication C code

```c
#define IND(A, x, y, d) A[(x)*(d)+(y)]

uint64_t testMM(const int x, const int y, const int z)
{
    double *A; double *B; double *C;
    long started, ended;
    float timeTaken;
    int i, j, k;
    srand(getSeed());
    A = (double *)malloc(sizeof(double)*x*y);
    B = (double *)malloc(sizeof(double)*x*z);
    C = (double *)malloc(sizeof(double)*y*z);
    for (i = 0; i < x*z; i++) B[i] = (double) rand() ;
    for (i = 0; i < y*z; i++) C[i] = (double) rand() ;
    for (i = 0; i < x*y; i++) A[i] = 0 ;
    started = example_get_time();
    for (i = 0; i < x; i++)
        for (j = 0; j < y; j++)
            for (k = 0; k < z; k++)
                // A[i][j] += B[i][k] + C[k][j];
                IND(A,i,j,y) += IND(B,i,k,z) * IND(C,k,j,z);
    ended = example_get_time();
    timeTaken = (ended - started)/1.f;
    return timeTaken;
}
```
Contiguous accesses are better:

- Data fetch as cache line (Core 2 Duo 64 byte per cache line)
- With contiguous data, a single cache fetch supports 8 reads of doubles.
- Transposing the matrix $C$ should reduce L1 cache misses!
float testMM(const int x, const int y, const int z)
{
    double *A; double *B; double *C; double *Cx;
    long started, ended; float timeTaken; int i, j, k;
    A = (double *)malloc(sizeof(double)*x*y);
    B = (double *)malloc(sizeof(double)*x*z);
    C = (double *)malloc(sizeof(double)*y*z);
    Cx = (double *)malloc(sizeof(double)*y*z);
    srand(getSeed());
    for (i = 0; i < x*z; i++) B[i] = (double) rand();
    for (i = 0; i < y*z; i++) C[i] = (double) rand();
    for (i = 0; i < x*y; i++) A[i] = 0;
    started = example_get_time();
    for(j =0; j < y; j++)
    for(k=0; k < z; k++)
        IND(Cx,j,k,z) = IND(C,k,j,y);
    for (i = 0; i < x; i++)
    for (j = 0; j < y; j++)
        for (k = 0; k < z; k++)
            IND(A, i, j, y) += IND(B, i, k, z) *IND(Cx, j, k, z);
    ended = example_get_time();
    timeTaken = (ended - started)/1.f;
    return timeTaken;
}
Issues with data reuse


- Computing a $32 \times 32$-block of $A$, so computing again 1024 coefficients: 1024 accesses in $A$, $384 \times 32$ in $B$ and $32 \times 384$ in $C$. Total = 25,600.

- The iteration space is traversed so as to reduce memory accesses.
float testMM(const int x, const int y, const int z)
{
    double *A; double *B; double *C;
    long started, ended; float timeTaken; int i, j, k, i0, j0, k0;
    A = (double *)malloc(sizeof(double)*x*y);
    B = (double *)malloc(sizeof(double)*x*z);
    C = (double *)malloc(sizeof(double)*y*z);
    srand(getSeed());
    for (i = 0; i < x*z; i++) B[i] = (double) rand() ;
    for (i = 0; i < y*z; i++) C[i] = (double) rand() ;
    for (i = 0; i < x*y; i++) A[i] = 0 ;
    started = example_get_time();
    for (i = 0; i < x; i += BLOCK_X)
        for (j = 0; j < y; j += BLOCK_Y)
            for (k = 0; k < z; k += BLOCK_Z)
                for (i0 = i; i0 < min(i + BLOCK_X, x); i0++)
                    for (j0 = j; j0 < min(j + BLOCK_Y, y); j0++)
                        for (k0 = k; k0 < min(k + BLOCK_Z, z); k0++)
                            IND(A,i0,j0,y) += IND(B,i0,k0,z) * IND(C,k0,j0,y);
    ended = example_get_time();
    timeTaken = (ended - started)/1.f;
    return timeTaken;
}
Transposing and blocking for optimizing data locality

```c
float testMM(const int x, const int y, const int z)
{
    double *A; double *B; double *C;
    long started, ended; float timeTaken; int i, j, k, i0, j0, k0;
    A = (double *)malloc(sizeof(double)*x*y);
    B = (double *)malloc(sizeof(double)*x*z);
    C = (double *)malloc(sizeof(double)*y*z);
    srand(getSeed());
    for (i = 0; i < x*z; i++) B[i] = (double) rand() ;
    for (i = 0; i < y*z; i++) C[i] = (double) rand() ;
    for (i = 0; i < x*y; i++) A[i] = 0 ;
    started = example_get_time();
    for (i = 0; i < x; i += BLOCK_X)
        for (j = 0; j < y; j += BLOCK_Y)
            for (k = 0; k < z; k += BLOCK_Z)
                for (i0 = i; i0 < min(i + BLOCK_X, x); i0++)
                    for (j0 = j; j0 < min(j + BLOCK_Y, y); j0++)
                        for (k0 = k; k0 < min(k + BLOCK_Z, z); k0++)
                            IND(A,i0,j0,y) += IND(B,i0,k0,z) * IND(C,j0,k0,z);
    ended = example_get_time();
    timeTaken = (ended - started)/1.f;
    return timeTaken;
}
```
### Experimental results

