Parallel Disk-Based Computation and Computational Group Theory

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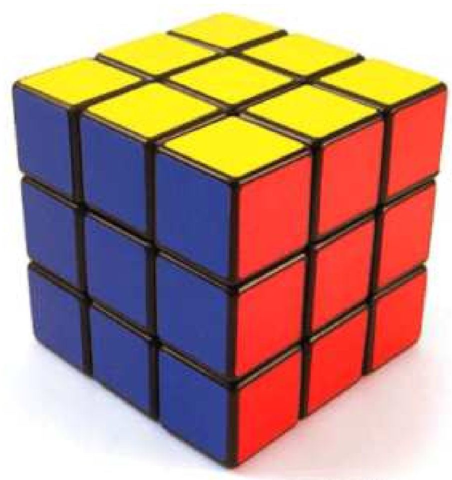


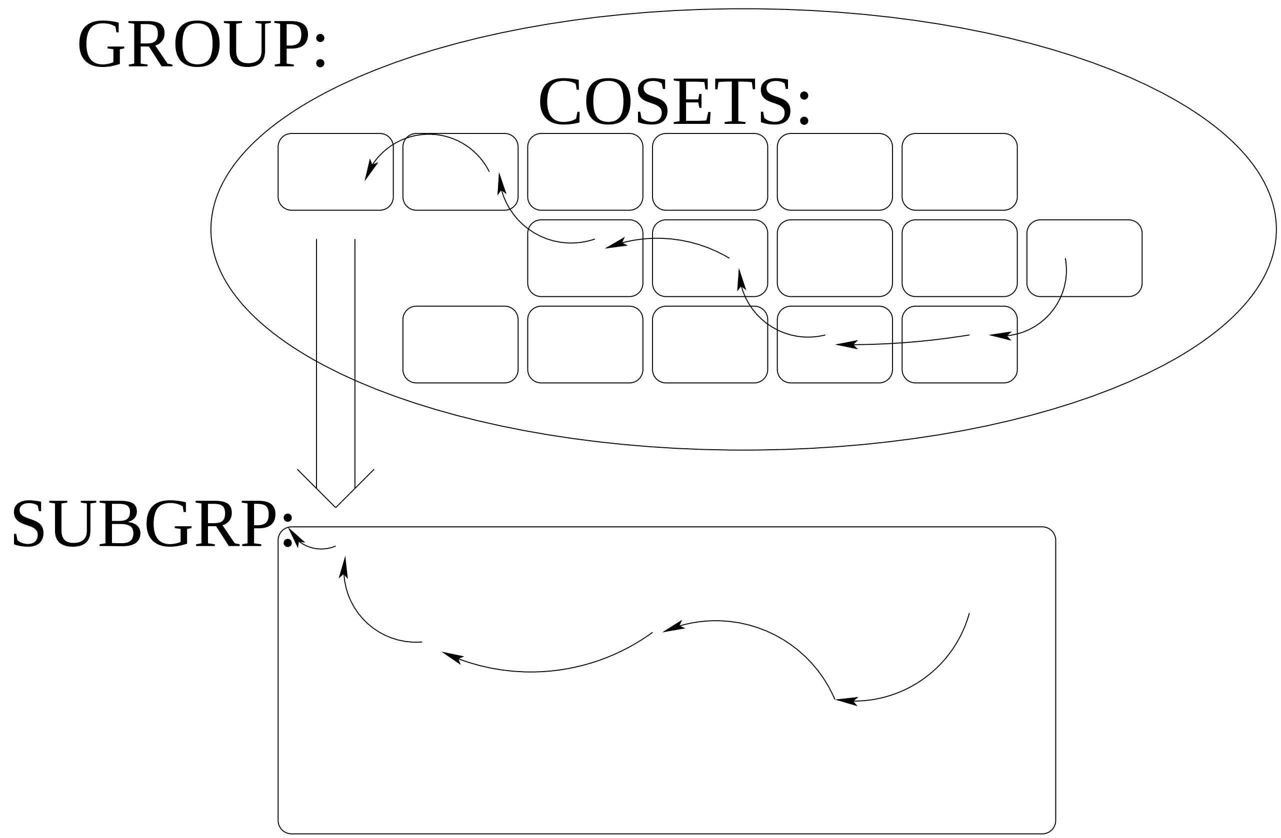
Applications from Computational Group Theory (2003–2007)

