

CS4442/9542b  
Artificial Intelligence II  
prof. Olga Veksler

*Lecture 10*

*Natural Language Processing*

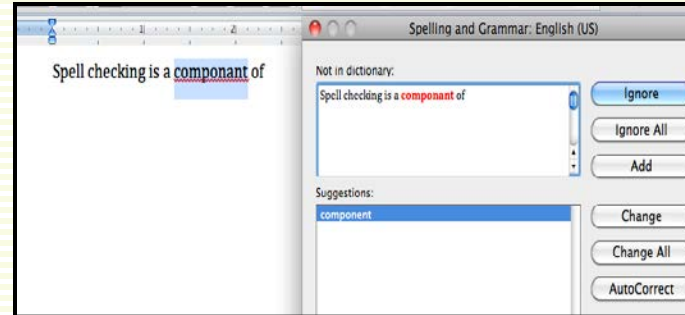
**Spelling Correction**

# Outline

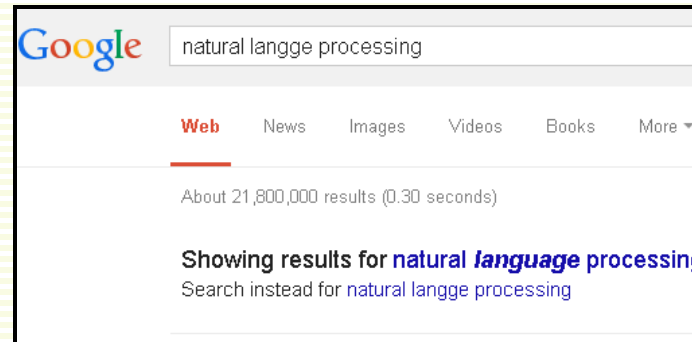
- Intro to spelling correction
- Types of spelling errors
  1. non word spelling errors
  2. real world spelling errors
- Spelling tasks:
  1. Detecting errors
    1. non word spelling errors: dictionary + context
    2. real word spelling errors: from context
  2. Correcting Errors
    - edit distance
      - Dynamic programming (DP) for computing minimum edit distance
    - noisy channel model

# Applications for Spelling Correction

- Word processing



- Web search



- Mobile devices, etc.

# Types of Spelling Errors

## 1. Non-word errors (not in dictionary)

- *graffe* → *giraffe*

## 2. Real-word errors (in dictionary)

- typographical errors
  - *three* → *there*
- cognitive errors (homophones)
  - *piece* → *peace*
  - *too* → *two*

# Non-Word Spelling Errors Detection/Correction

- Non-word spelling error detection
  - any word not in ***dictionary*** is an error
  - the larger the dictionary the better
- Non-word spelling error correction
  - generate ***candidates***
    - real words that are similar to error
  - choose the one which is best
    - shortest weighted edit distance
    - highest noisy channel probability

# Candidate Generation

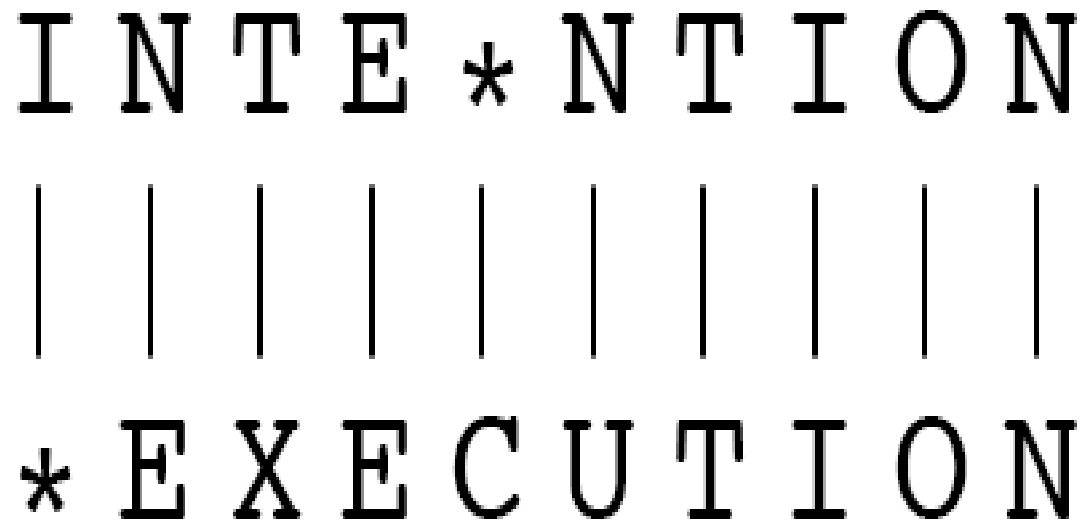
- Words with similar spelling
- The user typed **graffe**
- Which is closest
  - **graf, graft, grail, giraffe?**
- Use **edit distance** to compute  **$d(\text{graffe}, \text{graf})$**

# Damerau-Levenshtein Edit Distance

- The **minimum edit distance** between two strings is the minimum number of editing operations needed to transform one string into the other
- Editing operations:
  - insertion
  - deletion
  - substitution
  - for spelling, also want to include transposition of two letters

# Minimum Edit Distance

- Two strings and their **alignment**:



The diagram shows the alignment of the strings "INTENTION" and "\*EXECUTION". The characters are arranged in three rows. The first row contains the characters of "INTENTION". The second row contains vertical bars (|) that align with the characters in the first row. The third row contains the characters of "\*EXECUTION". The asterisk in the first row is aligned with the asterisk in the third row. The vertical bars connect the following characters: I to E, N to X, T to E, E to C, N to U, T to I, I to O, and O to N.

```
INTENTION
| | | | | | | |
*EXECUTION
```



# Minimum Edit Distance

```

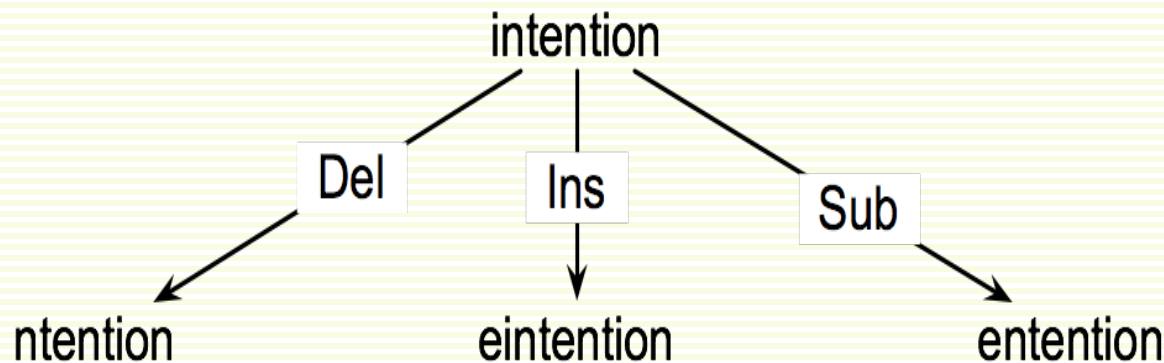
I N T E * N T I O N
| | | | | | | | |
* E X E C U T I O N
d s s   i s

