**CS2209A 2017**
**Applied Logic for Computer Science**

## Lecture 5

# Propositional Logic:

## Conditional statements

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# The card game

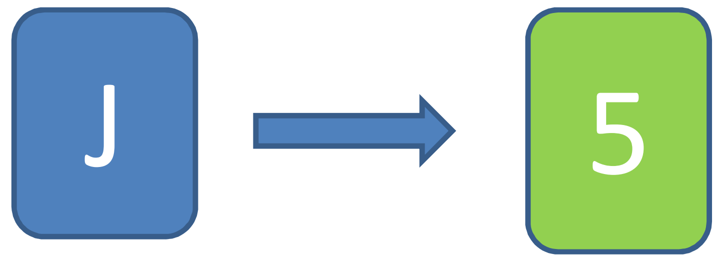
- You see the following cards. Each has a letter on one side and a number on the other.


- Which cards do you need to turn to check that "**if** a card has a J on it **then** it has a 5 on the other side"?

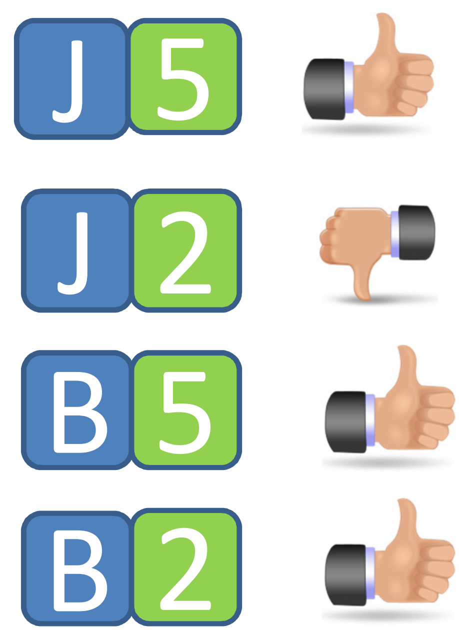
# “if … then” in logic

- This puzzle has a logical structure:

**“if *p* then *q*”**




- What circumstances make this **true**? Make this **false**?
  - *p* is true and *q* is true
  - ***p* is true and *q* is false**
  - *p* is false and *q* is true
  - *p* is false and *q* is false



# The card game

- You see the following cards. Each has a letter on one side and a number on the other.


- Which cards do you need to turn to check that “**if** a card has a J on it **then** it has a 5 on the other side”?
- **Those having a letter J and those having a number but not 5**

# Recap: “if and only if”

- **“If and only if”, iff,** $\leftrightarrow$
  - A $\leftrightarrow$ B:
    - A if and only if B
    - A $\rightarrow$ B and B $\rightarrow$ A
    - A and B are either both true or both false

- $(\boldsymbol{A} \leftrightarrow \boldsymbol{B}) \equiv (\boldsymbol{A} \rightarrow \boldsymbol{B}) \wedge (\boldsymbol{B} \rightarrow \boldsymbol{A})$

# Recap: **iff**

- $(\boldsymbol{A} \leftrightarrow \boldsymbol{B}) \equiv (\boldsymbol{A} \rightarrow \boldsymbol{B}) \wedge (\boldsymbol{B} \rightarrow \boldsymbol{A})$
- **Useful fact**: proving that $\boldsymbol{F} \equiv \boldsymbol{G}$ can be done by proving that $\boldsymbol{F} \leftrightarrow \boldsymbol{G}$ **is a tautology**
- Let $(\boldsymbol{A} \leftrightarrow \boldsymbol{B})$ be ***F*** and $(\boldsymbol{A} \rightarrow \boldsymbol{B}) \wedge (\boldsymbol{B} \rightarrow \boldsymbol{A})$ be ***G***

| A | B | $\boldsymbol{A} \leftrightarrow \boldsymbol{B}$ | $\boldsymbol{A} \rightarrow \boldsymbol{B}$ | $\boldsymbol{B} \rightarrow \boldsymbol{A}$ | $(\boldsymbol{A} \rightarrow \boldsymbol{B}) \wedge (\boldsymbol{B} \rightarrow \boldsymbol{A})$ | F $\leftrightarrow$ G |
|---|---|---|---|---|---|---|
| *True* | *True* | Ture | True | True | True | True |
| *True* | *False* | False | False | True | False | True |
| *False* | *True* | False | True | False | False | True |
| *False* | *False* | True | True | True | True | True |

# Contrapositive

- Let $\boldsymbol{A} \rightarrow \boldsymbol{B}$ be an **implication** (if A then B).
  - If a card has a J on one side then it has 5 on the other.
- Its **contrapositive** is $\neg\boldsymbol{B} \rightarrow \neg\boldsymbol{A}$.
  - If a card does not have 5 on one side then it cannot have J on the other.
- Contrapositive is equivalent to the original implication: $(\boldsymbol{A} \rightarrow \boldsymbol{B}) \equiv (\neg\boldsymbol{B} \rightarrow \neg\boldsymbol{A})$
  - This is why we need to check cards with numbers other than 5
  - Proof: $\neg B \rightarrow \neg A \equiv \neg\neg B \vee \neg A \equiv B \vee \neg A$
    $\equiv \neg A \vee B \equiv A \rightarrow B$

