Multithreaded Parallelism and Performance Measures

Marc Moreno Maza

University of Western Ontario, London, Ontario (Canada)

CS 3101

Plan

1. Parallelism Complexity Measures
2. cilk for Loops
3. Scheduling Theory and Implementation
4. Measuring Parallelism in Practice
5. Announcements

We shall also call this model multithreaded parallelism.
a **strand** is a maximal sequence of instructions that ends with a **spawn**, **sync**, or **return** (either explicit or implicit) statement.

At runtime, the **spawn** relation causes procedure instances to be structured as a rooted tree, called **spawn tree** or **parallel instruction stream**, where dependencies among strands form a dag.

We define several performance measures. We assume an ideal situation: no cache issues, no interprocessor costs:

- $T_p$ is the minimum running time on $p$ processors
- $T_1$ is called the **work**, that is, the sum of the number of instructions at each node.
- $T_\infty$ is the minimum running time with infinitely many processors, called the **span**

### The critical path length

Assuming all strands run in unit time, the longest path in the DAG is equal to $T_\infty$. For this reason, $T_\infty$ is also referred to as the **critical path length**.

### Work law

We have: $T_p \geq T_1/p$.

Indeed, in the best case, $p$ processors can do $p$ works per unit of time.
### Parallelism Complexity Measures

#### Span law

We have: \( T_p \geq T_\infty \).

Indeed, \( T_p < T_\infty \) contradicts the definitions of \( T_p \) and \( T_\infty \).

#### Speedup on \( p \) processors

- \( T_1 / T_p \) is called the **speedup on \( p \) processors**
- A parallel program execution can have:
  - **linear speedup**: \( T_1 / T_p = \Theta(p) \)
  - **superlinear speedup**: \( T_1 / T_p = \omega(p) \) (not possible in this model, though it is possible in others)
  - **sublinear speedup**: \( T_1 / T_p = o(p) \)

### Parallelism

Because the **Span Law** dictates that \( T_p \geq T_\infty \), the maximum possible speedup given \( T_1 \) and \( T_\infty \) is

\[
\frac{T_1}{T_\infty} = \text{parallelism} = \text{the average amount of work per step along the span.}
\]

- For Fib(4), we have \( T_1 = 17 \) and \( T_\infty = 8 \) and thus \( T_1 / T_\infty = 2.125 \).
- What about \( T_1(\text{Fib}(n)) \) and \( T_\infty(\text{Fib}(n)) \)?

---

The Fibonacci example

1
2
7
8
4
6
2
7
3
5

- For \( \text{Fib}(4) \), we have \( T_1 = 17 \) and \( T_\infty = 8 \) and thus \( T_1 / T_\infty = 2.125 \).
- What about \( T_1(\text{Fib}(n)) \) and \( T_\infty(\text{Fib}(n)) \)?
## Series Composition

- **Work?**
- **Span?**

### Work:
\[ T_1(A \cup B) = T_1(A) + T_1(B) \]

### Span:
\[ T_\infty(A \cup B) = T_\infty(A) + T_\infty(B) \]

## Parallel Composition

- **Work?**
- **Span?**

### Work:
\[ T_1(A \cup B) = T_1(A) + T_1(B) \]

### Span:
\[ T_\infty(A \cup B) = \max(T_\infty(A), T_\infty(B)) \]
cilk for Loops

Plan

1. Parallelism Complexity Measures
2. cilk for Loops
3. Scheduling Theory and Implementation
4. Measuring Parallelism in Practice
5. Announcements

(Moreno Maza) Multithreaded Parallelism and Performance Measures CS 3101

For loop parallelism in Cilk++

\[
\begin{pmatrix}
  a_{11} & a_{12} & \cdots & a_{1n} \\
  a_{21} & a_{22} & \cdots & a_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{n1} & a_{n2} & \cdots & a_{nn}
\end{pmatrix} \rightarrow \begin{pmatrix}
  a_{11} & a_{12} & \cdots & a_{1n} \\
  a_{12} & a_{22} & \cdots & a_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{n1} & a_{n2} & \cdots & a_{nn}
\end{pmatrix}
\]

\(A\) \rightarrow \(A^T\)

cilk_for (int i=1; i<n; ++i) {
  for (int j=0; j<i; ++j) {
    double temp = A[i][j];
    A[i][j] = A[j][i];
    A[j][i] = temp;
  }
}

The iterations of a cilk_for loop execute in parallel.