```

- If each operation has cost of 1
  - distance between these is 5
- If substitutions cost 2 (Levenshtein)
  - distance between them is 8

# How to Find Min Edit Distance?

- Naïve approach
- Searching for a path (sequence of edits) from the start string to the final string:
  - **Initial state:** the word we're transforming
  - **Operators:** insert, delete, substitute
  - **Goal state:** the word we're trying to get to
  - **Path cost:** what we want to minimize: the number of edits



# Minimum Edit as Search

- But the space of all edit sequences is huge!
  - we can't afford to navigate naively
  - many distinct paths wind up at the same state
  - **dynamic programming** for efficiency

# Defining Min Edit Distance

- For two strings
  - $X$  of length  $n$
  - $Y$  of length  $m$
- Define  $D(i, j)$ 
  - the edit distance between  $X[1..i]$  and  $Y[1..j]$ 
    - i.e., the first  $i$  characters of  $X$  and the first  $j$  characters of  $Y$
  - the edit distance between  $X$  and  $Y$  is thus  $D(n, m)$

# Dynamic Programming for Min Edit Distance

- Tabular computation of  $D(n, m)$
- Solve problems by combining solutions to subproblems
- Bottom-up
  - Initialize: compute  $D(i, j)$  for small  $i, j$
  - Iterate: compute larger  $D(i, j)$  based on previously computed smaller values

# Defining Min Edit Distance (Levenshtein)

- Initialization

$$D(i, 0) = i$$

$$D(0, j) = j$$

$$D(3, 0) = 3$$

**int**ention



~~ex~~ecution

$$D(0, 5) = 5$$

~~in~~tention



**ex**ecution

# Defining Min Edit Distance (Levenshtein)

- Initialization

$$D(i, 0) = i$$

$$D(0, j) = j$$

- Recurrence Relation (iteration)

```
for each  $i = 1 \dots m$   
  for each  $j = 1 \dots n$ 
```

$$D(i, j) = \min \begin{cases} D(i-1, j) + 1 \\ D(i, j-1) + 1 \\ D(i-1, j-1) + \begin{cases} 2 & \text{if } x(i) \neq y(j) \\ 0 & \text{if } x(i) = y(j) \end{cases} \end{cases}$$

# Defining Min Edit Distance (Levenshtein)

- The smallest of:

$$D(\mathbf{int}, \mathbf{exec}) = \mathbf{del}[\mathbf{t}] + D(\mathbf{in}, \mathbf{exec})$$

$$D(\mathbf{int}, \mathbf{exec}) = D(\mathbf{int}, \mathbf{exe}) + \mathbf{ins}[\mathbf{c}]$$

$$D(\mathbf{int}, \mathbf{exec}) = \mathbf{substitute}[\mathbf{t}, \mathbf{c}] + D(\mathbf{in}, \mathbf{exe})$$

- Recurrence Relation (iteration)

```
for each  $i = 1 \dots m$   
  for each  $j = 1 \dots n$ 
```

$$D(i, j) = \min \begin{cases} D(i-1, j) + 1 \\ D(i, j-1) + 1 \\ D(i-1, j-1) + \begin{cases} 2 & \text{if } \mathbf{x}(i) \neq \mathbf{y}(j) \\ 0 & \text{if } \mathbf{x}(i) = \mathbf{y}(j) \end{cases} \end{cases}$$



# Defining Min Edit Distance (Levenshtein)

- Initialization

$$D(i, 0) = i$$

$$D(0, j) = j$$

- Recurrence Relation (iteration)

for each  $i = 1 \dots m$   
for each  $j = 1 \dots n$

$$D(i, j) = \min \begin{cases} D(i-1, j) + 1 \\ D(i, j-1) + 1 \\ D(i-1, j-1) + \begin{cases} 2 & \text{if } x(i) \neq y(j) \\ 0 & \text{if } x(i) = y(j) \end{cases} \end{cases}$$

- Termination

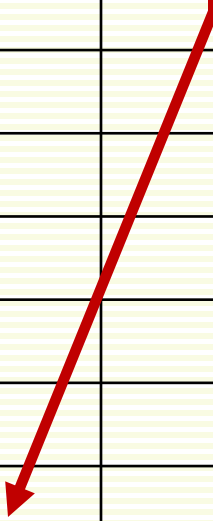
$D(n, m)$  is distance

# Edit Distance Table: Initialization

<b>N</b>	<b>9</b>									
<b>O</b>	<b>8</b>									
<b>I</b>	<b>7</b>									
<b>T</b>	<b>6</b>									
<b>N</b>	<b>5</b>									
<b>E</b>	<b>4</b>									
<b>T</b>	<b>3</b>									
<b>N</b>	<b>2</b>									
<b>I</b>	<b>1</b>									
<b>#</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
	<b>#</b>	<b>E</b>	<b>X</b>	<b>E</b>	<b>C</b>	<b>U</b>	<b>T</b>	<b>I</b>	<b>O</b>	<b>N</b>

# Edit Distance Table: Iterations

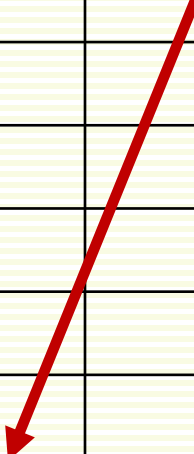
<b>N</b>	<b>9</b>									
<b>O</b>	<b>8</b>									
<b>I</b>	<b>7</b>									
<b>T</b>	<b>6</b>									
<b>N</b>	<b>5</b>									
<b>E</b>	<b>4</b>									
<b>T</b>	<b>3</b>									
<b>N</b>	<b>2</b>									
<b>I</b>	<b>1</b>									
<b>#</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
	<b>#</b>	<b>E</b>	<b>X</b>	<b>E</b>	<b>C</b>	<b>U</b>	<b>T</b>	<b>I</b>	<b>O</b>	<b>N</b>

$$D(i, j) = \min \begin{cases} D(i-1, j) + 1 \\ D(i, j-1) + 1 \\ D(i-1, j-1) + \begin{cases} 2 & \text{if } X(i) \neq Y(j) \\ 0 & \text{if } X(i) = Y(j) \end{cases} \end{cases}$$


# Edit Distance Table: Iterations

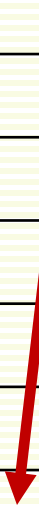
N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1	2								
#	0	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

$$D(i, j) = \min \begin{cases} D(i-1, j) + 1 \\ D(i, j-1) + 1 \\ D(i-1, j-1) + \begin{cases} 2 & \text{if } X(i) \neq Y(j) \\ 0 & \text{if } X(i) = Y(j) \end{cases} \end{cases}$$



