CS2214A – Discrete Structures for Computing

Introductory Lecture

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What is Discrete Mathematics?

1. Discrete mathematics is the part of mathematics devoted to the study of discrete (as opposed to continuous) objects.
2. Calculus deals with continuous objects and is not part of discrete mathematics.
3. Examples of discrete objects:
   a. integers,
   b. steps taken by a computer program,
   c. distinct paths to travel from point A to point B on a map along a road network,
   d. ways to pick a winning set of numbers in a lottery.
4. A course in discrete mathematics provides the mathematical background needed for all subsequent courses in computer science and for all subsequent courses in the many branches of discrete mathematics.

1 **Statement**: If n is an odd integer, then n^2 is odd.

2 **Proof**: If n is odd then there exists an integer k such that $n = 2k + 1$ and we have:

$$n^2 = (2k + 1)^2$$
$$= 4k^2 + 4k + 1$$
$$= 2(2k^2 + 2k) + 1$$
$$= 2r + 1$$

Therefore, there exists an integer r such that $n^2 = 2r + 1$ and n^2 is odd.

1 **Statement**: $1 = 2$.

2 **Proof**: There exist two integers a and b so that $a = b$ and we deduce:

$$a = b$$
$$\longrightarrow \quad a^2 = a \times b$$
$$\longrightarrow \quad a^2 - b^2 = a \times b - b^2$$
$$\longrightarrow \quad (a - b)(a + b) = b(a - b)$$
$$\longrightarrow \quad a + b = b$$
$$\longrightarrow \quad 2b = b$$
$$\longrightarrow \quad 2 = 1$$

1. Knights and Knaves: Some remote island is populated by two kinds of people: knights, who always tell the truth, and knaves, who always lie. Suppose you met two islanders, call them A and B, and you hear A saying "at least one of us is a knave".

2. Can you tell which of A and B is a knight and which is a knave?

1. Bertrand Russell and the Pope: "If $2 + 2 = 5$, then I am the Pope."

2. True or False?

Assume n is a positive integer and A is an integer array of size n.

```
c := A[0];
i := 1;
while (i ≠ n) {
    if (c < A[i]) then
        c := A[i];
    else
        i := i + 1;
}
```

What does the above program really do?

1. Consider 4 Boolean variables A, B, C and D.

2. Design a logic circuit for which the negation of $(A + B + \overline{C}) \cdot D$ is true whenever A, B, C and D are all true.

$\overline{(A \cdot B + \overline{C}) \cdot D}$
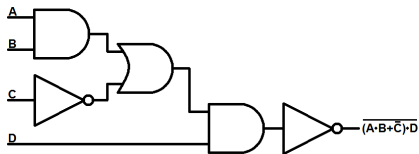
Kinds of Problems Solved Using Discrete Mathematics

1. How many ways can a password be chosen following specific rules?
2. How many valid Internet addresses are there?
3. What is the probability of winning a particular lottery?
4. Is there a link between two computers in a network?
5. How can I identify spam email messages?
6. How can I encrypt a message so that no unintended recipient can read it?
7. How can we build a circuit that adds two integers?

Kinds of Problems Solved Using Discrete Mathematics

1. What is the shortest path between two cities using a transportation system?
2. Find the shortest tour that visits each of a group of cities only once and then ends in the starting city.
3. How can we represent English sentences so that a computer can reason with them?
4. How can we prove that there are infinitely many prime numbers?
5. How can a list of integers be sorted so that the integers are in increasing order?
6. How many steps are required to do such a sorting?
7. How can it be proved that a sorting algorithm always correctly sorts a list?

Goals of a Course in Discrete Mathematics

1. **Mathematical Reasoning**: Ability to read, understand, and construct mathematical arguments and proofs.
2. **Combinatorial Analysis**: Techniques for counting objects of different kinds.
3. **Discrete Structures**: Abstract mathematical structures that represent objects and the relationships between them.
   Examples are sets, permutations, relations, graphs, trees, and finite state machines.

Goals of a Course in Discrete Mathematics

1. **Algorithmic Thinking**:
   a. One way to solve many problems is to specify an algorithm.
   b. An algorithm is a sequence of steps that can be followed to solve any instance of a particular problem.
   c. Algorithmic thinking involves specifying algorithms, analyzing the memory and time required by an execution of the algorithm, and verifying that the algorithm will produce the correct answer.

2. **Applications and Modeling**:
   a. It is important to appreciate and understand the wide range of applications of the topics in discrete mathematics and develop the ability to develop new models in various domains.
   b. concepts from discrete mathematics have not only been used to address problems in computing, but have been applied to solve problems in many areas such as chemistry, biology, linguistics, geography, business, etc.

Discrete Mathematics is a Gateway Course

1. Topics in discrete mathematics will be important in many courses that you will take in the future:
    a. **Computer Science**: Computer Architecture, Data Structures, Algorithms, Programming Languages, Compilers, Computer Security, Databases, Artificial Intelligence, Networking, Graphics, Game Design, Theory of Computation,
    b. **Mathematics**: Logic, Set Theory, Probability, Number Theory, Abstract Algebra, Combinatorics, Graph Theory, Game Theory, Network Optimization,
        - The concepts learned will also be helpful in continuous areas of mathematics.
    c. **Other Disciplines**: You may find concepts learned here useful in courses in philosophy, economics, linguistics, and other departments.