Implementation of for loops in Cilk++

Up to details (next week!) the previous loop is compiled as follows, using a divide-and-conquer implementation:

\[
\text{void recur(int lo, int hi)} \{ 
  \text{if (hi > lo) \{ // coarsen} 
  \text{int mid = lo + (hi - lo)/2;}
  \text{cilk_spawn recur(lo, mid);} 
  \text{recur(mid+1, hi);} 
  \text{cilk_sync;}
  \} \text{ else}
  \text{for (int j=0; j<hi; ++j) \{}
  \text{double temp = A[i][j];}
  \text{A[i][j] = A[j][i];}
  \text{A[j][i] = temp;}
  \}
\}
\]

Analysis of parallel for loops

Here we do not assume that each strand runs in unit time.

- **Span of loop control**: \(\Theta(\log(n))\)
- **Max span of an iteration**: \(\Theta(n)\)
- **Span**: \(\Theta(n)\)
- **Work**: \(\Theta(n^2)\)
- **Parallelism**: \(\Theta(n)\)
Parallelizing the inner loop

cilk_for (int i=1; i<n; ++i) {
    cilk_for (int j=0; j<i; ++j) {
        double temp = A[i][j];
        A[i][j] = A[j][i];
        A[j][i] = temp;
    }
}

- **Span of outer loop control**: $\Theta(\log(n))$
- **Max span of an inner loop control**: $\Theta(\log(n))$
- **Span of an iteration**: $\Theta(1)$
- **Span**: $\Theta(\log(n))$
- **Work**: $\Theta(n^2)$
- **Parallelism**: $\Theta(n^2/\log(n))$

But! More on this next week . . .

---

A **scheduler**'s job is to map a computation to particular processors. Such a mapping is called a **schedule**.

- If decisions are made at runtime, the scheduler is **online**, otherwise, it is **offline**.
- Cilk++'s scheduler maps strands onto processors dynamically at runtime.

- A strand is **ready** if all its predecessors have executed.
- A scheduler is **greedy** if it attempts to do as much work as possible at every step.
In any greedy schedule, there are two types of steps:

- **complete step**: There are at least $p$ strands that are ready to run. The greedy scheduler selects any $p$ of them and runs them.
- **incomplete step**: There are strictly less than $p$ threads that are ready to run. The greedy scheduler runs them all.

For any greedy schedule, we have $T_p \leq \frac{T_1}{p} + T_\infty$

- #complete steps $\leq \frac{T_1}{p}$, by definition of $T_1$.
- #incomplete steps $\leq T_\infty$. Indeed, let $G'$ be the subgraph of $G$ that remains to be executed immediately prior to a incomplete step.
  1. During this incomplete step, all strands that can be run are actually run.
  2. Hence removing this incomplete step from $G'$ reduces $T_\infty$ by one.

Cilk/Cilk++ randomized work-stealing scheduler load-balances the computation at run-time. Each processor maintains a ready deque:

- A ready deque is a double ended queue, where each entry is a procedure instance that is ready to execute.
- Adding a procedure instance to the bottom of the deque represents a procedure call being spawned.
- A procedure instance being deleted from the bottom of the deque represents the processor beginning/resuming execution on that procedure.
- Deletion from the top of the deque corresponds to that procedure instance being stolen.

A mathematical proof guarantees near-perfect linear speed-up on applications with sufficient parallelism, as long as the architecture has sufficient memory bandwidth.

