Implantation and new applications of the Cartan's equivalence method

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The Cartan's equivalence method is an algorithm which allows to decide if two systems of differential equations can be mapped on each other by a local diffeomorphism from a given (pseudo)group of transformations. This problem amounts to the local classification of G-structures and therefore to the computation of a complete set of invariants of these ones. Although Elie Cartan has treated, since 1905, a great number of examples, this method remained misunderstood for a long time. In the 50's, two of the Cartan's students (C. Erhesmann and S. S. Chern) gave a theoretical foundation for a good part of Cartan's computations by developping the theory of jet spaces and G-structures. More recently, R. Gardner, N. Kamran and P. Olver (see the book "equivalence, invariants and symmetry") endeavored to bring out the algorithmic aspect of Cartan's method. The Maple implantation we propose allows to treat examples that remained out of capability up to now. Thus will be presented :

- some classification results about third order ordinary differential equations under the group of contact transformations,
- some classification results about fourth order ordinary differential equations as well as some systems of ordinary differential equations,
- the study of systems of completely integrable second order partial differential equations systems with one dependent variable and n independent variables.

One of the essential optimisation is the use of derivations that do not necesserally commute pairwise. The equivalence problems are also tackled using differential algebra technics.