# Edit Distance Table: Iterations

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1	2								
#	0	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

$$D(i, j) = \min \begin{cases} D(i-1, j) + 1 \\ D(i, j-1) + 1 \\ D(i-1, j-1) + \begin{cases} 2 & \text{if } X(i) \neq Y(j) \\ 0 & \text{if } X(i) = Y(j) \end{cases} \end{cases}$$


# Edit Distance Table: Iterations

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1	2	<b>3</b>							
#	<b>0</b>	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

# Edit Distance Table: Termination

N	9			$D(i, j) = \min \begin{cases} D(i-1, j) + 1 \\ D(i, j-1) + 1 \\ D(i-1, j-1) + \begin{cases} 2 & \text{if } X(i) \neq Y(j) \\ 0 & \text{if } X(i) = Y(j) \end{cases} \end{cases}$						
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1	2	3	4	5	6	7	6	7	8
#	0	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

# Edit Distance Table: Termination

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2	<b>3</b>								
I	1	2	3	4	5	6	7	6	7	8
#	0	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N



# Edit Distance Table: Termination

N	9	8	9	10	11	12	11	10	9	<b>8</b>
O	8	7	8	9	10	11	10	9	8	9
I	7	6	7	8	9	10	9	8	9	10
T	6	5	6	7	8	9	8	9	10	11
N	5	4	5	6	7	8	9	10	11	10
E	4	3	4	5	6	7	8	9	10	9
T	3	4	5	6	7	8	7	8	9	8
N	2	3	4	5	6	7	8	7	8	7
I	1	2	3	4	5	6	7	6	7	8
#	<b>0</b>	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

# MinEdit with Backtrace

- Keep pointer to extract the optimal alignment

<b>n</b>	9	↓ 8	↙↖↓ 9	↙↖↓ 10	↙↖↓ 11	↙↖↓ 12	↓ 11	↓ 10	↓ 9	↙ 8	
<b>o</b>	8	↓ 7	↙↖↓ 8	↙↖↓ 9	↙↖↓ 10	↙↖↓ 11	↓ 10	↓ 9	↙ 8	← 9	
<b>i</b>	7	↓ 6	↙↖↓ 7	↙↖↓ 8	↙↖↓ 9	↙↖↓ 10	↓ 9	↙ 8	← 9	← 10	
<b>t</b>	6	↓ 5	↙↖↓ 6	↙↖↓ 7	↙↖↓ 8	↙↖↓ 9	↙ 8	← 9	← 10	↖↓ 11	
<b>n</b>	5	↓ 4	↙↖↓ 5	↙↖↓ 6	↙↖↓ 7	↙↖↓ 8	↙↖↓ 9	↙↖↓ 10	↙↖↓ 11	↙↓ 10	
<b>e</b>	4	↙ 3	← 4	↙↖ 5	← 6	← 7	↖↓ 8	↙↖↓ 9	↙↖↓ 10	↓ 9	
<b>t</b>	3	↙↖↓ 4	↙↖↓ 5	↙↖↓ 6	↙↖↓ 7	↙↖↓ 8	↙ 7	↖↓ 8	↙↖↓ 9	↓ 8	
<b>n</b>	2	↙↖↓ 3	↙↖↓ 4	↙↖↓ 5	↙↖↓ 6	↙↖↓ 7	↙↖↓ 8	↓ 7	↙↖↓ 8	↙ 7	
<b>i</b>	1	↙↖↓ 2	↙↖↓ 3	↙↖↓ 4	↙↖↓ 5	↙↖↓ 6	↙↖↓ 7	↙ 6	← 7	← 8	
<b>#</b>	<b>0</b>	1	2	3	4	5	6	7	8	9	
	<b>#</b>	<b>e</b>	<b>x</b>	<b>e</b>	<b>c</b>	<b>u</b>	<b>t</b>	<b>i</b>	<b>o</b>	<b>n</b>	

# Performance

- Time:  $O(nm)$
- Space:  $O(nm)$

# Weighted Edit Distance

- Why add weights to computation?
- Some letters are more likely to be mistyped than others
- Collect statistics on how often
  - one letter is substituted with another
  - on deletions
  - on insertions
  - on transpositions

# Confusion Matrix for Spelling Errors

sub[X, Y] = Substitution of X (incorrect) for Y (correct)

X	Y (correct)																									
	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
a	0	0	7	1	342	0	0	2	118	0	1	0	0	3	76	0	0	1	35	9	9	0	1	0	5	0
b	0	0	9	9	2	2	3	1	0	0	0	5	11	5	0	10	0	0	2	1	0	0	8	0	0	0
c	6	5	0	16	0	9	5	0	0	0	1	0	7	9	1	10	2	5	39	40	1	3	7	1	1	0
d	1	10	13	0	12	0	5	5	0	0	2	3	7	3	0	1	0	43	30	22	0	0	4	0	2	0
e	388	0	3	11	0	2	2	0	89	0	0	3	0	5	93	0	0	14	12	6	15	0	1	0	18	0
f	0	15	0	3	1	0	5	2	0	0	0	3	4	1	0	0	0	6	4	12	0	0	2	0	0	0
g	4	1	11	11	9	2	0	0	0	1	1	3	0	0	2	1	3	5	13	21	0	0	1	0	3	0
h	1	8	0	3	0	0	0	0	0	0	2	0	12	14	2	3	0	3	1	11	0	0	2	0	0	0
i	103	0	0	0	146	0	1	0	0	0	0	6	0	0	49	0	0	0	2	1	47	0	2	1	15	0
j	0	1	1	9	0	0	1	0	0	0	0	2	1	0	0	0	0	0	5	0	0	0	0	0	0	0
k	1	2	8	4	1	1	2	5	0	0	0	0	5	0	2	0	0	0	6	0	0	0	4	0	0	3
l	2	10	1	4	0	4	5	6	13	0	1	0	0	14	2	5	0	11	10	2	0	0	0	0	0	0
m	1	3	7	8	0	2	0	6	0	0	4	4	0	180	0	6	0	0	9	15	13	3	2	2	3	0
n	2	7	6	5	3	0	1	19	1	0	4	35	78	0	0	7	0	28	5	7	0	0	1	2	0	2
o	91	1	1	3	116	0	0	0	25	0	2	0	0	0	0	14	0	2	4	14	39	0	0	0	18	0
p	0	11	1	2	0	6	5	0	2	9	0	2	7	6	15	0	0	1	3	6	0	4	1	0	0	0
q	0	0	1	0	0	0	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
r	0	14	0	30	12	2	2	8	2	0	5	8	4	20	1	14	0	0	12	22	4	0	0	1	0	0
s	11	8	27	33	35	4	0	1	0	1	0	27	0	6	1	7	0	14	0	15	0	0	5	3	20	1
t	3	4	9	42	7	5	19	5	0	1	0	14	9	5	5	6	0	11	37	0	0	2	19	0	7	6
u	20	0	0	0	44	0	0	0	64	0	0	0	0	2	43	0	0	4	0	0	0	0	2	0	8	0
v	0	0	7	0	0	3	0	0	0	0	0	1	0	0	1	0	0	0	8	3	0	0	0	0	0	0
w	2	2	1	0	1	0	0	2	0	0	1	0	0	0	0	7	0	6	3	3	1	0	0	0	0	0
x	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0	0
y	0	0	2	0	15	0	1	7	15	0	0	0	2	0	6	1	0	7	36	8	5	0	0	1	0	0
z	0	0	0	7	0	0	0	0	0	0	0	7	5	0	0	0	0	2	21	3	0	0	0	0	3	0