Group	Space Size	State Size	Total Storage
Fischer Fi_{23}	1.17×10^{10}	100 bytes	1 TB
"Baby Monster"	1.35×10^{10}	548 bytes	7 TB
Janko J_4	1.31×10^{11}	64 bytes	8 TB
Rubik (sym. cosets)	1.4×10^{12}	6 bytes	8 TB

(joint with Eric Robinson, Dan Kunkle)

Solution of Rubik's Cube: Cosets in Mathematical Group Theory

Rubik's Cube: 26 Moves Suffice to Solve Rubik's Cube (2007)
7 terabytes (temporary storage), 10^{12} states, 6 bytes per state
Purposely chose a method requiring very large storage.
Further details at: `http://www.ccs.neu.edu/home/gene/papers/rubik-slides.pdf`



History of Rubik's Cube

- Invented in late 1970s in Hungary.
- 1982: "God's Number" (number of moves needed) was known by authors of conjecture to be between 17 and 52.
- 1990: C., Finkelstein, and Sarawagi showed 11 moves suffice for Rubik's $2 \times 2 \times 2$ cube (corner cubies only); but apparently known by hobbyists in 1982
- 1995: Reid showed 29 moves suffice (lower bound of 20 already known)
- 2006: Radu showed 27 moves suffice
- 2007 Kunkle and C. showed 26 moves suffice (and computation is still proceeding)
- D. Kunkle and G. Cooperman, "Twenty-Size Moves Suffice for Rubik's Cube", *International Symposium on Symbolic and Algebraic Computation* (ISSAC-07), 2007, ACM Press, pp. 235–242.
- 2008 Rokicki, 22 moves suffice (50 core-years, contributed by John Welborn and Sony Pictures Imageworks) `http://cubezzz.homelinux.org/drupal/?q=node/view/121`

The End of the World (as we know it) is Coming

- The world is changing, as we near the end of Moore's Law.
  - Memory chips are no longer twice as dense every 18 months.
  - Large RAM is still available on server-class motherboards.
  - But the commodity market doesn't want to pay that premium.
  - So, those of us doing large search and scientific computations are being left out in the cold. We still need those ever larger memories – especially as the trend toward multi-core CPUs places ever more pressure on RAM.
  - Heterogeneous computing (cell processors, NVIDIA G8, etc.) only make it worse: The newfound power for regular computations (e.g. SIMD) are causing us to run out of RAM faster than ever before!!!

What To Do When You Run out of RAM?

1. Shared-memory many-CPU computers usually have more RAM (expensive, limited to 100 GB or 1 TB of RAM)
2. Use aggregate RAM of a computer cluster (needs parallel programming)
3. Use the disk on a single computer (much slower than RAM)
4. **Use the many parallel local disks (or SAN) of a cluster** (ALMOST FREE!!)

A Fourth Major Use for Disks:

1. File System
2. Database (relational or other)
3. Virtual Memory
4. **Parallel Disks as extension of RAM** (similar goal to virtual memory: minimally invasive modification of seqeuntial program, but good performance)

Our Solution

- Disk is the New RAM
- Bandwidth of Disk: ˜ 100 MB/s
- Bandwidth of 50 Disks: 50×100 MB/s = 5 GB/s
- Bandwidth of RAM: approximately 5 GB/s

- Conclusion:

1. **CLAIM:** A computer cluster of 50 quad-core nodes, each with 500 GB of mostly idle disk space, is a good approximation to a shared memory computer with 200 CPU cores and a *single* subsystem with 25 TB of shared memory.
   (The arguments also work for a SAN with multiple access nodes, but we consider local disks for simplicity.)
2. The disks of a cluster can serve as if they were RAM.
3. The traditional RAM can then serve as if it were cache.

