CS4442/9542b
Artificial Intelligence II
prof. Olga Veksler

Lecture 16

Computer Vision

Stereo

Some slides are from S. Seitz, S. Narasimhan, K. Grauman

Outline

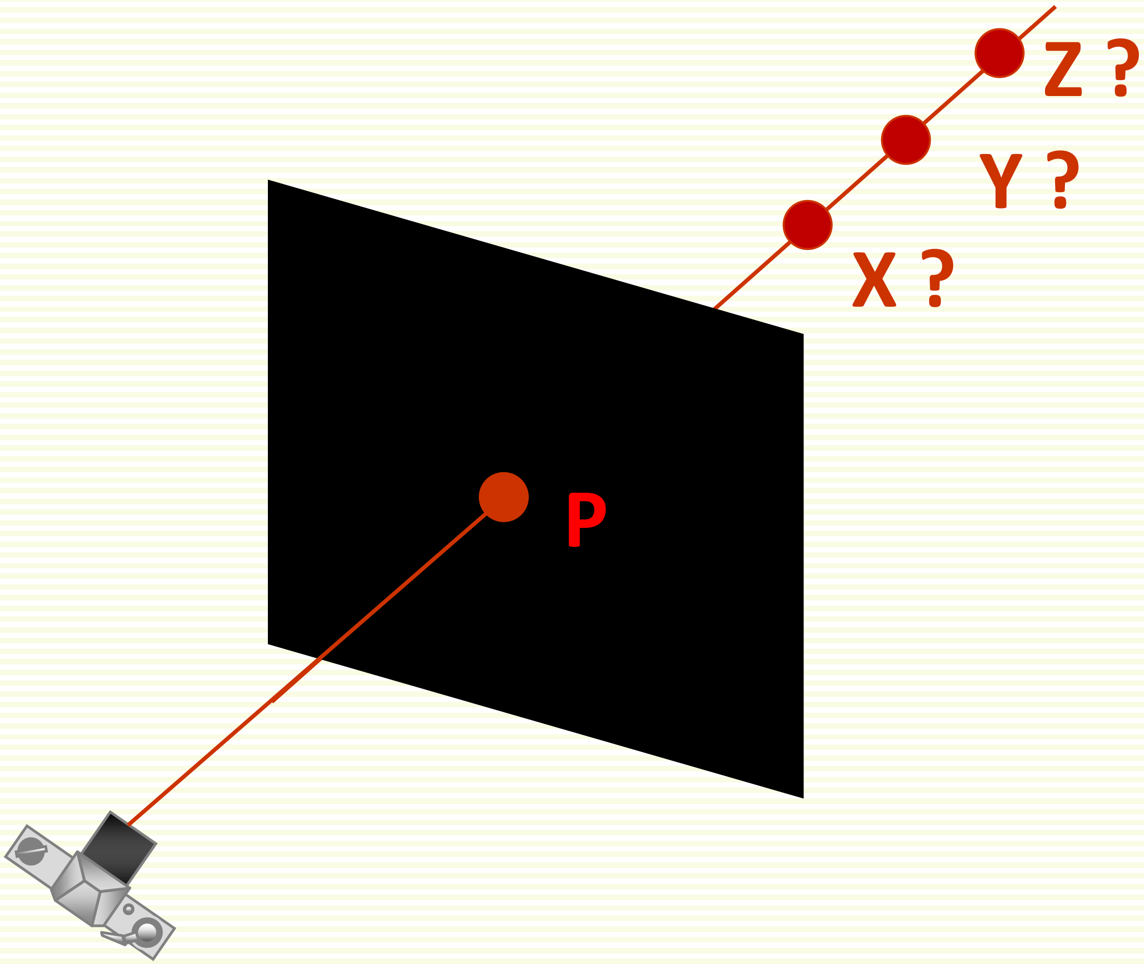
- Cues for 3D reconstruction
- Stereo Cues
- Stereo Reconstruction
    1) camera calibration and rectification
        - an easier, mostly solved problem
    2) stereo correspondence
        - a harder problem

2D Images

- World is 3D
- In 2D images, depth (the third coordinate) is largely lost
  - includes human retina

