# High-performance computing and symbolic computation

Marc Moreno Maza

Ontario Research Center for Computer Algebra (ORCCA)
University of Western Ontario, Canada

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# Research themes and team members

- Symbolic computation: computing exact solutions of algebraic problems on computers with applications to sciences and engineering.
- High-performance computing: making best use of modern computer architectures, in particular hardware accelerators (multi-cores GPUs)

## Current students

PDF: Masoud Ataei,
PhD: Ali Asadi, Egor Chesakov, Davood Mohajerani, Robert Moir, Mehdi Samadieh, Steven Thornton,
MSc: Alex Brandt, Colin Costello, Delaram TalaAshrafi, Yiming Guan, Amha Tsegaye, Lin-Xiao Wang, Haoze Yuan.

## Alumni

Parisa Alvandi ( U. Waterloo , Canada) Moshin Ali ( ANU , Australia) Jinlong Cai ( Oracle , USA) Changbo Chen ( Chinese Acad. of Sc. ) Xiaohui Chen ( AMD , Canada) Svyatoslav Covanov ( U. Lorraine , France) Akpodigha Filatei ( Guaranty Turnkey Systems ltd , Nigeria) Oleg Golubitsky ( Google Canada ) Sardar A. Haque ( Qassim University, , Saudi Arabia) Zunaid Haque ( IBM Canada ) Rui-Juan Jing ( Chinese Acad. of Sc. ) Mahsa Kazemi ( Isfahan U. of Tech. , Iran) François Lemaire ( U. Lille 1 , France) Farnam Mansouri ( Microsoft , Canada) Liyun Li ( Banque de Montréal , Canada) Xin Li ( U. Carlos III , Spain) Wei Pan ( Intel Corp. , USA) Sushek Shekar ( Ciena , Canada) Paul Vrbik ( U. Newcastle , Australia) Ning Xie ( Huawei , Canada) Yuzhen Xie ( Critical Outcome Technologies , Canada) Li Zhang ( IBM Canada )
...

# Solving polynomial systems symbolically

$R := PolynomialRing([x, y, z]);\ F := [5*x^{\wedge}2 + 2*x*z^{\wedge}2 + 5*y^{\wedge}6 + 15*y^{\wedge}4 + 5*z^{\wedge}2 - 15*y^{\wedge}5 - 5*y^{\wedge}3];$

$$polynomial\_ring$$

$$[5x^2 + 2xz^2 + 5y^6 + 15y^4 + 5z^2 - 15y^5 - 5y^3] \qquad (1)$$

$RealTriangularize(F, R, output = record);$

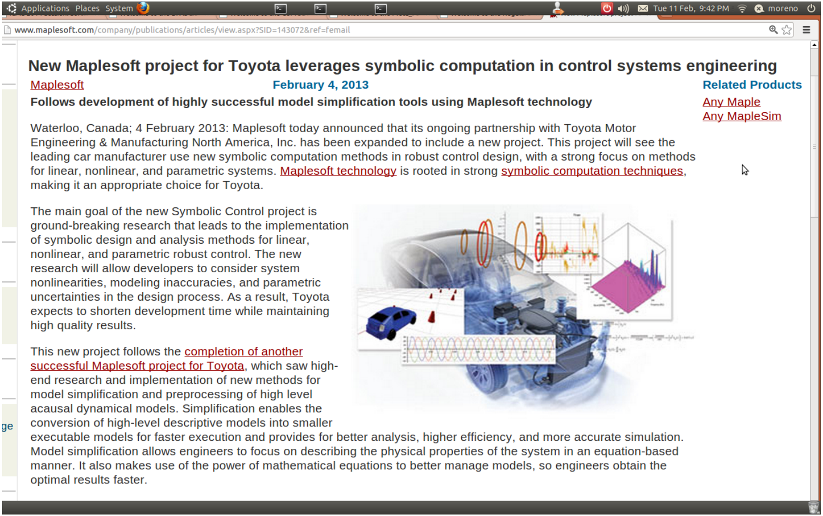
$$\left\{\begin{array}{l} 5x^2 + 2z^2x + 5y^6 + 15y^4 - 5y^3 - 15y^5 + 5z^2 = 0 \\ 25y^6 - 75y^5 + 75y^4 - z^4 - 25y^3 + 25z^2 < 0 \end{array}\right., \qquad (2)$$

$$\left\{\begin{array}{c} 5x + z^2 = 0 \\ 25y^6 - 75y^5 + 75y^4 - 25y^3 - z^4 + 25z^2 = 0 \\ 64z^4 - 1600z^2 + 25 > 0 \\ z \neq 0 \\ z - 5 \neq 0 \\ z + 5 \neq 0 \end{array}\right., \left\{\begin{array}{c} x = 0 \\ y - 1 = 0 \\ z = 0 \end{array}\right., \left\{\begin{array}{c} x = 0 \\ y = 0 \\ z = 0 \end{array}\right., \left\{\begin{array}{c} x + 5 = 0 \\ y - 1 = 0 \\ z - 5 = 0 \end{array}\right.,$$

$$\left\{\begin{array}{c} x + 5 = 0 \\ y = 0 \\ z - 5 = 0 \end{array}\right., \left\{\begin{array}{c} x + 5 = 0 \\ y - 1 = 0 \\ z + 5 = 0 \end{array}\right., \left\{\begin{array}{c} x + 5 = 0 \\ y = 0 \\ z + 5 = 0 \end{array}\right., \left\{\begin{array}{c} 5x + z^2 = 0 \\ 2y - 1 = 0 \\ 64z^4 - 1600z^2 + 25 = 0 \end{array}\right.$$

Figure: The *RegularChains* solver designed in our UWO lab is at the heart of Maple, which has about 5,000,000 licences world-wide.