What About Disk Latency?

- Unfortunately, putting 50 disks on it, doesn't speed up the latency.
- So, re-organize the data structures and low-level algorithms.
- Our group has five years of case histories applying this to computational group theory — but each case requires months of development and debugging.
- We're now developing both higher level abstractions for run-time libraries, and a language extension that will make future development much faster.

LONGER-TERM GOALS

- Why do it?

  1. State space search occurs across a huge number of scientific disciplines.
  2. Because the world is running out of RAM!
     - A commodity motherboard holds only 4 GB RAM.
     - We now have 4- and 8-core motherboards, but no one will be putting eight times as much RAM on a *commodity* motherboard.

Applications Benefiting from Disk-Based Parallel Computation

Symbolic Algebra:

Applications occur wherever there is intermediate swell!

	Application
1.	Gröbner bases
2.	Term Rewriting (Knuth-Bendix)
3.	Term Manipulations (differential operators, polynomial multiplications, ...)
4.	Large Search Applications (widespread)
5.	Numerous examples from computational group theory
6.	Others

Applications Benefiting from Disk-Based Parallel Computation

	Discipline	Example Application
1.	Verification	Symbolic Computation using BDDs
2.	Verification	Explicit State Verification
3.	Comp. Group Theory	Search and Enumeration in Mathematical Structures
4.	Coding Theory	Search for New Codes
5.	Security	Exhaustive Search for Passwords
6.	Semantic Web	RDF query language; OWL Web Ontology Language
7,	Artificial Intelligence	Planning
8.	Proteomics	Protein folding via a kinetic network model
9.	Operations Research	Branch and Bound
10.	Operations Research	Integer Programming (applic. of Branch-and-Bound)
11.	Economics	Dynamic Programming
12.	Numerical Analysis	ATLAS, PHiPAC, FFTW, and other adaptive software
13.	Engineering	Sensor Data
14.	A.I. Search	Rubik’s Cube

LONGER-TERM GOALS (cont.)

- The world is changing, as we near the end of Moore's Law.
  - Memory chips are no longer twice as dense every 18 months.
  - Large RAM is still available on server-class motherboards.
  - But the commodity market doesn't want to pay that premium.
  - So, those of us doing large search and scientific computations are being left out in the cold. We still need those ever larger memories – especially as the trend toward multi-core CPUs places ever more pressure on RAM.
- Our solution is to use disk as the new RAM! (See next slide.)

Central Claim

Suppose one had a single computer with 25 terabytes of RAM and 200 CPU cores. Does that satisfy your need for computers with more RAM?

CLAIM: A computer cluster of 50 quad-core nodes, each with a 500 GB local disk, is a good approximation of the above computer. (The arguments also work for a SAN with multiple access nodes, but we discuss local disks for simplicity.)

When is a cluster like a 25 TB shared memory computer?

- Assume 500 GB/node of free disk space
- Assume 50 nodes,
- The bandwidth of 50 disks is $50 \times 100 MB/s = 5 GB/s$.
- The bandwidth of a *single* RAM subsystem is about $5 GB/s$.

CLAIM: *You probably have the 25 TB of temporary disk space lying idle on your own* recent-model *computer cluster. You just didn't know it.*
(Or were you just not telling other people about the space, so you could use if for yourself?)

The economics of disks are such that one saves very little by buying less than 500 GB disk per node. It's common to buy the 500 GB disk, and reserve the extra space for expansion.

When is a cluster NOT like a 25 TB shared memory computer?

1. We require a *parallel* program. (We must access the local disks of many cluster nodes in parallel.)
2. The latency problem of disk.
3. Can the network keep up with the disk?