Computing the product of two $n \times n$ matrices on my laptop (Core2 Duo CPU P8600 @ 2.40GHz, L1 cache of 3072 KB, 4 GBytes of RAM)

<table>
<thead>
<tr>
<th>$n$</th>
<th>naive</th>
<th>transposed</th>
<th>speedup</th>
<th>$64 \times 64$-tiled</th>
<th>speedup</th>
<th>t. &amp; t.</th>
<th>speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td>128</td>
<td>7</td>
<td>3</td>
<td>speedup</td>
<td>7</td>
<td>speedup</td>
<td>2</td>
<td>speedup</td>
</tr>
<tr>
<td>256</td>
<td>26</td>
<td>43</td>
<td></td>
<td>155</td>
<td></td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>512</td>
<td>1805</td>
<td>265</td>
<td>6.81</td>
<td>1928</td>
<td>0.936</td>
<td>187</td>
<td>9.65</td>
</tr>
<tr>
<td>1024</td>
<td>24723</td>
<td>3730</td>
<td>6.62</td>
<td>14020</td>
<td>1.76</td>
<td>1490</td>
<td>16.59</td>
</tr>
<tr>
<td>2048</td>
<td>271446</td>
<td>29767</td>
<td>9.11</td>
<td>112298</td>
<td>2.41</td>
<td>11960</td>
<td>22.69</td>
</tr>
<tr>
<td>4096</td>
<td>2344594</td>
<td>238453</td>
<td>9.83</td>
<td>1009445</td>
<td>2.32</td>
<td>101264</td>
<td>23.15</td>
</tr>
</tbody>
</table>

Timings are in milliseconds.

The cache-oblivious multiplication (more on this later) runs within 12978 and 106758 for $n = 2048$ and $n = 4096$ respectively.
Other performance counters

Hardware count events

- **CPI**  *Clock cycles Per Instruction*: the number of clock cycles that happen when an instruction is being executed. With pipelining we can improve the CPI by exploiting instruction level parallelism.

- **L1 and L2 Cache Miss Rate**.

- **Instructions Retired**: In the event of a misprediction, instructions that were scheduled to execute along the mispredicted path must be canceled.

<table>
<thead>
<tr>
<th></th>
<th>CPI</th>
<th>L1 Miss Rate</th>
<th>L2 Miss Rate</th>
<th>Percent SSE Instructions</th>
<th>Instructions Retired</th>
</tr>
</thead>
<tbody>
<tr>
<td>In C</td>
<td>4.78</td>
<td>0.24</td>
<td>0.02</td>
<td>43%</td>
<td>13,137,280,000</td>
</tr>
<tr>
<td>Transposed</td>
<td>1.13</td>
<td>0.15</td>
<td>0.02</td>
<td>50%</td>
<td>13,001,486,336</td>
</tr>
<tr>
<td>Tiled</td>
<td>0.49</td>
<td>0.02</td>
<td>0</td>
<td>39%</td>
<td>18,044,811,264</td>
</tr>
</tbody>
</table>
Analyzing cache misses in the naive and transposed multiplication

- Let $A$, $B$ and $C$ have format $(m, n)$, $(m, p)$ and $(p, n)$ respectively.
- $A$ is scanned once, so $mn/L$ cache misses if $L$ is the number of coefficients per cache line.
- $B$ is scanned $n$ times, so $mnp/L$ cache misses if the cache cannot hold a row.
- $C$ is accessed “nearly randomly” (for $m$ large enough) leading to $mnp$ cache misses.
- Since $2mnp$ arithmetic operations are performed, this means roughly one cache miss per flop!
- If $C$ is transposed, then the ratio improves to 1 for $L$. 

$$A = \begin{bmatrix} 
\vdots \\
\vdots \\
\vdots 
\end{bmatrix}$$

$$B = \begin{bmatrix} 
\vdots \\
\vdots \\
\vdots 
\end{bmatrix}$$

$$C = \begin{bmatrix} 
\vdots \\
\vdots \\
\vdots 
\end{bmatrix}$$
Let $A$, $B$ and $C$ have format $(m, n)$, $(m, p)$ and $(p, n)$ respectively. Assume all tiles are square of order $B$ and three fit in cache. If $C$ is transposed, then loading three blocks in cache cost $3B^2/L$. This process happens $n^3/B^3$ times, leading to $3n^3/(BL)$ cache misses. Three blocks fit in cache for $3B^2 < Z$, if $Z$ is the cache size. So $O(n^3/(\sqrt{ZL}))$ cache misses, if $B$ is well chosen, which is optimal.
Summary and notes
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A multi-core processor is an integrated circuit to which two or more individual processors (called cores in this sense) have been attached.
Cores on a multi-core device can be coupled tightly or loosely:
- may share or may not share a cache,
- implement inter-core communications methods or message passing.