# Application to mathematical sciences and engineering



www.maplesoft.com/company/publications/articles/view.aspx?SID=143072&ref=femail

## New Maplesoft project for Toyota leverages symbolic computation in control systems engineering

Maplesoft February 4, 2013

**Follows development of highly successful model simplification tools using Maplesoft technology**

Waterloo, Canada; 4 February 2013: Maplesoft today announced that its ongoing partnership with Toyota Motor Engineering & Manufacturing North America, Inc. has been expanded to include a new project. This project will see the leading car manufacturer use new symbolic computation methods in robust control design, with a strong focus on methods for linear, nonlinear, and parametric systems. Maplesoft technology is rooted in strong symbolic computation techniques, making it an appropriate choice for Toyota.

The main goal of the new Symbolic Control project is ground-breaking research that leads to the implementation of symbolic design and analysis methods for linear, nonlinear, and parametric robust control. The new research will allow developers to consider system nonlinearities, modeling inaccuracies, and parametric uncertainties in the design process. As a result, Toyota expects to shorten development time while maintaining high quality results.

This new project follows the completion of another successful Maplesoft project for Toyota, which saw high-end research and implementation of new methods for model simplification and preprocessing of high level acausal dynamical models. Simplification enables the conversion of high-level descriptive models into smaller executable models for faster execution and provides for better analysis, higher efficiency, and more accurate simulation. Model simplification allows engineers to focus on describing the physical properties of the system in an equation-based manner. It also makes use of the power of mathematical equations to better manage models, so engineers obtain the optimal results faster.

**Related Products**

Any Maple

Any MapleSim

Figure: Toyota engineers use our software to design control systems

# High-performance computing: parallel program translation

```
int main(){
  int sum_a=0, sum_b=0;
  int a[ 5 ] = {0,1,2,3,4};
  int b[ 5 ] = {0,1,2,3,4};
  #pragma omp parallel
  {
    #pragma omp sections
    {
      #pragma omp section
      {
        for(int i=0; i<5; i++)
          sum_a += a[ i ];
      }
      #pragma omp section
      {
        for(int i=0; i<5; i++)
           sum_b += b[ i ];
      }  }  }
}
```

```
int main()
{
  int sum_a=0, sum_b=0;
  int a[ 5 ] = {0,1,2,3,4};
  int b[ 5 ] = {0,1,2,3,4};

  meta_fork shared(sum_a){
    for(int i=0; i<5; i++)
       sum_a += a[ i ];
      }

  meta_fork shared(sum_b){
    for(int i=0; i<5; i++)
       sum_b += b[ i ];
      }

  meta_join;
}
```

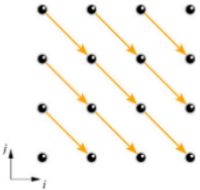
```
void fork_func0(int* sum_a,int* a)
{
        for(int i=0; i<5; i++)
          (*sum_a) += a[ i ];
}
void fork_func1(int* sum_b,int* b)
{
        for(int i=0; i<5; i++)
           (*sum_b) += b[ i ];
}
int main()
{
  int sum_a=0, sum_b=0;
  int a[ 5 ] = {0,1,2,3,4};
  int b[ 5 ] = {0,1,2,3,4};
  cilk_spawn fork_func0(&sum_a,a);
  cilk_spawn fork_func1(&sum_b,b);
  cilk_sync;
}
```

Our lab develops a compilation platform for translating parallel programs from one language to another; above we translate from `OpenMP` to `CilkPlus` through `MetaFork`. This project is supported by IBM Canada.

# High-performance computing: automatic parallelization

## Serial dense univariate polynomial multiplication

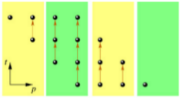
```
for(i=0; i<=n; i++){
  for(j=0; j<=n; j++)
    c[i+j] += a[i] * b[j];
}
```




Dependence analysis suggests to set $t(i,j) = n - j$ and $p(i,j) = i + j$. Then, the work is decomposed into *blocks* having good data locality.

## GPU-like multi-threaded dense univariate polynomial multiplication

```
meta_for (b=0; b<= 2 n / B; b++) {
    for (u=0; u<=min(B-1, 2*n - B * b); u++) {
       p = b * B + u;
       for (t=max(0,n-p); t<=min(n,2*n-p) ;t++)
           c[p] = c[p] + a[t+p-n] * b[n-t];
   }
}
```




| | | | | | | | |
|---|---|---|---|---|---|---|---|
| p: | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| r: | 0 | 0 | 1 | 1 | 0 | 0 | 1 |
| o: | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| b: | 0 | 0 | 1 | 1 | 2 | 2 | 3 |
| s: | 0 | 0 | 0 | 0 | 1 | 1 | 1 |

We use symbolic computation to automatically translate serial programs to GPU-like programs.

# Research projects with publicly available software

Basic Polynomial Algebra Subprograms

www.bpaslib.org

www.metafork.org

www.cumodp.org

www.regularchains.org

## Courses

CS 6652: . . . Symbolic solvers . . .

- Driving application: automatic parallelization
- Related topics: scientifi computing, program analysis, computer algebra, linear & non-linear optimization
- Objects of study: exact representation of real numbers on computers, real or integer solutions of (parametric) polynomial systems
- languages: `Maple`, `C/C++`.

CS 9535/4402: Distributed and Parallel Systems

- multi-core, GPGPU, hierarchical memory,
- fork-join concurrency, SIMD, message passing
- parallel algorithms: design and complexity analysis
- scheduling (work-stealing scheduler) and synchronization
- languages: `Julia`, `CilkPlus`, `CUDA`, `MPI`.