Solutions to the Latency of Disk Do Exist

1. For duplicates on frontier in state space search: *Delayed Duplicate Detection* implies waiting until many nodes of the next frontier (and duplicates from previous iterations) have been discovered. Then remove duplicates.

2. For hash tables, wait until there are millions of hash queries. Then sort on the hash index, and scan the disk to resolve queries.

3. For pointer-chasing, wait until millions of pointers are available for chasing. Then sort and scan the disk to dereference pointers.

Example Times for Different Phases of Computation

Baby Monster group: 4.2×10^{33} elements
Goal: Construct 13,571,955,000 “points” of permutation representation

Manager	Disk Time	CPU/RAM Time	Network Time
Read/Write	0.5 days	0 days	—
Computation	0 days	3 days	2 days
Check	0 days	0 days	—
Hash	$\ll$ 1 day	$\ll$ 1 day	—
Formatting/Sorting	$\ll$ 1 day	$\ll$ 1 day	—
Duplicate Elimination	$\ll$ 1 day	$<$ 1 day	—
Rebuilding	$\ll$ 1 day	6 days	—
Approximate Total	*2 days*	*10 days*	*2 days*

NOTE: Between CPU time and RAM bandwidth, the computation is primarily limited by RAM bandwidth. *(Disk time not the bottleneck.)*

Using faster CPUs has almost no benefit!
(Only faster RAM helps.)

“A Comparative Analysis of Parallel Disk-Based Methods for Enumerating Implicit Graphs”, Eric Robinson, Daniel Kunkle and Gene Cooperman, *Proc. of 2007 International Workshop on Parallel Symbolic and Algebraic Computation* (PASCO ’07), ACM Press, 2007, pp. 78–87

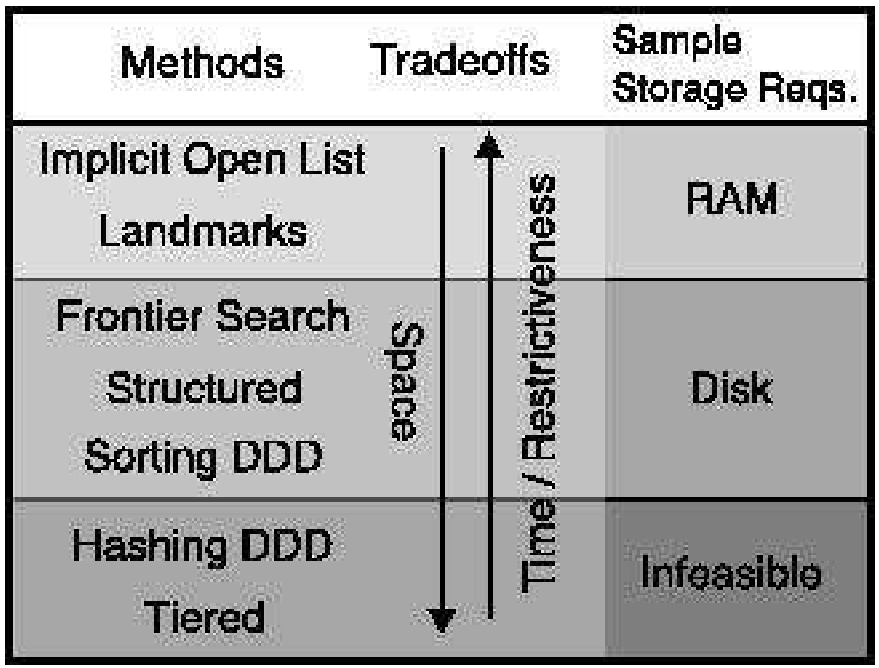
SPDE: System for Parallel Disk-Based Enumeration

SPDE Software provides infrastructure

1. Free and Open Source Software (currently beta; see Eric Robinson for copies)
2. Modules added for different search strategies

Space-Time Tradeoffs using Additional Disk

- Use even more disk space in order to speed up the algorithm.