Cores on a multi-core implement the same architecture features as single-core systems such as instruction pipeline parallelism (ILP), vector-processing, hyper-threading, etc.
Cache Coherence (1/6)

Figure: Processor $P_1$ reads $x=3$ first from the backing store (higher-level memory)
Next, Processor $P_2$ loads $x=3$ from the same memory.
Cache Coherence (3/6)

Figure: Processor $P_4$ loads $x=3$ from the same memory
Cache Coherence (4/6)

Figure: Processor $P_2$ issues a write $x=5$
Figure: Processor $P_2$ writes $x=5$ in his local cache
Figure: Processor $P_1$ issues a read $x$, which is now invalid in its cache
In this cache coherence protocol each block contained inside a cache can have one of three possible states:

- **M**: the cache line has been *modified* and the corresponding data is inconsistent with the backing store; the cache has the responsibility to write the block to the backing store when it is evicted.

- **S**: this block is unmodified and is *shared*, that is, exists in at least one cache. The cache can evict the data without writing it to the backing store.

- **I**: this block is *invalid*, and must be fetched from memory or another cache if the block is to be stored in this cache.

These coherency states are maintained through communication between the caches and the backing store.

The caches have different responsibilities when blocks are read or written, or when they learn of other caches issuing reads or writes for a block.
True Sharing and False Sharing

- **True sharing:**
  - True sharing cache misses occur whenever two processors access the same data word
  - True sharing requires the processors involved to explicitly synchronize with each other to ensure program correctness.
  - A computation is said to have **temporal locality** if it re-uses much of the data it has been accessing.
  - Programs with high temporal locality tend to have less true sharing.

- **False sharing:**
  - False sharing results when different processors use different data that happen to be co-located on the same cache line
  - A computation is said to have **spatial locality** if it uses multiple words in a cache line before the line is displaced from the cache
  - Enhancing spatial locality often minimizes false sharing

See *Data and Computation Transformations for Multiprocessors* by J.M. Anderson, S.P. Amarasinghe and M.S. Lam
http://suif.stanford.edu/papers/anderson95/paper.html
Multi-core processor (cntd)

**Advantages:**
- Cache coherency circuitry operate at higher rate than off-chip.
- Reduced power consumption for a dual core vs two coupled single-core processors (better quality communication signals, cache can be shared)

**Challenges:**
- Adjustments to existing software (including OS) are required to maximize performance
- Production yields down (an Intel quad-core is in fact a double dual-core)
- Two processing cores sharing the same bus and memory bandwidth may limit performances
- High levels of false or true sharing and synchronization can easily overwhelm the advantage of parallelism
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From Cilk to Cilk++ and Cilk Plus

- Cilk has been developed since 1994 at the MIT Laboratory for Computer Science by Prof. Charles E. Leiserson and his group, in particular by Matteo Frigo.
- Besides being used for research and teaching, Cilk was the system used to code the three world-class chess programs: Tech, Socrates, and Cilkchess.
- Over the years, the implementations of Cilk have run on computers ranging from networks of Linux laptops to an 1824-nodes Intel Paragon.
- From 2007 to 2009 Cilk has lead to Cilk++, developed by Cilk Arts, an MIT spin-off, which was acquired by Intel in July 2009 and became Cilk Plus, see http://www.cilk.com/
- Cilk++ can be freely downloaded at http://software.intel.com/en-us/articles/download-intel-cilk-sdk/
- Cilk is still developed at MIT http://supertech.csail.mit.edu/cilk/
Cilk++ (and Cilk Plus)

- Cilk++ (resp. Cilk) is a **small set of linguistic extensions to C++** (resp. C) supporting **fork-join parallelism**

- Both Cilk and Cilk++ feature a **provably efficient work-stealing scheduler**.

- Cilk++ provides a **hyperobject library** for parallelizing code with global variables and performing reduction for data aggregation.

- Cilk++ includes the Cilkscreen race detector and the Cilkview performance analyzer.
Nested Parallelism in Cilk ++

```c
int fib(int n)
{
    if (n < 2) return n;
    int x, y;
    x = cilk_spawn fib(n-1);
    y = fib(n-2);
    cilk_sync;
    return x+y;
}
```

- The named child function `cilk_spawn fib(n-1)` may execute in parallel with its parent.
- Cilk++ keywords `cilk_spawn` and `cilk_sync` grant permissions for parallel execution. They do not command parallel execution.
The iterations of a `cilk_for` loop may execute in parallel.
Serial Semantics (1/2)

- Cilk (resp. Cilk++) is a multithreaded language for parallel programming that generalizes the semantics of C (resp. C++) by introducing linguistic constructs for parallel control.

- Cilk (resp. Cilk++) is a **faithful extension** of C (resp. C++):
  - The C (resp. C++) elision of a Cilk (resp. Cilk++) is a correct implementation of the semantics of the program.
  - Moreover, on one processor, a parallel Cilk (resp. Cilk++) program scales down to run nearly as fast as its C (resp. C++) elision.