A spawn/return in Cilk is over 100 times faster than a Pthread create/exit and less than 3 times slower than an ordinary C function call on a modern Intel processor.
Scheduling Theory and Implementation

The work-stealing scheduler (3/13)

spawn
call
call
call

Call!
P
P
P

(Moreno Maza) Multithreaded Parallelism and Performance

Scheduling Theory and Implementation

The work-stealing scheduler (4/13)

spawn
call
 spawn
call
call
call
spawn
spawn
spawn
call
spawn
call
spawn
call
call
P
spawn
P
 P
P
call
call
spawn
spawn
Spawn!
P
 P
 P
P

(Moreno Maza) Multithreaded Parallelism and Performance

Scheduling Theory and Implementation

The work-stealing scheduler (5/13)

spawn
call
 ... Call!
P
 P
 P
P

(Moreno Maza) Multithreaded Parallelism and Performance

Scheduling Theory and Implementation

The work-stealing scheduler (6/13)

spawn
call
 spawn
spawn
call
call
call
spawn
spawn
call
spawn
call
spawn
call
spawn
P
spawn
spawn
P
 P
P
call
call
spawn
spawn
Return!
P
 P
 P
P

(Moreno Maza) Multithreaded Parallelism and Performance

Scheduling Theory and Implementation
The work-stealing scheduler (7/13)

The work-stealing scheduler (8/13)

The work-stealing scheduler (9/13)

The work-stealing scheduler (10/13)
The work-stealing scheduler (11/13)

Performances of the work-stealing scheduler

Assume that

- each strand executes in unit time,
- for almost all “parallel steps” there are at least $p$ strands to run,
- each processor is either working or stealing.

Then, the randomized work-stealing scheduler is expected to run in

$$T_P = T_1/p + O(T_\infty)$$

- During a steal-free parallel steps (steps at which all processors have work on their deque) each of the $p$ processors consumes 1 work unit.
- Thus, there is at most $T_1/p$ steal-free parallel steps.
- During a parallel step with steals each thief may reduce by 1 the running time with a probability of $1/p$.
- Thus, the expected number of steals is $O(p T_\infty)$.
- Therefore, the expected running time

$$T_P = (T_1 + O(p T_\infty))/p = T_1/p + O(T_\infty).$$

(Moreno Maza) Multithreaded Parallelism and Performance IV
CS 3101 37 / 56

(Moreno Maza) Multithreaded Parallelism and Performance IV
CS 3101 38 / 56

(Moreno Maza) Multithreaded Parallelism and Performance IV
CS 3101 39 / 56

(Moreno Maza) Multithreaded Parallelism and Performance IV
CS 3101 40 / 56
Overheads and burden

- Obviously $T_{1/p} + T_\infty$ will under-estimate $T_p$ in practice.
- Many factors (simplification assumptions of the fork-join parallelism model, architecture limitation, costs of executing the parallel constructs, overheads of scheduling) will make $T_p$ larger in practice.
- One may want to estimate the impact of those factors:
  1. by improving the estimate of the randomized work-stealing complexity result
  2. by comparing a Cilk++ program with its C++ elision
  3. by estimating the costs of spawning and synchronizing
- Cilk++ estimates $T_p$ as $T_p = T_{1/p} + 1.7 \text{burden} - \text{span}$, where burden_span is 15000 instructions times the number of continuation edges along the critical path.

Plan

- Parallelism Complexity Measures
- Cilk_for Loops
- Scheduling Theory and Implementation
- Measuring Parallelism in Practice
- Announcements

Cilkview

- Cilkview computes work and span to derive upper bounds on parallel performance
- Cilkview also estimates scheduling overhead to compute a burdened span for lower bounds.

The Fibonacci Cilk++ example

Code fragment

```c
long fib(int n)
{
    if (n < 2) return n;
    long x, y;
    x = cilk_spawn fib(n-1);
    y = fib(n-2);
    cilk_sync;
    return x + y;
}
```
**Fibonacci program timing**

The environment for benchmarking:
- model name: Intel(R) Core(TM)2 Quad CPU Q6600 @ 2.40GHz
- L2 cache size: 4096 KB
- memory size: 3 GB

