Problem 1 Suppose that a Bayesian spam filter is trained on a set of 500 spam messages and 200 messages that are not spam. The word “exciting” appears in 40 spam messages and in 25 messages that are not spam. Would an incoming message be rejected as spam if it contains the word “exciting” and the threshold for rejecting spam is 0.9? (Assume, for simplicity, that the message is equally likely to be spam as it is not to be spam.)

Solution 1 Let $S$ be the event that the message is spam. Let $E$ be the event that the message contains the word “exciting”. We have:

\[
P(S|E) = \frac{P(E|S)p(S)}{P(E|S)p(S) + P(E|\overline{S})p(\overline{S})}
\]

\[
= \frac{40/500 \times 1/2}{40/500 \times 1/2 + 25/200 \times 1/2}
\]

\[
= \frac{16}{41}.
\]

Problem 2 Let $A$ be the set of all ordered pairs of integers for which the second element of the pair is nonzero. Symbolically,

\[
A = \mathbb{Z} \times (\mathbb{Z} \setminus \{0\}).
\]

Define a binary relation $R$ on $A$ as follows: For all $(a, b), (c, d) \in A$,

\[
(a, b) R (c, d) \iff ad = bc.
\]

1. Is $R$ reflexive?
2. Is $R$ symmetric?
3. Is $R$ antisymmetric?
4. Is $R$ transitive?
5. Is $R$ an equivalence relation, a partial order, neither, or both?
6. List four distinct elements in the equivalence class $[(1, 3)]$.
7. List four distinct elements in the equivalence class $[(2, 5)]$.
8. Describe the equivalence classes of $R$.

Solution 2 For a similar problem, see Assignment 4.

Problem 3 For each of the following two graphs, determine whether or not it has an Euler circuit. Justify your answers. If the graph has an Euler
circuit, use the algorithm described in class to find it, including drawings of intermediate subgraphs. If the graph has an Euler path, use the algorithm described in class to find it, including drawings of intermediate subgraphs.

Solution 3  1. Removing (b, c) makes every degree even. Now we build an Euler circuit around b. We start with b, i, h, a, b We insert at i the Euler circuit around i: i, c, d, e, j, d, g, i We insert at a the Euler circuit around a: a, d, i, a.

2. Every degree even, hence there exists an Euler circuit. Now we build an Euler circuit around a. We start with the “frontier” : a, b, c, d, e, j, o, n, m, l, k, f. Next, we go around b using 4 vertical edges and the circular edges. We are left with two triangles