# Converse and Inverse

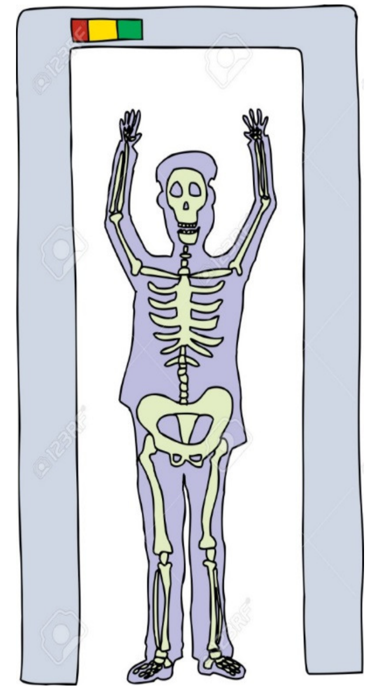
- Let $\boldsymbol{A} \rightarrow \boldsymbol{B}$ be an **implication** (if A then B).
  - If a card has a J on one side then it has 5 on the other.
- Its **converse** is $\boldsymbol{B} \rightarrow \boldsymbol{A}$
  - If a card has 5 on one side, then it has J on the other.
- Its **inverse** is $\neg\boldsymbol{A} \rightarrow \neg\,\boldsymbol{B}$
  - If a card does not have J on one side, it cannot have 5 on the other.
- Converse is **not equivalent** to the original implication!
  - **For A=true, B=false,** $\boldsymbol{A} \rightarrow \boldsymbol{B}$ **is false, but** $\mathbf{B} \rightarrow \boldsymbol{A}$ **is true**.
- Converse is **not equivalent** to the negation of $A \rightarrow B$
  - **For A=true, B=true,** $\boldsymbol{B} \rightarrow \boldsymbol{A}$ **is true, but** $\neg(\boldsymbol{A} \rightarrow \boldsymbol{B})$ **is false.**
- **Converse** is **equivalent** to the **inverse**. **Why?**
  - $(\neg\boldsymbol{A} \rightarrow \neg\,\boldsymbol{B}\,)$ is the **contrapositive** of $(\boldsymbol{B} \rightarrow \boldsymbol{A})$

# More on If and only if

- $A \leftrightarrow B$ ("A if and only if B") is true exactly when both the **implication $A \rightarrow B$** and its **converse $B \rightarrow A$** (equivalently, **inverse $\neg A \rightarrow \neg B$**) are true
  - Come to UCC 146 for the tutorial session **if and only if** you have a time conflict at 3:30pm-4:30pm on Tuesdays.
  - If you have a time conflict at 3:30pm-4:30pm on Tuesdays,
    - then come to NS 1 for the tutorial hour at 9:00pm-10:00pm
  - And if you don't have a time conflict at 3:30pm-4:30pm on Tuesdays,
    - then come to UCC 146 (not to NS 1)

# Contrapositive vs. Converse

- **"If a person is carrying a weapon, then airport metal detector will ring".**
  - Same  as "If the airport metal detector does not ring, then the person is not carrying a weapon".
  - Not the same as: "If the airport metal detector rings, then the person is carrying a weapon."
- "If the person is sick, then the test is positive".
- "If he is a murderer, his fingerprints are on the knife".



# Proof vs. Disproof


- To **prove** that something is **(always) true**:
  - Make sure it holds in **every scenario**
    - $\neg B \to \neg A$ is equivalent to $A \to B$, because

      $\neg B \to \neg A \equiv \neg\neg B \vee \neg A \equiv B \vee \neg A \equiv \neg A \vee B \equiv A \to B$
    - So $(\neg B \to \neg A) \leftrightarrow (A \to B)$ is a **tautology**.
    - I have classes every day that starts with T. I have classes on Tuesday and Thursday (and Monday, but that's irrelevant).
  - Or assume it does not hold, and then get something strange as a consequence:
    - To show **A is true**, enough to show $\boldsymbol{\neg A \to FALSE}$.
    - Prove "the number of prime numbers is infinite".
      Suppose there are finitely many prime numbers. What divides the number that's a product of all primes +1?

# Proof vs. Disproof


- To **disprove** that something is **always true**, enough to give just one scenario where it is false (find a falsifying assignment).
  - To disprove that $A \to B \equiv B \to A$
    - Take $A = true, B = false$,
    - Then $A \to B$ is false, but $\mathrm{B} \to A$ is true.
  - To disprove that $B \to A \equiv \neg (A \to B)$
    - Take A=true, B=true
    - Then $\mathrm{B} \to A$ is true, but $\neg(A \to B)$ is false.
  - I have classes every day! – No, you don't have classes on Saturday
  - Women don't do Computer Science! – Me?

# Treasure hunt

- In the back of an old cupboard you discover a note signed by a pirate famous for his bizarre sense of humor and love of logical puzzles. In the note he wrote that he had hidden a treasure somewhere on the property. He listed 5 true statements and challenged the reader to use them to figure out the location of the treasure

# Treasure hunt

1. If this house is next to a lake, then a treasure is not in the kitchen
2. If the tree in the font yard is an elm, then the treasure is in the kitchen
3. This house is next to a lake
4. The tree in the front yard is an elm, or the treasure  is buried under the flagpole
5. If the tree in the back yard is an oak, then the treasure is in the garage.