<table>
<thead>
<tr>
<th># of int</th>
<th>timing(s)</th>
<th>timing(s)</th>
<th>speedup</th>
<th>timing(s)</th>
<th>speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td># cores = 1</td>
<td># cores = 2</td>
<td># cores = 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.086</td>
<td>0.046</td>
<td>1.870</td>
<td>0.025</td>
<td>3.440</td>
</tr>
<tr>
<td>35</td>
<td>0.776</td>
<td>0.436</td>
<td>1.780</td>
<td>0.206</td>
<td>3.767</td>
</tr>
<tr>
<td>40</td>
<td>8.931</td>
<td>4.842</td>
<td>1.844</td>
<td>2.389</td>
<td>3.723</td>
</tr>
<tr>
<td>45</td>
<td>105.263</td>
<td>54.017</td>
<td>1.949</td>
<td>27.200</td>
<td>3.870</td>
</tr>
<tr>
<td>50</td>
<td>1165.000</td>
<td>665.115</td>
<td>1.752</td>
<td>340.638</td>
<td>3.420</td>
</tr>
</tbody>
</table>

**Quicksort**

code in cilk/examples/qsort

```c
void sample_qsort(int * begin, int * end)
{
    if (begin != end)
    {
        --end;
        int * middle = std::partition(begin, end,
                                      std::bind2nd(std::less<int>(), *end));
        using std::swap;
        swap(*end, *middle);
        cilk_spawn sample_qsort(begin, middle);
        sample_qsort(++middle, ++end);
        cilk_sync;
    }
}
```

**Quicksort timing**

Timing for sorting an array of integers:

<table>
<thead>
<tr>
<th># of int</th>
<th>timing(s)</th>
<th>timing(s)</th>
<th>speedup</th>
<th>timing(s)</th>
<th>speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td># cores = 1</td>
<td># cores = 2</td>
<td># cores = 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 \times 10^6</td>
<td>1.958</td>
<td>1.016</td>
<td>1.927</td>
<td>0.541</td>
<td>3.619</td>
</tr>
<tr>
<td>50 \times 10^6</td>
<td>10.518</td>
<td>5.469</td>
<td>1.923</td>
<td>2.847</td>
<td>3.694</td>
</tr>
<tr>
<td>100 \times 10^6</td>
<td>21.481</td>
<td>11.096</td>
<td>1.936</td>
<td>5.954</td>
<td>3.608</td>
</tr>
<tr>
<td>500 \times 10^6</td>
<td>114.300</td>
<td>57.996</td>
<td>1.971</td>
<td>31.086</td>
<td>3.677</td>
</tr>
</tbody>
</table>

**Matrix multiplication**

Timing of multiplying a $687 \times 837$ matrix by a $837 \times 1107$ matrix

<table>
<thead>
<tr>
<th># of int</th>
<th>timing(s)</th>
<th>timing(s)</th>
<th>speedup</th>
<th>timing(s)</th>
<th>speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td># cores = 1</td>
<td># cores = 2</td>
<td># cores = 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.273</td>
<td>1.165</td>
<td>0.721</td>
<td>1.674</td>
<td>0.399</td>
</tr>
<tr>
<td>16</td>
<td>1.270</td>
<td>1.787</td>
<td>0.711</td>
<td>1.408</td>
<td>0.349</td>
</tr>
<tr>
<td>32</td>
<td>1.280</td>
<td>1.757</td>
<td>0.729</td>
<td>1.223</td>
<td>0.308</td>
</tr>
<tr>
<td>48</td>
<td>1.258</td>
<td>1.760</td>
<td>0.715</td>
<td>1.164</td>
<td>0.293</td>
</tr>
<tr>
<td>64</td>
<td>1.258</td>
<td>1.798</td>
<td>0.700</td>
<td>1.159</td>
<td>0.291</td>
</tr>
<tr>
<td>80</td>
<td>1.252</td>
<td>1.773</td>
<td>0.706</td>
<td>1.267</td>
<td>0.320</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>threshold</th>
<th>iterative</th>
<th>recursive</th>
</tr>
</thead>
<tbody>
<tr>
<td>st(s)</td>
<td>pt(s)</td>
<td>su</td>
</tr>
<tr>
<td>10</td>
<td>1.273</td>
<td>1.165</td>
</tr>
<tr>
<td>16</td>
<td>1.270</td>
<td>1.787</td>
</tr>
<tr>
<td>32</td>
<td>1.280</td>
<td>1.757</td>
</tr>
<tr>
<td>48</td>
<td>1.258</td>
<td>1.760</td>
</tr>
<tr>
<td>64</td>
<td>1.258</td>
<td>1.798</td>
</tr>
<tr>
<td>80</td>
<td>1.252</td>
<td>1.773</td>
</tr>
</tbody>
</table>