- To obtain the serialization of a Cilk++ program
  ```
  #define cilk_for for
  #define cilk_spawn
  #define cilk_sync
  ```
Serial Semantics (2/2)

```c
int fib (int n) {
    if (n<2) return (n);
    else {
        int x,y;
        x = cilk_spawn fib(n-1);
        y = fib(n-2);
        cilk_sync;
        return (x+y);
    }
}
```

Cilk++ source

Serialization
A **scheduler**’s job is to map a computation to particular processors. Such a mapping is called a **schedule**.

- If decisions are made at runtime, the scheduler is **online**, otherwise, it is **offline**.
- Cilk++’s scheduler maps strands onto processors dynamically at runtime.
The Cilk++ Platform

```c
int fib (int n) {
    if (n<2) return (n);
    else {
        int x,y;
        x = cilk_spawn fib(n-1);
        y = fib(n-2);
        cilk_sync;
        return (x+y);
    }
}
```

Serializtion

Conventional Regression Tests

Reliable Single-Threaded Code

Cilk++ Compiler

Conventional Compiler

Cilk++ source

Hyperobject Library

Hyperobject Library

Cilkview Scalability Analyzer

Cilkview Scalability Analyzer

Cilkscreen Race Detector

Cilkscreen Race Detector

Runtime System

Exceptional Performance

Parallel Regression Tests

Reliable Multi-Threaded Code
## Benchmarks for the parallel version of the cache-oblivious mm

Multiplying a $4000 \times 8000$ matrix by a $8000 \times 4000$ matrix

- on 32 cores = 8 sockets x 4 cores (Quad Core AMD Opteron 8354) per socket.
- The 32 cores share a L3 32-way set-associative cache of 2 Mbytes.

<table>
<thead>
<tr>
<th>#core</th>
<th>Elision (s)</th>
<th>Parallel (s)</th>
<th>speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>420.906</td>
<td>51.365</td>
<td>8.19</td>
</tr>
<tr>
<td>16</td>
<td>432.419</td>
<td>25.845</td>
<td>16.73</td>
</tr>
<tr>
<td>24</td>
<td>413.681</td>
<td>17.361</td>
<td>23.83</td>
</tr>
<tr>
<td>32</td>
<td>389.300</td>
<td>13.051</td>
<td>29.83</td>
</tr>
</tbody>
</table>
So does the (tuned) cache-oblivious matrix multiplication
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The fork-join parallelism model

Example:

```
int fib (int n) {
    if (n<2) return (n);
    else {
        int x,y;
        x = cilk_spawn fib(n-1);
        y = fib(n-2);
        cilk_sync;
        return (x+y);
    }
}
```

"Processor oblivious"

"The computation dag unfolds dynamically."

We shall also call this model **multithreaded parallelism**.
We define several performance measures. We assume an ideal situation: no cache issues, no interprocessor costs:

- \( T_p \) is the minimum running time on \( p \) processors.
- \( T_1 \) is called the **work**, that is, the sum of the number of instructions at each node.
- \( T_\infty \) is the minimum running time with infinitely many processors, called the **span**.
Assuming all strands run in unit time, the longest path in the DAG is equal to $T_\infty$. For this reason, $T_\infty$ is also referred to as the **critical path length**.
We have: $T_p \geq T_1/p$.
Indeed, in the best case, $p$ processors can do $p$ works per unit of time.
We have: $T_p \geq T_\infty$.

Indeed, $T_p < T_\infty$ contradicts the definitions of $T_p$ and $T_\infty$. 
**Speedup on $p$ processors**

- $T_1/T_p$ is called the **speedup on $p$ processors**

- A parallel program execution can have:
  - **linear speedup**: $T_1/T_p = \Theta(p)$
  - **superlinear speedup**: $T_1/T_p = \omega(p)$ (not possible in this model, though it is possible in others)
  - **sublinear speedup**: $T_1/T_p = o(p)$
Series composition

- Work?
- Span?
Series composition

- **Work:** $T_1(A \cup B) = T_1(A) + T_1(B)$
- **Span:** $T_\infty(A \cup B) = T_\infty(A) + T_\infty(B)$
Parallel composition

- Work?
- Span?
Parallel composition

- **Work**: $T_1(A \cup B) = T_1(A) + T_1(B)$
- **Span**: $T_\infty(A \cup B) = \max(T_\infty(A), T_\infty(B))$
Some results in the fork-join parallelism model

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Work</th>
<th>Span</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merge sort</td>
<td>Θ(n lg n)</td>
<td>Θ(lg^3 n)</td>
</tr>
<tr>
<td>Matrix multiplication</td>
<td>Θ(n^3)</td>
<td>Θ(lg n)</td>
</tr>
<tr>
<td>Strassen</td>
<td>Θ(n^{lg 7})</td>
<td>Θ(lg^2 n)</td>
</tr>
<tr>
<td>LU–decomposition</td>
<td>Θ(n^3)</td>
<td>Θ(n lg n)</td>
</tr>
<tr>
<td>Tableau construction</td>
<td>Θ(n^2)</td>
<td>Ω(n^{lg 3})</td>
</tr>
<tr>
<td>FFT</td>
<td>Θ(n lg n)</td>
<td>Θ(lg^2 n)</td>
</tr>
<tr>
<td>Breadth–first search</td>
<td>Θ(E)</td>
<td>Θ(d lg V)</td>
</tr>
</tbody>
</table>