# Confusion Matrix for Spelling Errors

**sub[X, Y] = Sub**

X											v	w	x	y	z
	a	b	c	d	e	f	g	h	i		0	1	0	5	0
a	0	0	7	1	342	0	0	2	118		0	8	0	0	0
b	0	0	9	9	2	2	3	1	0		3	7	1	1	0
c	6	5	0	16	0	9	5	0	0		0	4	0	2	0
d	1	10	13	0	12	0	5	5	0		0	2	0	18	0
e	388	0	3	11	0	2	2	0	89		0	1	0	0	0
f	0	15	0	3	1	0	5	2	0		0	2	0	3	0
g	4	1	11	11	9	2	0	0	0		0	0	0	0	0
h	1	8	0	3	0	0	0	0	0		0	5	3	20	1
w	2	2	1	0	1	0	0	2	0	0	1	0	0	0	0
x	0	0	0	2	0	0	0	0	0	0	0	0	0	9	0
y	0	0	2	0	15	0	1	7	15	0	0	2	0	6	1
z	0	0	0	7	0	0	0	0	0	0	7	5	0	0	0

# Weighted Min Edit Distance

- Initialization

$$D(0,0) = 0$$

$$D(i,0) = D(i-1,0) + \text{del}[x(i)] \quad 1 < i \leq n$$

$$D(0,j) = D(0,j-1) + \text{ins}[y(j)] \quad 1 < j \leq m$$

- Recurrence Relation

$$D(i,j) = \min \begin{cases} D(i-1,j) + \text{del}[x(i)] \\ D(i,j-1) + \text{ins}[y(j)] \\ D(i-1,j-1) + \text{sub}[x(i),y(j)] \end{cases}$$

- Termination

$D(n,m)$  is distance

# Noisy Channel Model: Intuition

send  $\mathbf{w}$   receive  $\mathbf{x}$

- We see an observation  $\mathbf{x}$  of a misspelled word  $\mathbf{w}$
- Decode the correct word

$$\hat{\mathbf{w}} = \underset{\mathbf{w} \in \mathbf{V}}{\operatorname{argmax}} \mathbf{P}(\mathbf{w} \mid \mathbf{x})$$

best guess

prob of  $\mathbf{w}$  given misspelled  $\mathbf{x}$

search over vocabulary  $\mathbf{V}$



# How to Model $P(\mathbf{w} | \mathbf{x})$ ?

- $P(\mathbf{w} | \mathbf{x})$  is not easy to model

send  $\mathbf{w}$   receive  $\mathbf{x}$

- But  $P(\mathbf{x} | \mathbf{w})$  is easier to model
- Can study the properties of noisy channel
  - how often one intends to write  $\mathbf{w}$  but writes  $\mathbf{x}$  instead?
  - example: how often want to write “the” but write “hta”

# Noisy Channel

- Since  $P(\mathbf{x} | \mathbf{w})$  is easier to model, rewrite:

$$\hat{\mathbf{w}} = \operatorname{argmax}_{\mathbf{w} \in V} P(\mathbf{w} | \mathbf{x})$$

$$= \operatorname{argmax}_{\mathbf{w} \in V} \frac{P(\mathbf{x} | \mathbf{w})P(\mathbf{w})}{P(\mathbf{x})}$$

$$= \operatorname{argmax}_{\mathbf{w} \in V} \underbrace{P(\mathbf{x} | \mathbf{w})}_{\substack{\text{noisy} \\ \text{channel} \\ \text{model}}} \underbrace{P(\mathbf{w})}_{\substack{\text{language} \\ \text{model}}}$$

# How to Model $P(\mathbf{w})$ ?

- Use one of the language models we have learned
- Unigram probabilities if  $\mathbf{w}$  is just a word
- If modeling error in context, use bigram or trigram model
  - Sentence: Like ther books
  - Non-word = ther
  - Both there and their have edit distance of 1 to ther
  - Both  $P(\text{their})$  and  $P(\text{there})$ , have high probability
  - Add context:  $P(\text{like there books})$  vs.  $P(\text{like their books})$
- With smoothing if necessary
  - Add-Delta, Good-Turing, etc

# Modeling Noisy Channel $P(\mathbf{x}|\mathbf{w})$

- Also called error probability model
- Build model from **marked** training data, much like for modeling language
  - need texts with marked mistakes/corrections
- Assume one mistake per word for simplicity
  - Misspelled word  $\mathbf{x}$  has characters  $\mathbf{x}_1 \mathbf{x}_2 \dots \mathbf{x}_m$
  - Dictionary word  $\mathbf{w}$  has characters  $\mathbf{w}_1 \mathbf{w}_2 \dots \mathbf{w}_n$
  - Can get from  $\mathbf{w}$  to  $\mathbf{x}$  with either one insertion, or one deletion, etc
- $P(\mathbf{x}|\mathbf{w})$  is probability of this one mistake

# Modeling Noisy Channel $P(x|w)$

- Get counts from marked data
- Here  $y$  and  $z$  are characters

`del[z,y]:`      `count(zy typed as z)`

`ins[z,y]:`      `count(z typed as zy)`

`sub[z,y]:`      `count(z typed as y)`

`trans[z,y]:`    `count(zy typed as yz)`

- Generate confusion matrix from marked data
  - Peter Norvig, etc.

