CS3101 Due: Tuesday 8-th of April 2014

Problem Set 3

CS3101 *Submission instructions on last page*

PROBBLEM 1. [100 points]

Let A be a $n \times n$ invertible lower triangular matrix. A simple divide-and-conquer strategy to invert A is described below. Let A be partitioned into $(n/2) \times (n/2)$ blocks as follows:

$$A = \begin{bmatrix} A_1 & 0 \\ A_2 & A_3 \end{bmatrix}, \tag{1}$$

where n is assumed to be a power of 2. Clearly A_1 and A_3 are invertible lower triangular matrices. A^{-1} is given by

$$A^{-1} = \begin{bmatrix} A_1^{-1} & 0 \\ -A_3^{-1}A_2A_1^{-1} & A_3^{-1} \end{bmatrix} \tag{2}$$

Therefore, we can obtain the inverse of A by recursively computing the inverses of A_1 and A_3, and by performing two $(n/2) \times (n/2)$ matrix multiplications to generate the term $-A_3^{-1}A_2A_1^{-1}$. This divide-and-conquer method leads to the following questions.

1. Write a `Cilk++`-like "program" to parallelize the above algorithm. This "program" should be quite informal: the only things that really matter are input sizes in the recursive calls and, where the `cilk_spawn` and `cilk_sync` constructs are used. You can make use of

   - a subroutine MULT(C, A, B, n) computing the product of two square matrices A and B (and writing the product AB to C) of order n in work $M_1(n) = \Theta(n^3)$ and with span $M_\infty(n) = \Theta(\log^2(n))$.
   - a subroutine CONSTRUCT$(E_{1,1}, E_{1,2}, E_{2,1}, E_{2,2}, n)$ which takes fours square matrices $E_{1,1}, E_{1,2}, E_{2,1}, E_{2,2}$ of order n and returns the square matrix E of order $2n$ defines as follows:
     $$E = \begin{bmatrix} E_{1,1} & E_{1,2} \\ E_{2,1} & E_{2,2} \end{bmatrix}, \tag{3}$$
     we shall assume that the work and span of this second subroutine can be neglected in our analysis.

2. Analyze the work and span of your parallel program.

3. Recalculate the span if we have $M_\infty(n) = \Theta(n)$ instead of $M_\infty(n) = \Theta(\log^2(n))$.

Note: Knowing how to invert a matrix is not necessary for this exercise: Equation(2) and the fact that a 1×1 matrix is inverted just as a number are sufficient information on the subject.

Submission instructions.

Format: Problems 1 involves only answering questions in plain English. Those answers should be gathered in a PDF document To summarize, each assignment submission consists of a singl PDF file: `Pb1.pdf`.

Submission: The assignment should be returned to the instructor by email.

Collaboration. You are expected to do this assignment *on your own* without assistance from anyone else in the class. However, you can use literature and if you do so, briefly list your references in the assignment. Be careful! You might find on the web solutions to our problems that are not appropriate. For instance, because the cache memory model is different. So please, avoid those traps and work out the solutions by yourself. You should not hesitate to contact the instructor if you have any question regarding this assignment. I will be more than happy to help.

Marking. This assignment will be marked out of 100. A 10 % bonus will be given if your answers are clearly organized, precise and concise. Messy assignments (unclear statements, lack of correctness in the reasoning, many typographical or language mistakes) may give rise to a 10 % malus.