We shall prove those results in the next lectures.
### For loop parallelism in Cilk++

The iterations of a `cilk_for` loop execute in parallel.

```cilt
```cilk_for (int i=1; i<n; ++i) {
    for (int j=0; j<i; ++j) {
        double temp = A[i][j];
        A[i][j] = A[j][i];
        A[j][i] = temp;
    }
}
```
Implementation of for loops in Cilk++

Up to details (next week!) the previous loop is compiled as follows, using a divide-and-conquer implementation:

```c
void recur(int lo, int hi) {
    if (hi > lo) { // coarsen
        int mid = lo + (hi - lo)/2;
        cilk_spawn recur(lo, mid);
        recur(mid+1, hi);
        cilk_sync;
    } else
        for (int j=lo; j<hi; ++j) {
            double temp = A[hi][j];
            A[hi][j] = A[j][hi];
            A[j][hi] = temp;
        }
}
```
Analysis of parallel for loops

Here we do not assume that each strand runs in unit time.

- **Span of loop control**: $\Theta(\log(n))$
- **Max span of an iteration**: $\Theta(n)$
- **Span**: $\Theta(n)$
- **Work**: $\Theta(n^2)$
- **Parallelism**: $\Theta(n)$
For loops in the fork-join parallelism model: another example

cilk_for (int i = 1; i <= 8; i ++){
    f(i);
}

A cilk_for loop executes recursively as 2 for loops of $n/2$ iterations, adding a span of $\Theta(\log(n))$.

Figure: DAG for a cilk_for with 8 iterations.
The work-stealing scheduler (1/11)
The work-stealing scheduler (2/11)
The work-stealing scheduler (3/11)
The work-stealing scheduler (4/11)
The work-stealing scheduler (5/11)
The work-stealing scheduler (6/11)
The work-stealing scheduler (7/11)
The work-stealing scheduler (8/11)
The work-stealing scheduler (9/11)
The work-stealing scheduler (10/11)
The work-stealing scheduler (11/11)
Performances of the work-stealing scheduler

Assume that

- each strand executes in unit time,
- for almost all “parallel steps” there are at least \( p \) strands to run,
- each processor is either working or stealing.

Then, the randomized work-stealing scheduler is expected to run in

\[
T_P = T_1/p + O(T_\infty)
\]
Overheads and burden

- Many factors (simplification assumptions of the fork-join parallelism model, architecture limitation, costs of executing the parallel constructs, overheads of scheduling) will make $T_p$ larger in practice than $T_1/p + T_\infty$.

- One may want to estimate the impact of those factors:
  1. by improving the estimate of the randomized work-stealing complexity result
  2. by comparing a Cilk++ program with its C++ elision
  3. by estimating the costs of spawning and synchronizing

- Cilk++ estimates $T_p$ as $T_p = T_1/p + 1.7 \text{ burden}\_\text{span}$, where burden\_span is 15000 instructions times the number of continuation edges along the critical path.
Cilkview computes work and span to derive upper bounds on parallel performance.

Cilkview also estimates scheduling overhead to compute a burdened span for lower bounds.
The Fibonacci Cilk++ example

Code fragment

```c
long fib(int n)
{
  if (n < 2) return n;
  long x, y;
  x = cilk_spawn fib(n-1);
  y = fib(n-2);
  cilk_sync;
  return x + y;
}
```
Fibonacci program timing

The environment for benchmarking:
- model name: Intel(R) Core(TM)2 Quad CPU Q6600 @ 2.40GHz
- L2 cache size: 4096 KB
- memory size: 3 GB

<table>
<thead>
<tr>
<th>n</th>
<th>#cores = 1</th>
<th>#cores = 2</th>
<th>#cores = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.086</td>
<td>0.046</td>
<td>1.870</td>
</tr>
<tr>
<td>35</td>
<td>0.776</td>
<td>0.436</td>
<td>1.780</td>
</tr>
<tr>
<td>40</td>
<td>8.931</td>
<td>4.842</td>
<td>1.844</td>
</tr>
<tr>
<td>45</td>
<td>105.263</td>
<td>54.017</td>
<td>1.949</td>
</tr>
<tr>
<td>50</td>
<td>1165.000</td>
<td>665.115</td>
<td>1.752</td>
</tr>
</tbody>
</table>
Quicksort

code in cilk/examples/qsort