2D Images

- Depth is inherently ambiguous from a single view



Street Pavement Art

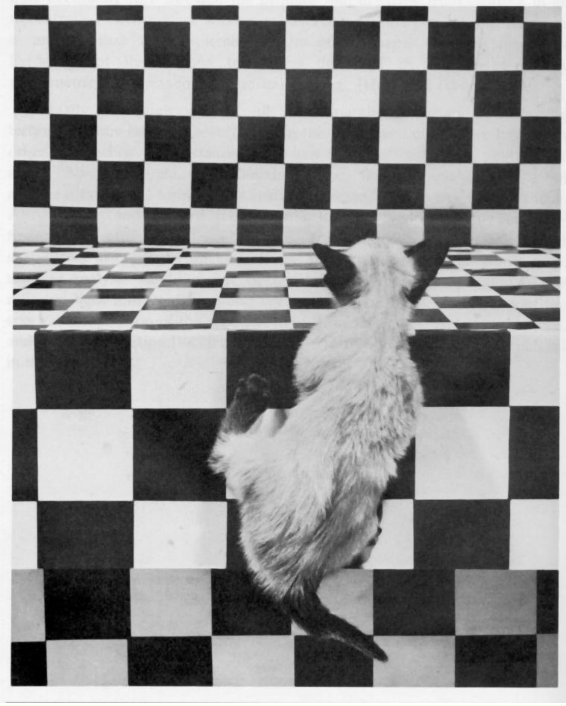
- Viewed from the “right” side

Street Pavement Art

- Viewed from the “wrong” side

Babies and Animals Perceive Depth

- Yet we perceive the world in 3D



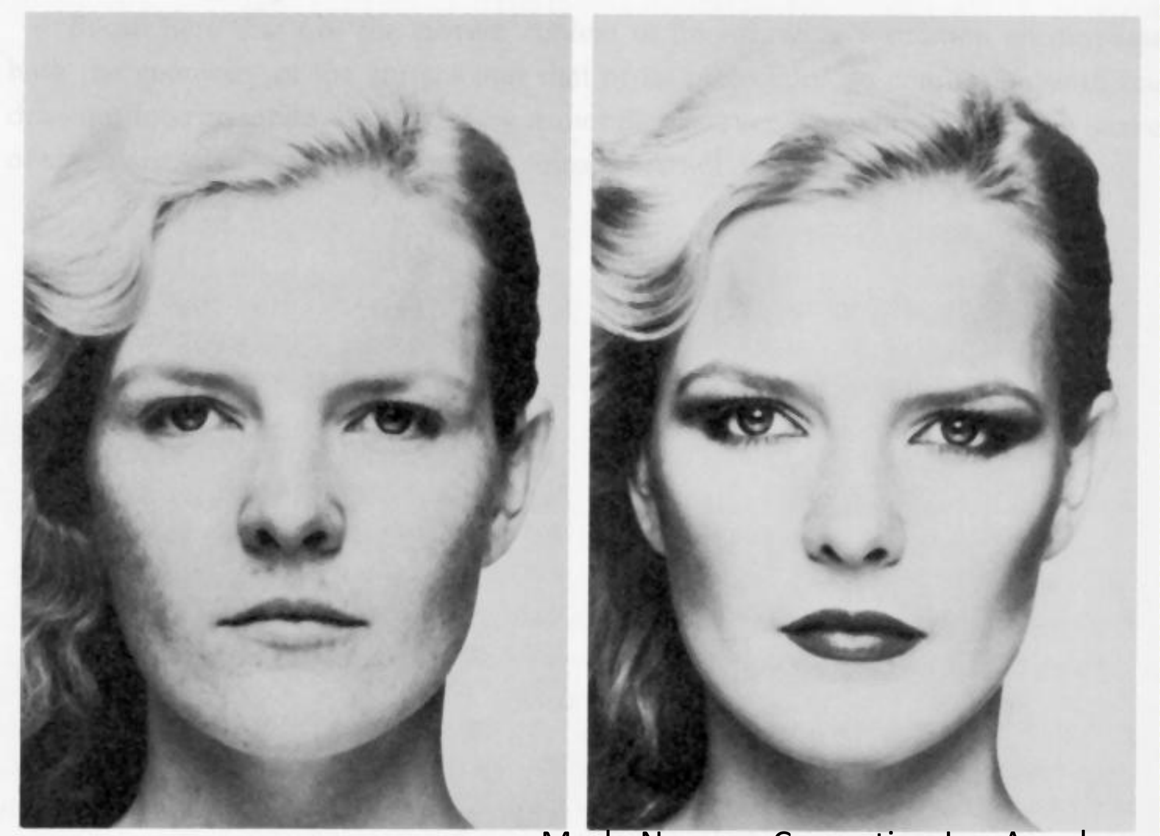
The Visual Cliff, by William Vandivert, 1960

