Guidelines. The quiz consists of four questions. All answers should be written in the answer boxes. No justifications for the answers are needed, unless explicitly required. You are expected to do this quiz on your own without assistance from anyone else in the class. If possible, please avoid pencils and use pens with dark ink. Thank you.

PRAM Cheat Sheet

- The input of a PRAM program consists of \( n \) items stored in \( M[1], \ldots, M[n] \).
- The output of a PRAM program consists of \( n' \) items stored in \( n' \) memory cells, say \( M[n+1], \ldots, M[n+n'] \).
- Special assumptions (EREW, CREW, COMMON CRCW, etc.) have to be made in order to resolve shared memory access conflicts.
- The only way processors can exchange data is by writing into and reading from shared memory cells.
- The Parallel Time, denoted by \( T(n, p) \), is the time elapsed from the start of a parallel computation to the moment where the last processor finishes, on an input data of size \( n \), and using \( p \) processors.
- The parallel efficiency, denoted by \( E(n, p) \), is

\[
E(n, p) = \frac{SU(n)}{pT(n, p)},
\]

where \( SU(n) \) is a lower bound for a sequential execution.
Question 1. On a CREW-PRAM Machine, what does the pseudo-code do?

A[1..6] := [0,0,0,0,0,1];
Active Processors P[1], ..., P[5];
// id the index of one of the active processor
for each 1 <= step <= 5 do
print A;
Question 2. Consider the following PRAM algorithm written for the EREW sub-model. Recall that with EREW, no two processors are allowed to read or write the same shared memory cell simultaneously.

Input: 2n integer numbers stored in $M[1], \ldots, M[2n]$, where $n \geq 2$ is a power of 2.

Output: The maximum of those numbers, written at $M[2n + 1]$.

Program:

Active Processors P[1], ..., P[n];

\[
\text{step} := 0; \\
\text{jump} := 2^\text{step}; \\
\text{while jump} \leq n \text{ do } \{ \\
\hspace{1em} // \text{id the index of one of the active processor} \\
\hspace{1em} \text{id} \text{ mod jump} = 0 \\
\hspace{1em} \quad M[2 \times \text{id}] := \max(M[2 \times \text{id}], M[2 \times \text{id} - \text{jump}]); \\
\hspace{1em} \text{step} := \text{step} + 1; \\
\hspace{1em} \text{jump} := 2^\text{step}; \\
\} \\
\text{if (id} = n) \text{ then } M[2n+1] := M[2n];
\]

Answer each of the following questions:

(1) What is $T(2n, n)$, $SU(n)$, $S(2n, n)$?

(2) What is $W(2n, n)$, $E(2n, n)$?

(3) Propose a variant of the algorithm for an input of size $n$ using $p$ processors, for a well-chosen value of $p$, such that we have $S(2n, p) = 50\%$.

For (3), you are asked to justify your answer.
Question 3. Given $n$ integer numbers stored in $M[1], \ldots, M[n]$, where $n \geq 2$, propose a COMMON CRCW-PRAM program computing the maximum of those numbers and for which $T(n, p)$ is constant (that is, independent of $n$ and $p$) where $p$ is the number of active processors (that you should specify). The resulting maximum must be written in $M[n + 1]$. You are also asked to estimate $E(n, p)$.

Recall that COMMON CRCW allows (1) simultaneous reads of the same memory cell and, (2) simultaneous writes of the same memory provided that all all values to be written are equal.