```c
void sample_qsort(int * begin, int * end) {
    if (begin != end) {
        --end;
        int * middle = std::partition(begin, end,
                                       std::bind2nd(std::less<int>(), *end));
        using std::swap;
        swap(*end, *middle);
        cilk_spawn sample_qsort(begin, middle);
        sample_qsort(++middle, ++end);
        cilk_sync;
    }
}
```
# Quicksort timing

Timing for sorting an array of integers:

<table>
<thead>
<tr>
<th># of int</th>
<th>#cores = 1 timing(s)</th>
<th>#cores = 2 timing(s)</th>
<th>speedup</th>
<th>#cores = 4 timing(s)</th>
<th>speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10 \times 10^6$</td>
<td>1.958</td>
<td>1.016</td>
<td>1.927</td>
<td>0.541</td>
<td>3.619</td>
</tr>
<tr>
<td>$50 \times 10^6$</td>
<td>10.518</td>
<td>5.469</td>
<td>1.923</td>
<td>2.847</td>
<td>3.694</td>
</tr>
<tr>
<td>$100 \times 10^6$</td>
<td>21.481</td>
<td>11.096</td>
<td>1.936</td>
<td>5.954</td>
<td>3.608</td>
</tr>
<tr>
<td>$500 \times 10^6$</td>
<td>114.300</td>
<td>57.996</td>
<td>1.971</td>
<td>31.086</td>
<td>3.677</td>
</tr>
</tbody>
</table>
Matrix multiplication

Code in cilk/examples/matrix

Timing of multiplying a $687 \times 837$ matrix by a $837 \times 1107$ matrix

<table>
<thead>
<tr>
<th>threshold</th>
<th>iterative</th>
<th>recursive</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>st(s)</td>
<td>pt(s)</td>
</tr>
<tr>
<td>10</td>
<td>1.273</td>
<td>1.165</td>
</tr>
<tr>
<td>16</td>
<td>1.270</td>
<td>1.787</td>
</tr>
<tr>
<td>32</td>
<td>1.280</td>
<td>1.757</td>
</tr>
<tr>
<td>48</td>
<td>1.258</td>
<td>1.760</td>
</tr>
<tr>
<td>64</td>
<td>1.258</td>
<td>1.798</td>
</tr>
<tr>
<td>80</td>
<td>1.252</td>
<td>1.773</td>
</tr>
</tbody>
</table>

st = sequential time; pt = parallel time with 4 cores; su = speedup
Using `cilk_for` to perform operations over an array in parallel:

```c
static const int COUNT = 4;
static const int ITERATION = 1000000;
long arr[COUNT];
long do_work(long k){
    long x = 15;
    static const int nn = 87;
    for (long i = 1; i < nn; ++i)
        x = x / i + k % i;
    return x;
}
int cilk_main(){
    for (int j = 0; j < ITERATION; j++)
        cilk_for (int i = 0; i < COUNT; i++)
            arr[i] += do_work( j * i + i + j);
}
1) Parallelism Profile

Work: 6,480,811,250 ins
Span: 2,116,811,250 ins
Burdened span: 31,920,811,250 ins
Parallelism: 3.06
Burdened parallelism: 0.20
Number of spawns/syncs: 3,000,000
Average instructions / strand: 720
Strands along span: 4,000,001
Average instructions / strand on span: 529

2) Speedup Estimate

2 processors: 0.21 - 2.00
4 processors: 0.15 - 3.06
8 processors: 0.13 - 3.06
16 processors: 0.13 - 3.06
32 processors: 0.12 - 3.06
A simple fix

Inverting the two for loops

```c
int cilk_main()
{
    cilk_for (int i = 0; i < COUNT; i++)
        for (int j = 0; j < ITERATION; j++)
            arr[i] += do_work( j * i + i + j);
}
```
1) Parallelism Profile

- Work: 5,295,801,529 ins
- Span: 1,326,801,107 ins
- Burdened span: 1,326,830,911 ins
- Parallelism: 3.99
- Burdened parallelism: 3.99
- Number of spawns/syncs: 3
- Average instructions / strand: 529,580,152
- Strands along span: 5
- Average instructions / strand on span: 265,360,221

2) Speedup Estimate

- 2 processors: 1.40 - 2.00
- 4 processors: 1.76 - 3.99
- 8 processors: 2.01 - 3.99
- 16 processors: 2.17 - 3.99
- 32 processors: 2.25 - 3.99
## Timing

<table>
<thead>
<tr>
<th>version</th>
<th>#cores = 1</th>
<th>#cores = 2</th>
<th>#cores = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>timing(s)</td>
<td>timing(s)</td>
<td>timing(s)</td>
</tr>
<tr>
<td>original</td>
<td>7.719</td>
<td>9.611</td>
<td>10.758</td>
</tr>
<tr>
<td>improved</td>
<td>7.471</td>
<td>3.724</td>
<td>1.888</td>
</tr>
<tr>
<td></td>
<td></td>
<td>speedup</td>
<td>speedup</td>
</tr>
<tr>
<td></td>
<td>0.803</td>
<td>2.006</td>
<td>3.957</td>
</tr>
</tbody>
</table>
Plan

1. Data locality and cache misses
   - Hierarchical memories
   - Impact on our programs

2. Multicore programming
   - Multicore architectures
   - Cilk / Cilk++ / Cilk Plus
   - The fork-join multithreaded programming model
   - Practical issues and optimization tricks
Example 1: a small loop with grain size = 1

Code:

```c
const int N = 100 * 1000 * 1000;

void cilk_for_grainsize_1()
{
    #pragma cilk_grainsize = 1
    cilk_for (int i = 0; i < N; ++i)
    {
        fib(2);
    }
}
```

