CS3101 Due: Wednesday 10th of April

Problem Set 3

CS3101 *Submission instructions on last page*

PROBBLEM 1. [60 points] Let A be a $n \times n$ invertible lower triangular matrix. A simple divide-and-conquer strategy to invert A is described below.

Let A be partitioned into $(n/2) \times (n/2)$ blocks as follows:

$$A = \left[\begin{array}{cc} A_1 & 0 \\ A_2 & A_3 \end{array} \right] , \qquad (1)$$

where n is assumed to be a power of 2. Clearly A_1 and A_3 are invertible lower triangular matrices. A^{-1} is given by

$$A^{-1} = \left[\begin{array}{cc} A_1^{-1} & 0 \\ -A_3^{-1}A_2A_1^{-1} & A_3^{-1} \end{array} \right] \qquad (2)$$

Therefore, we can obtain the inverse of A by recursively computing the inverses of A_1 and A_3, and by performing two $(n/2) \times (n/2)$ matrix multiplications to generate the term $-A_3^{-1}A_2A_1^{-1}$. This divide-and-conquer method leads to the following questions.

Question 1. [15 points] Write a `Cilk++` program to parallelize the above algorithm for matrices with `float` coefficients. You can make use of the code for matrix multiplication. More details will be given during the lab sessions of March 21 and March 28. available in the `Cilk++` distribution.

Question 2. [10 points] Analyze the work and span of the matrix multiplication code that you are using

Question 3. [20 points]Analyze the work and span of your parallel program for inverting lower triangular matrices.

Question 4. [15 points] Collect performance results with `Cilkview` for input random dense lower triangular matrices of order 2^{10}, 2^{11} and 2^{12}.

PROBBLEM 2. [40 points] In this problem, we consider the multiplication of two $n \times n$ matrices. To be simple, we assume that n = 2^p for some integer p.

Question 1. [10 points] Write a pseudo-code with $O(n^3)$ processors by CREW-PRAM model. What is the time complexity? What is the cost? What is the efficiency?

Question 2. [15 points] Following Question 1, can you reduce the number of processors to $O(N^3/\log n)$ using a CREW-PRAM program. What is the time complexity? What is the cost? What is the efficiency?

Question 3. [15 points] Design an algorithm to complete this in $O(\log(n))$ time by an EREW-PRAM program. How many processors you need? What is the cost? What is the efficiency?

Submission instructions.

Format: The answers to the problem questions should be typed.

- If these are programs, input test files and a `Makefile` (for compiling and running) are required.
- If these are algorithms or complexity analyzes, LaTeX is highly recommended; in any case a PDF file should gather all these answers.

All the files should be archived using the UNIX `tar` command.

Submission: The assignment should be returned to the instructor by email.

Collaboration. You are expected to do this assignment *on your own* without assistance from anyone else in the class. However, you can use literature and if you do so, briefly list your references in the assignment. Be careful! You might find on the web solutions to our problems that are not appropriate. For instance, because the parallelism model is different. So please, avoid those traps and work out the solutions by yourself. You should not hesitate to contact me if you have any questions regarding this assignment. I will be more than happy to help.

Marking. This assignment will be marked out of 100. A 10 % bonus will be given if your paper is clearly organized, the answers are precise and concise, the typography and the language are in good order. Messy assignments (unclear statements, lack of correctness in the reasoning, many typographical and language mistakes) may give rise to a 10 % malus.