# Confusion Matrix for Modeling $P(x|w)$

**sub[X, Y] = Substitution of X (incorrect) for Y (correct)**

X	Y (correct)																									
	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
a	0	0	7	1	342	0	0	2	118	0	1	0	0	3	76	0	0	1	35	9	9	0	1	0	5	0
b	0	0	9	9	2	2	3	1	0	0	0	5	11	5	0	10	0	0	2	1	0	0	8	0	0	0
c	6	5	0	16	0	9	5	0	0	0	1	0	7	9	1	10	2	5	39	40	1	3	7	1	1	0
d	1	10	13	0	12	0	5	5	0	0	2	3	7	3	0	1	0	43	30	22	0	0	4	0	2	0
e	388	0	3	11	0	2	2	0	89	0	0	3	0	5	93	0	0	14	12	6	15	0	1	0	18	0
f	0	15	0	3	1	0	5	2	0	0	0	3	4	1	0	0	0	6	4	12	0	0	2	0	0	0
g	4	1	11	11	9	2	0	0	0	1	1	3	0	0	2	1	3	5	13	21	0	0	1	0	3	0
h	1	8	0	3	0	0	0	0	0	0	2	0	12	14	2	3	0	3	1	11	0	0	2	0	0	0
i	103	0	0	0	146	0	1	0	0	0	0	6	0	0	49	0	0	0	2	1	47	0	2	1	15	0
j	0	1	1	9	0	0	1	0	0	0	0	2	1	0	0	0	0	0	5	0	0	0	0	0	0	0
k	1	2	8	4	1	1	2	5	0	0	0	0	5	0	2	0	0	0	6	0	0	0	4	0	0	3
l	2	10	1	4	0	4	5	6	13	0	1	0	0	14	2	5	0	11	10	2	0	0	0	0	0	0
m	1	3	7	8	0	2	0	6	0	0	4	4	0	180	0	6	0	0	9	15	13	3	2	2	3	0
n	2	7	6	5	3	0	1	19	1	0	4	35	78	0	0	7	0	28	5	7	0	0	1	2	0	2
o	91	1	1	3	116	0	0	0	25	0	2	0	0	0	0	14	0	2	4	14	39	0	0	0	18	0
p	0	11	1	2	0	6	5	0	2	9	0	2	7	6	15	0	0	1	3	6	0	4	1	0	0	0
q	0	0	1	0	0	0	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
r	0	14	0	30	12	2	2	8	2	0	5	8	4	20	1	14	0	0	12	22	4	0	0	1	0	0
s	11	8	27	33	35	4	0	1	0	1	0	27	0	6	1	7	0	14	0	15	0	0	5	3	20	1
t	3	4	9	42	7	5	19	5	0	1	0	14	9	5	5	6	0	11	37	0	0	2	19	0	7	6
u	20	0	0	0	44	0	0	0	64	0	0	0	0	2	43	0	0	4	0	0	0	0	2	0	8	0
v	0	0	7	0	0	3	0	0	0	0	0	1	0	0	1	0	0	0	8	3	0	0	0	0	0	0
w	2	2	1	0	1	0	0	2	0	0	1	0	0	0	0	7	0	6	3	3	1	0	0	0	0	0
x	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0	0
y	0	0	2	0	15	0	1	7	15	0	0	0	2	0	6	1	0	7	36	8	5	0	0	1	0	0
z	0	0	0	7	0	0	0	0	0	0	0	7	5	0	0	0	0	2	21	3	0	0	0	0	3	0

# Modeling Noisy Channel $P(\mathbf{x}|\mathbf{w})$

- Assume can get from  $\mathbf{x}$  to  $\mathbf{w}$  with one error

$$P(\mathbf{x}|\mathbf{w}) = \begin{cases} \frac{\text{del}[\mathbf{w}_{i-1}, \mathbf{w}_i]}{\text{count}[\mathbf{w}_{i-1}, \mathbf{w}_i]} & \text{if deletion} \\ \frac{\text{ins}[\mathbf{w}_{i-1}, \mathbf{x}_i]}{\text{count}[\mathbf{w}_{i-1}]} & \text{if insertion} \\ \frac{\text{sub}[\mathbf{x}_i, \mathbf{w}_i]}{\text{count}[\mathbf{w}_i]} & \text{if substitution} \\ \frac{\text{trans}[\mathbf{w}_{i-1}, \mathbf{w}_i]}{\text{count}[\mathbf{w}_{i-1}, \mathbf{w}_i]} & \text{if transposition} \end{cases}$$

# Non-Word Spelling Error Example

**"acress"**



# Non-Word Spelling Error Example

**"acress"**

- 80% of errors are within edit distance 1
- Almost all errors within edit distance 2
- Can also allow insertion of **space** or **hyphen**

`thisidea` → `this idea`

`inlaw` → `in-law`

# Words within distance 1 of **acress**

Error	Candidate Correction	Correct Letter	Error Letter	Type
acress	actress	t	-	deletion
acress	creess	-	a	insertion
acress	caress	ca	ac	transposition
acress	access	c	r	substitution
acress	across	o	e	substitution
acress	acres	-	s	insertion
acress	acres	-	s	insertion

# Unigram Probability $P(w)$

word	Frequency of word	$P(\text{word})$
actress	9,321	.0000230573
cress	220	.0000005442
caress	686	.0000016969
access	37,038	.0000916207
across	<b>120,844</b>	<b>.0002989314</b>
acres	12,874	.0000318463

- Counts from 404,253,213 words in Corpus of Contemporary English (COCA)

# Noisy Channel Model $P(x|w)$ for `acress`

Candidate Correction	Correct Letter	Error Letter	$x w$	$P(x \text{word})$
<code>actress</code>	<code>t</code>	<code>-</code>	<code>c ct</code>	<b>.000117</b>
<code>cress</code>	<code>-</code>	<code>a</code>	<code>a #</code>	.00000144
<code>caress</code>	<code>ca</code>	<code>ac</code>	<code>ac ca</code>	.00000164
<code>access</code>	<code>c</code>	<code>r</code>	<code>r c</code>	.000000209
<code>across</code>	<code>o</code>	<code>e</code>	<code>e o</code>	.00000093
<code>acres</code>	<code>-</code>	<code>s</code>	<code>es e</code>	.00000321
<code>acres</code>	<code>-</code>	<code>s</code>	<code>ss s</code>	.00000342

# Combining $P(w)$ and $P(w|x)$ for `acress`

Candidate Correction	Correct Letter	Error Letter	$x w$	$P(x word)$	$P(word)$	$10^9 * P(x w)P(w)$
<code>actress</code>	<code>t</code>	-	<code>c ct</code>	.000117	.0000231	<b>2.7</b>
<code>acress</code>	-	<code>a</code>	<code>a #</code>	.00000144	.000000544	<b>0.00078</b>
<code>caress</code>	<code>ca</code>	<code>ac</code>	<code>ac ca</code>	.00000164	.00000170	<b>0.0028</b>
<code>access</code>	<code>c</code>	<code>r</code>	<code>r c</code>	.000000209	.0000916	<b>0.019</b>
<b><code>across</code></b>	<b><code>o</code></b>	<b><code>e</code></b>	<b><code>e o</code></b>	<b>.0000093</b>	<b>.000299</b>	<b>2.8</b>
<code>acres</code>	-	<code>s</code>	<code>es e</code>	.0000321	.0000318	<b>1.0</b>
<code>acres</code>	-	<code>s</code>	<code>ss s</code>	.0000342	.0000318	<b>1.0</b>

# Adding Context

"a stellar and versatile actress whose combination of sass and glamour"