st = sequential time; pt = parallel time with 4 cores; su = speedup
The cilkview example from the documentation

Using cilk_for to perform operations over an array in parallel:

```c
static const int COUNT = 4;
static const int ITERATION = 1000000;
long arr[COUNT];
long do_work(long k){
    long x = 15;
    static const int nn = 87;
    for (long i = 1; i < nn; ++i)
        x = x / i + k % i;
    return x;
}
```

```c
int cilk_main(){
    for (int j = 0; j < ITERATION; j++)
        for (int i = 0; i < COUNT; i++)
            arr[i] += do_work( j * i + i + j);
}
```

A simple fix

Inverting the two for loops

```c
int cilk_main()
{
    for (int j = 0; j < ITERATION; j++)
        for (int i = 0; i < COUNT; i++)
            arr[i] += do_work( j * i + i + j);
}
```

1) Parallelism Profile
- Work: 6,480,801,250 ins
- Span: 2,116,801,250 ins
- Burdened span: 31,920,801,250 ins
- Parallelism: 3.06
- Burdened parallelism: 0.20
- Number of spawns/syncs: 3,000,000
- Average instructions / strand: 720
- Strands along span: 4,000,001
- Average instructions / strand on span: 529

2) Speedup Estimate
- 2 processors: 0.21 - 2.00
- 4 processors: 0.15 - 3.06
- 8 processors: 0.13 - 3.06
- 16 processors: 0.13 - 3.06
- 32 processors: 0.12 - 3.06

1) Parallelism Profile
- Work: 5,295,801,529 ins
- Span: 1,326,801,107 ins
- Burdened span: 1,326,830,911 ins
- Parallelism: 3.99
- Burdened parallelism: 3.99
- Number of spawns/syncs: 3
- Average instructions / strand: 529,580,152
- Strands along span: 5
- Average instructions / strand on span: 265,360,221

2) Speedup Estimate
- 2 processors: 1.40 - 2.00
- 4 processors: 1.76 - 3.99
- 8 processors: 2.01 - 3.99
- 16 processors: 2.17 - 3.99
- 32 processors: 2.25 - 3.99
## Timing

<table>
<thead>
<tr>
<th></th>
<th>#cores = 1</th>
<th>#cores = 2</th>
<th>#cores = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>version</td>
<td>timing(s)</td>
<td>timing(s)</td>
<td>speedup</td>
</tr>
<tr>
<td>original</td>
<td>7.719</td>
<td>9.611</td>
<td>0.803</td>
</tr>
<tr>
<td></td>
<td>10.758</td>
<td>0.718</td>
<td></td>
</tr>
<tr>
<td>improved</td>
<td>7.471</td>
<td>3.724</td>
<td>2.006</td>
</tr>
<tr>
<td></td>
<td>1.888</td>
<td>3.957</td>
<td></td>
</tr>
</tbody>
</table>

(Moreno Maza) Multithreaded Parallelism and Performance Measures CS 3101 53 / 56

## Plan

1. Parallelism Complexity Measures
2. cilk_for Loops
3. Scheduling Theory and Implementation
4. Measuring Parallelism in Practice
5. Announcements

## Acknowledgements

- Charles E. Leiserson (MIT) for providing me with the sources of its lecture notes.
- Matteo Frigo (Intel) for supporting the work of my team with Cilk++.
- Yuzhen Xie (UWO) for helping me with the images used in these slides.
- Liyun Li (UWO) for generating the experimental data.

## References