Expectations:

- Parallelism should be large, perhaps $\Theta(N)$ or $\Theta(N/\log N)$.
- We should see great speedup.
Speedup is indeed great...
...but performance is lousy
Recall how `cilk_for` is implemented

Source:

```c
    cilk_for (int i = A; i < B; ++i)
       BODY(i)
```

Implementation:

```c
void recur(int lo, int hi) {
    if ((hi - lo) > GRAINSIZE) {
        int mid = lo + (hi - lo) / 2;
        cilk_spawn recur(lo, mid);
        cilk_spawn recur(mid, hi);
    } else
       for (int i = lo; i < hi; ++i)
          BODY(i);
}
```

```c
recur(A, B);
```
Default grain size

Cilk++ chooses a grain size if you don’t specify one.

```c
void cilk_for_default_grainsize()
{
    cilk_for (int i = 0; i < N; ++i)
        fib(2);
}
```

Cilk++’s heuristic for the grain size:

\[
\text{grain size} = \min \left\{ \frac{N}{8P}, 512 \right\}.
\]

- Generates about \(8P\) parallel leaves.
- Works well if the loop iterations are not too unbalanced.
Speedup with default grain size
Large grain size

A large grain size should be even faster, right?

```c
void cilk_for_large_grainsize()
{
    #pragma cilk_grainsize = N
    cilk_for (int i = 0; i < N; ++i)
        fib(2);
}
```

Actually, no (except for noise):

<table>
<thead>
<tr>
<th>Grain size</th>
<th>Runtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.55 s</td>
</tr>
<tr>
<td>default (= 512)</td>
<td>2.44 s</td>
</tr>
<tr>
<td>(N (= 10^8))</td>
<td>2.42 s</td>
</tr>
</tbody>
</table>
Speedup with grain size $= N$
Trade-off between grain size and parallelism

Use Cilkview to understand the trade-off:

<table>
<thead>
<tr>
<th>Grain size</th>
<th>Parallelism</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6,951,154</td>
</tr>
<tr>
<td>default (= 512)</td>
<td>248,784</td>
</tr>
<tr>
<td>(N (= 10^8))</td>
<td>1</td>
</tr>
</tbody>
</table>

In Cilkview, \(P = 1\):

\[
\text{default grain size} = \min \left\{ \frac{N}{8P}, 512 \right\} = \min \left\{ \frac{N}{8}, 512 \right\}.
\]
Lessons learned

- Measure overhead before measuring speedup.
  - Compare 1-processor Cilk++ versus serial code.
- Small grain size $\Rightarrow$ higher work overhead.
- Large grain size $\Rightarrow$ less parallelism.
- The default grain size is designed for small loops that are reasonably balanced.
  - You may want to use a smaller grain size for unbalanced loops or loops with large bodies.
- Use Cilkview to measure the parallelism of your program.
Example 2: A for loop that spawns

Code:

```c
const int N = 10 * 1000 * 1000;

/* empty test function */
void f() { }

void for_spawn()
{
  for (int i = 0; i < N; ++i)
    cilk_spawn f();
}
```

Expectations:
- I am spawning $N$ parallel things.
- Parallelism should be $\Theta(N)$, right?
“Speedup” of `for_spawn()`
Insufficient parallelism

PPA analysis:
- PPA says that both work and span are $\Theta(N)$.
- Parallelism is $\approx 1.62$, independent of $N$.
- Too little parallelism: no speedup.

Why is the span $\Theta(N)$?

```c
for (int i = 0; i < N; ++i)
    cilk_spawn f();
```
Alternative: a `cilk_for` loop.

Code:

```c
/* empty test function */
void f() { }

void test_cilk_for()
{
    cilk_for (int i = 0; i < N; ++i)
    {
        f();
    }
}
```

PPA analysis:

The parallelism is about 2000 (with default grain size).

- The parallelism is high.
- As we saw earlier, this kind of loop yields good performance and speedup.
Lessons learned

- `cilk_for()` is different from `for(...) cilk_spawn`.
- The span of `for(...) cilk_spawn` is $\Omega(N)$.
- For simple flat loops, `cilk_for()` is generally preferable because it has higher parallelism.
- (However, `for(...) cilk_spawn` might be better for recursively nested loops.)
- Use Cilkview to measure the parallelism of your program.
Example 3: Vector addition

Code:

```c
const int N = 50 * 1000 * 1000;

double A[N], B[N], C[N];

void vector_add()
{
    cilk_for (int i = 0; i < N; ++i)
        A[i] = B[i] + C[i];
}
```

Expectations:

- Cilkview says that the parallelism is 68,377.
- This will work great!
**Speedup of vector_add()**

![Graph showing speedup vs number of processors. The speedup increases linearly with the number of processors.](image-url)
Bandwidth of the memory system

A typical machine: AMD Phenom 920 (Feb. 2009).