- Adding context to misspelled word really helps
- Use bigram language model
- Counts from CCAE with add-1 smoothing

$$P(\text{actress} | \text{versatile}) = .000021$$

$$P(\text{whose} | \text{actress}) = .0010$$

$$P(\text{versatile actress whose}) = .000021 * .0010 = 210 \times 10^{-10}$$

$$P(\text{across} | \text{versatile}) = .000021$$

$$P(\text{whose} | \text{across}) = .000006$$

$$P(\text{versatile across whose}) = .000021 * .000006 = 1 \times 10^{-10}$$

# Using Bigram Language Model

- Now multiply by noisy channel probabilities

$$P(\text{acress} \mid \text{actress}) = .000117$$

$$P(\text{acress} \mid \text{across}) = .0000093$$

- To finally get

$$\begin{aligned} P(\text{versatile actress whose})P(\text{acress} \mid \text{actress}) &= .000117 \times 210 \times 10^{-10} \\ &= .002457 \times 10^{-10} \end{aligned}$$

$$\begin{aligned} P(\text{versatile across whose})P(\text{acress} \mid \text{across}) &= .0000093 \times 1 \times 10^{-10} \\ &= .0000093 \times 10^{-10} \end{aligned}$$

# Real-Word Spelling Errors

leaving in about fifteen **minuets** to go to her house.

The design **an** construction of the system.

Can they **lave** him my messages?

The study was conducted mainly **be** John Black.

- Spelling errors that are words in dictionary
- 25-40% of spelling errors are real words
  - from [Kukich 1992]
- Use context of in this case



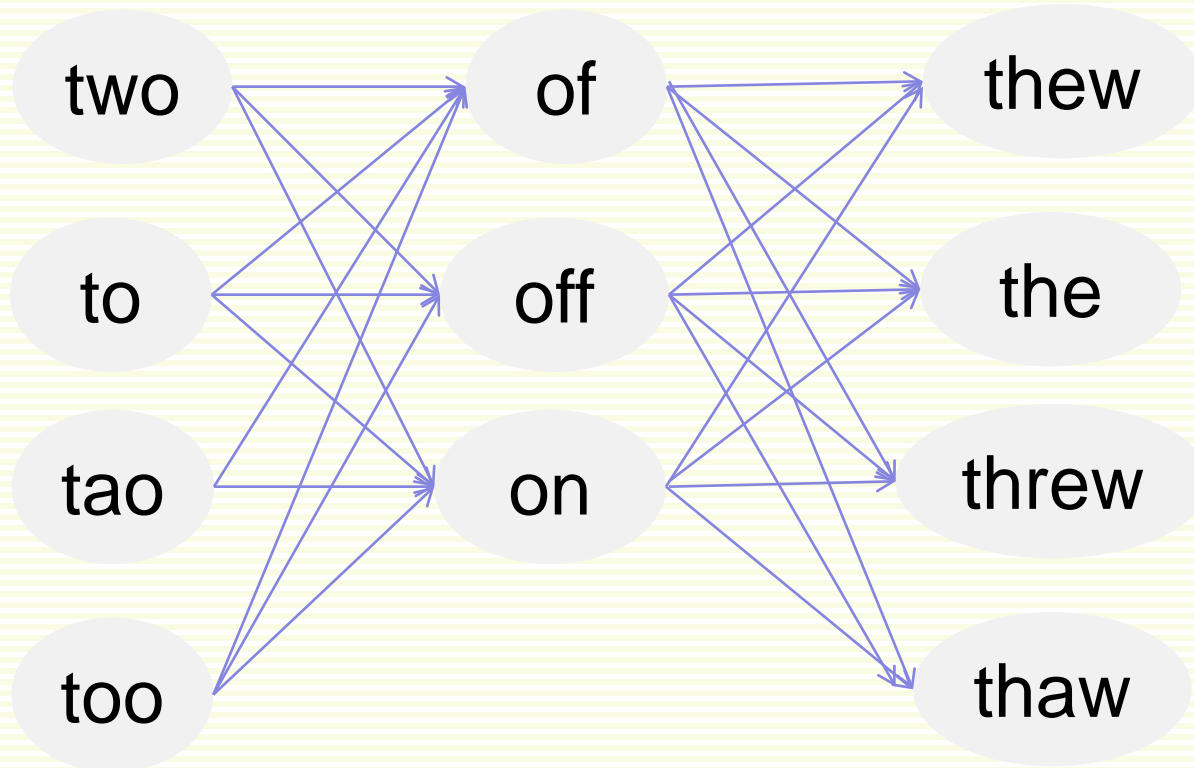
# Solving Real-World Spelling Errors

- For each word **w** in the sentence
  - Generate *candidate set*
    - the word itself
    - all single-letter edits that are English words
    - homophone words (similar pronunciations)
- Choose best combination word candidates
  - noisy channel model
  - task-specific classifier

# Example of Real-Word Spell Correction [P.Norvig]

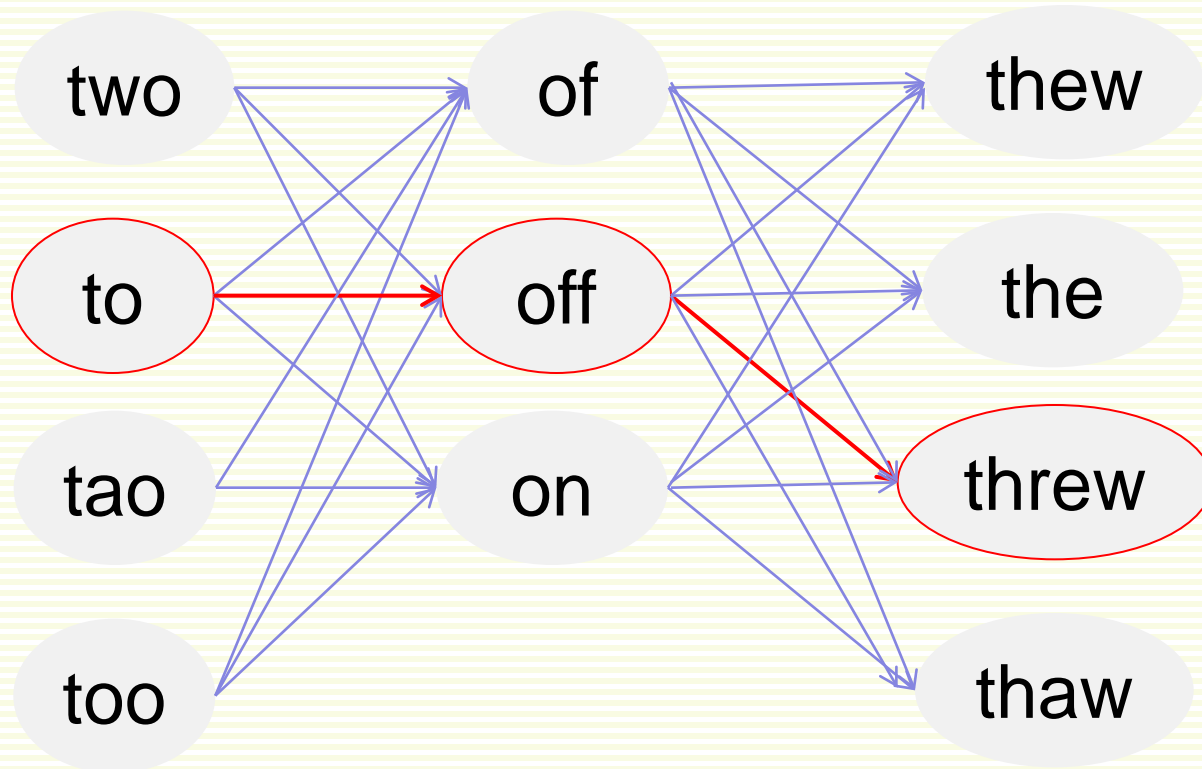
- Example: “two of thew”
  - Candidate (two) = {two, to, tao, too}
  - Candidate (of) = {of, off, on}
  - Candidate (thew) = {thew, the, threw, thaw}