Search Modules (not all implemented)

FROM: "A Comparative Analysis of Parallel Disk-Based Methods for Enumerating Implicit Graphs", Eric Robinson, Daniel Kunkle and Gene Cooperman, *Proc. of 2007 Int. Workshop on Parallel Symbolic and Algebraic Computation* (PASCO '07), ACM Press, 2007, pp. 78–87

1. Breadth-First Search (BFS) with perfect hash (traditional) — *requires perfect hash function*
2. Sorting-based Delayed Duplicate Detection (DDD) — *uses external sort to compare frontier against known states*
3. Hash-based Delayed Duplicate Detection (DDD) — *hash high bits to determines bin number; corresponding bins from frontier and known states loaded into RAM; detect duplicates in bins with hashing*
4. Level Modulo 3 — *assumes perfect hash function; store search depth modulo three; can easily distinguish previous level* (level - 1 mod 3) *and next level* (level + 1 mod 3)
5. Structured Duplicate Detection — *requires search graph with good locality; local search in RAM for duplicates eliminates most duplicates*

Search Modules (cont.)

6. Implicit Open List —
   allows multiple passes at a given level; needed when initial frontier (duplicates not yet detected) is too large for available storage

7. Landmarks — *store a fraction of states (e.g. states ending in three zeroes); these are the landmarks; Given any other state, a local search finds a landmark state. new state associated with nearest landmark*

8. Frontier Search — *requires that inverses of neighbor generators themselves be neighbor generators; need check for duplicates at current level, next level, or previous level; but by setting a generator used bit, one can avoid later searching the inverse generator (since it would only lead to the previous level)*

9. Tiered Duplicate Detection —
   like a 1-bit Bloom filters; use lossy hash function; if hashes to empty hash slot, it's new; if occupied slot, either a collision or a duplicate; save ambiguous state in second tier and use a different delayed duplicate detection method on this significantly smaller *set of states*

LONGER-TERM GOAL: Roomy Mini-Language

Well-understood building blocks already exist: external sorting, B-trees, Bloom filters, Delayed Duplicate Detection, Distributed Hash Trees (DHT)

GOAL:

1. Extend C/C++ with Roomy run-time library:
   Support for extensible arrays, lists (support append, concatenate, merge, extract. ...), hash arrays; Emphasize streaming access (using MapReduce, Reduce, Modify-in-Place, ...)
2. Minimally invasive: common data structures in user *sequential* code are replaced by Roomy data structures
3. Add Roomy Modules for Common Tasks:
   (a) Parallel Breadth-First and (approximate) Depth-First Search
   (b) Priority Queues
   (c) Table lookup/modify

First alpha release of Roomy expected by end of summer. (open source)

Example Roomy Code (under development)

```
// Function to be mapped over cur level to produce next level
void genNextLev(void* val) {
    /*
     *  User defined code to generate nbrs array inserted here.
     */
    for(int i=0; i<numNbrs; i++) {
        RoomyList_add(nextLevList, nbrs[i]);
    }
}

// Init lists for duplicates, current level, and next level
RoomyList* allLevList = RoomyList_make("allLev", eltSize);
RoomyList* curLevList = RoomyList_make("lev0", eltSize);
RoomyList* nextLevList = RoomyList_make("lev1", eltSize);

// Add start element
RoomyList_add(allLevList, startElt);
RoomyList_add(curLevList, startElt);

// Generate levels until no new states are found
while(RoomyList_size(curLevList)) {
    // generate next level from current
    RoomyList_map(curLevList, genNextLev);

    // detect duplicates
    RoomyList_removeDupes(nextLevList);
    RoomyList_removeAll(nextLevList, allLevList);
    RoomyList_addAll(allLevList, nextLevList);

    // rotate levels
    RoomyList_destroy(curLevList);
    curLevList = nextLevList;
    nextLevList = RoomyList_make(levName, eltSize);
}
```

QUESTIONS?