<table>
<thead>
<tr>
<th>Cache level</th>
<th>daxpy bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>19.6 GB/s per core</td>
</tr>
<tr>
<td>L2</td>
<td>18.3 GB/s per core</td>
</tr>
<tr>
<td>L3</td>
<td>13.8 GB/s shared</td>
</tr>
<tr>
<td>DRAM</td>
<td>7.1 GB/s shared</td>
</tr>
</tbody>
</table>

**daxpy**: \( x[i] = a \times x[i] + y[i] \), double precision.

The memory bottleneck:

- A single core can generally saturate most of the memory hierarchy.
- Multiple cores that access memory will conflict and slow each other down.
How do you determine if memory is a bottleneck?

Hard problem:
- No general solution.
- Requires guesswork.

Two useful techniques:
- Use a profiler such as the Intel VTune.
  - Interpreting the output is nontrivial.
  - No sensitivity analysis.
- Perturb the environment to understand the effect of the CPU and memory speeds upon the program speed.
How to perturb the environment

- Overclock/underclock the processor, e.g. using the power controls.
  - If the program runs at the same speed on a slower processor, then the memory is (probably) a bottleneck.
- Overclock/underclock the DRAM from the BIOS.
  - If the program runs at the same speed on a slower DRAM, then the memory is not a bottleneck.
- Add spurious work to your program while keeping the memory accesses constant.
- Run $P$ independent copies of the serial program concurrently.
  - If they slow each other down then memory is probably a bottleneck.
Perturbing `vector_add()`

```c
const int N = 50 * 1000 * 1000;

double A[N], B[N], C[N];

void vector_add()
{
    cilk_for (int i = 0; i < N; ++i) {
        A[i] = B[i] + C[i];
        fib(5); // waste time
    }
}
```
Speedup of perturbed vector_add()
Interpreting the perturbed results

The memory is a bottleneck:

- A little extra work (\texttt{fib(5)}) keeps 8 cores busy. A little more extra work (\texttt{fib(10)}) keeps 16 cores busy.
- Thus, we have enough parallelism.
- The memory is \textit{probably} a bottleneck. (If the machine had a shared FPU, the FPU could also be a bottleneck.)

OK, but how do you fix it?

- \texttt{vector\_add} cannot be fixed in isolation.
- You must generally restructure your program to increase the reuse of cached data. Compare the iterative and recursive matrix multiplication from yesterday.
- (Or you can buy a newer CPU and faster memory.)
Lessons learned

- Memory is a common bottleneck.
- One way to diagnose bottlenecks is to perturb the program or the environment.
- Fixing memory bottlenecks usually requires algorithmic changes.
Example 4: Nested loops

Code:

```
const int N = 1000 * 1000;

void inner_parallel()
{
    for (int i = 0; i < N; ++i)
        cilk_for (int j = 0; j < 4; ++j)
            fib(10); /* do some work */
}
```

Expectations:
- The inner loop does 4 things in parallel. The parallelism should be about 4.
- Cilkview says that the parallelism is 3.6.
- We should see some speedup.
“Speedup” of `inner_parallell()`
Interchanging loops

Code:

```c
const int N = 1000 * 1000;

void outer_parallel()
{
    cilk_for (int j = 0; j < 4; ++j)
    {
        for (int i = 0; i < N; ++i)
            fib(10); /* do some work */
    }
}
```

Expectations:

- The outer loop does 4 things in parallel. The parallelism should be about 4.
- Cilkview says that the parallelism is 4.
- Same as the previous program, which didn’t work.
Speedup of `outer_parallel()`
Parallelism vs. burdened parallelism

Parallelism:
The best speedup you can hope for.

Burdened parallelism:
Parallelism after accounting for the unavoidable migration overheads.

Depends upon:
- How well we implement the Cilk++ scheduler.
- How you express the parallelism in your program.

Cilkview prints the burdened parallelism:
- 0.29 for inner_parallel(), 4.0 for outer_parallel().
- In a good program, parallelism and burdened parallelism are about equal.
What is the burdened parallelism?

Code:

```c
A();
cilk_spawn B();
C();
D();
cilk_sync;
E();
```

Burdened critical path:

The **burden** is $\Theta(10000)$ cycles (locks, malloc, cache warmup, reducers, etc.)
The burden in our examples

$\Theta(N)$ spawns/syncs on the critical path (large burden):

```c
void inner_parallel()
{
    for (int i = 0; i < N; ++i)
        cilk_for (int j = 0; j < 4; ++j)
            fib(10); /* do some work */
}
```

$\Theta(1)$ spawns/syncs on the critical path (small burden):

```c
void outer_parallel()
{
    cilk_for (int j = 0; j < 4; ++j)
        for (int i = 0; i < N; ++i)
            fib(10); /* do some work */
}
```
Lessons learned

- Insufficient parallelism yields *no speedup*; high burden yields *slowdown*.
- Many spawns but small parallelism: suspect large burden.
- Cilkview helps by printing the burdened span and parallelism.
- The burden can be interpreted as the number of spawns/syncs on the critical path.
- If the burdened parallelism and the parallelism are approximately equal, your program is ok.
Summary and notes

We have learned to identify and (when possible) address these problems:

- High overhead due to small grain size in `cilk_for` loops.
- Insufficient parallelism.
- Insufficient memory bandwidth.
- Insufficient burdened parallelism.