# Graph of all Possible Candidate Combinations



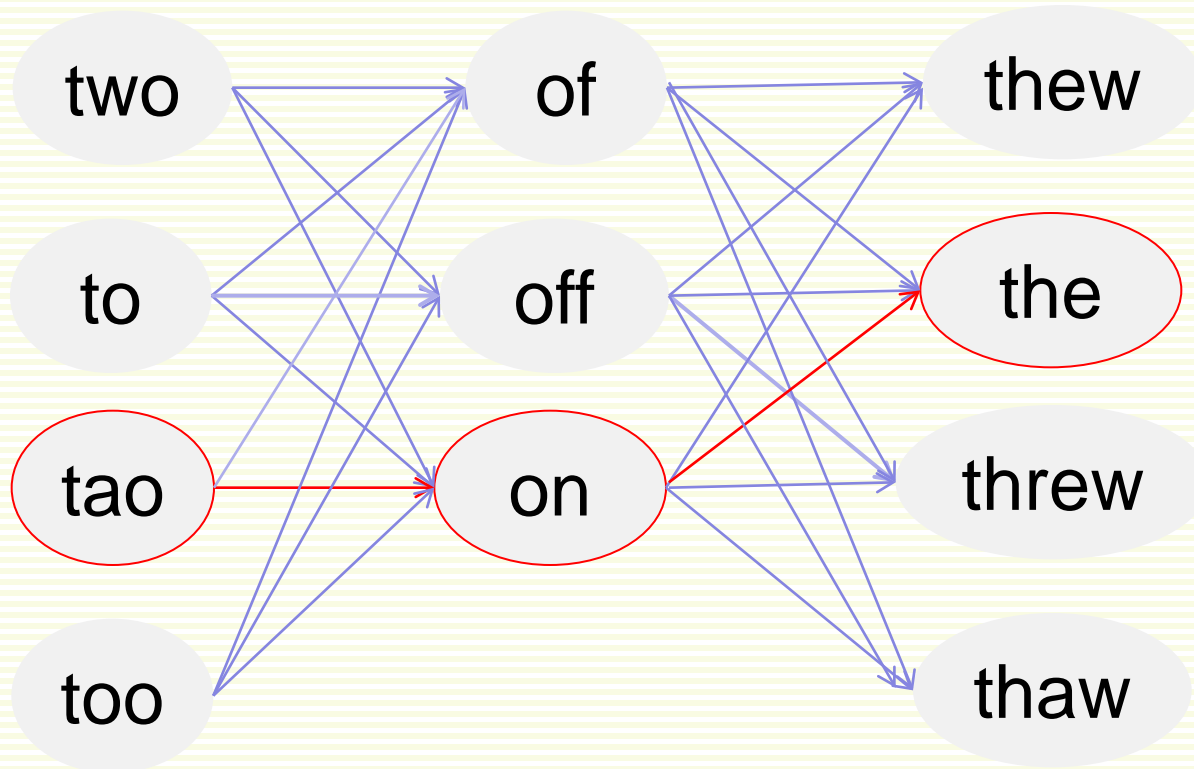
- Each path corresponds to a possible sentence

# Graph of all Possible Candidate Combinations



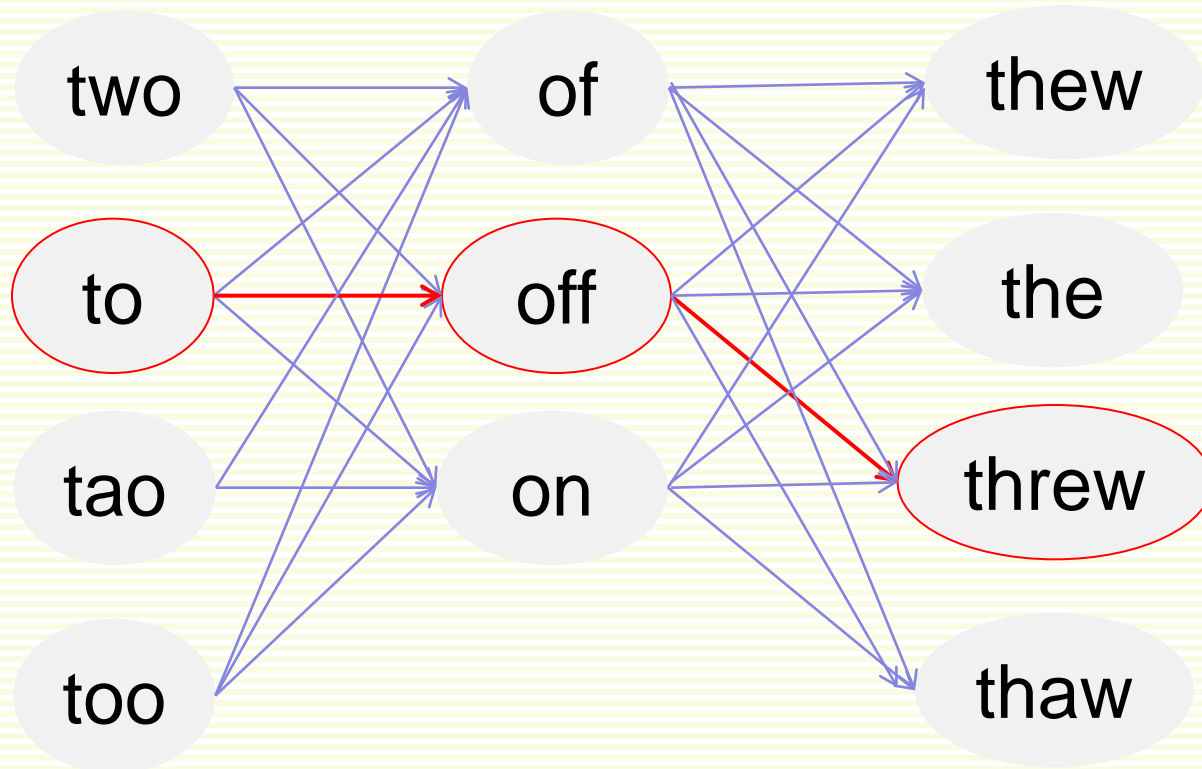
- One possible path (sentence)

# Graph of all Possible Candidate Combinations



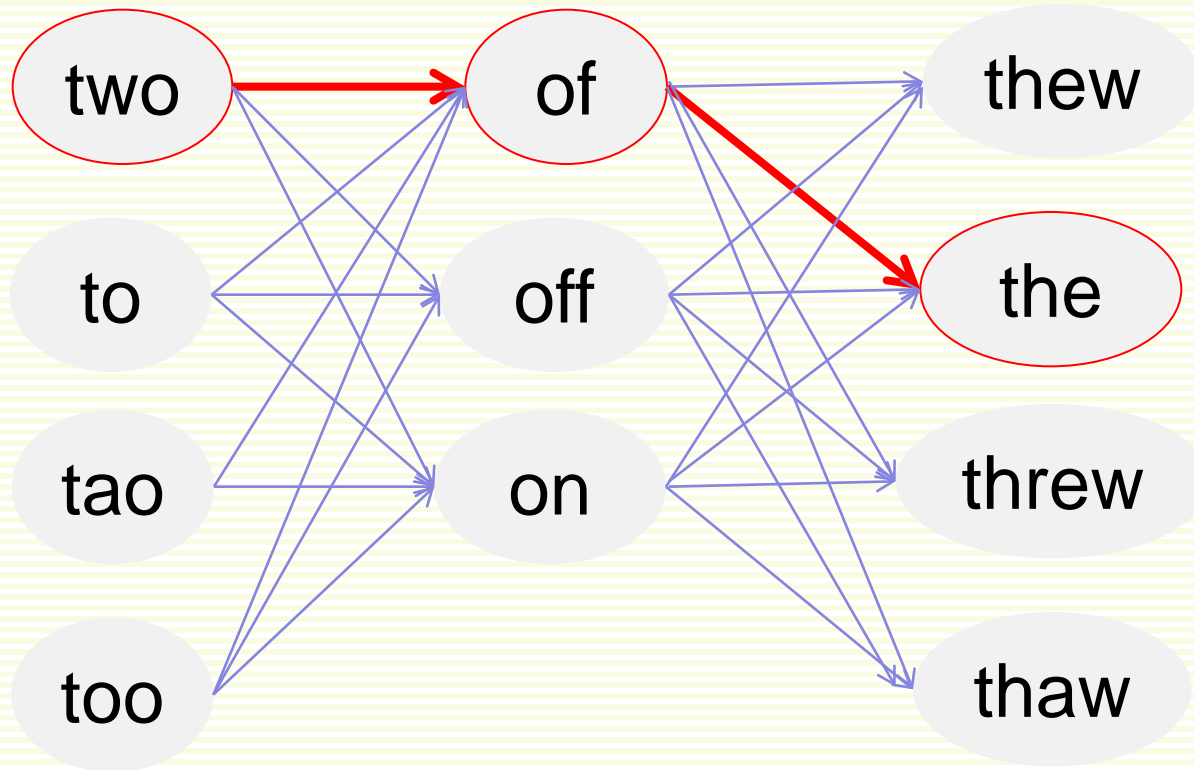
- Another possible path (sentence)

# Graph of all Possible Candidate Combinations



- Compute probability of each path (sentence)
- Choose the path (sentence) of highest probability

# Noisy Channel for Real-Word Spell Correction



- The best probability path (sentence), hopefully
- Can use DP (dynamic programming) to speed up computation

# Simplification: One Error per Sentence

- Or simplify to allow only one word to be replaced
  - assumes one error per sentence
- Example: “two of thew”
  - $C(\text{two}) = \{\text{two, to, tao, too}\}$
  - $C(\text{of}) = \{\text{of, off, on}\}$
  - $C(\text{thew}) = \{\text{thew, the, threw, thaw}\}$
- Check only 11 sentences:

Two of threw

Two of thew

Two of thew

To of thew

Two off thew

Two of the

Tao of thew

Two on thew

Two of threw

Too of thew

Two of thaw



# Where to Get Probabilities

- Language model  $\mathbf{P}(\mathbf{w})$ 
  - Same as before
    - Bigram or trigram
      - should not use unigram as it gives no context
- Noisy Channel Model  $\mathbf{P}(\mathbf{w} | \mathbf{x})$ 
  - same as for non-word spelling correction
  - plus need probability for no error,  $\mathbf{P}(\mathbf{w} | \mathbf{w})$
- Choose sentence that maximizes  $\mathbf{P}(\mathbf{w} | \mathbf{x})\mathbf{P}(\mathbf{w})$

# Probability of No Error

- Noisy channel probability for correctly typed word?
  - $P(\text{the}|\text{the})$
  - i.e. probability of no error
- Depends on the application
  - .90 (1 error in 10 words)
    - *for Olga*
  - .95 (1 error in 20 words)
  - .99 (1 error in 100 words)
  - .995 (1 error in 200 words)
    - *for English literature major*

# Peter Norvig's Example Continued

x	w	x w	P(x w)	P(w)	$10^9 P(x w)P(w)$
thew	the	ew e	0.000007	0.02	144
thew	thew		0.95	0.000000009	90
thew	thaw	e a	0.001	0.00000007	0.7
thew	threw	h hr	0.000008	0.0000004	0.03
thew	thwe	ew we	0.000003	0.000000004	0.0001

# HCI issues in spelling

- Autocorrect, f very confident in correction
  - Very common mistake: hte → the
- Less confident
  - Give one best correction
- Less confident
  - Give a correction list
- Unconfident
  - Just flag as an error

# State of the art noisy channel

- Never just multiply the prior and the error model
- Independence assumptions  $\rightarrow$  probabilities not commensurate
- Instead: weigh them

$$\hat{\mathbf{w}} = \operatorname{argmax}_{\mathbf{w} \in \mathcal{V}} \mathbf{P}(\mathbf{x} | \mathbf{w}) \mathbf{P}(\mathbf{w})^{\lambda}$$

- Learn  $\lambda$  from a validation set

# Classifier for Real-Word Spelling Correction

- Instead of just channel model and language model
- Use many features in a classifier
- Build a classifier for a specific pair like:

whether/weather

- “cloudy” within +/- 10 words
- \_\_\_ to VERB
- \_\_\_ or not

# Evaluation

- Some spelling error test sets
  - Wikipedia's list of common English misspelling
  - Aspell filtered version of that list
  - Birkbeck spelling error corpus
  - Peter Norvig's list of errors (includes Wikipedia and Birkbeck, for training or testing)