3D Shape from Images

- What image cues provide 3D information?
- Cues from a single image
- Cues from multiple images
    - Motion cues
    - Stereo cues
- Can we use these cues in a computer vision system?

Single Image 3D Cues: Shading

- Pixels covered by shadow are perceived to be further away



Merle Norman Cosmetics, Los Angeles

Single Image 3D Cues: Linear Perspective

- The further away are parallel lines, the closer they come together

Single Image 3D Cues: Relative Size

- If objects have the same size, those further away appear smaller

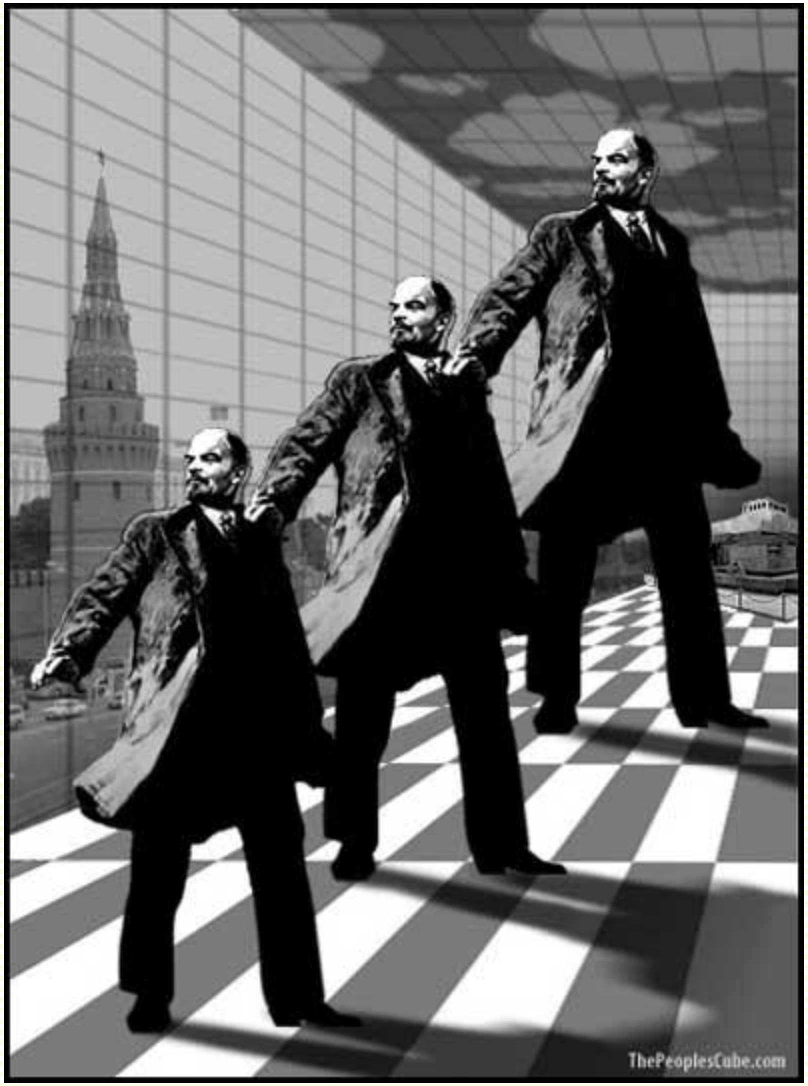
Single Image 3D Cues: Texture

- Further away texture appears finer (smaller scale)

Single Image 3D Cues: Known Size

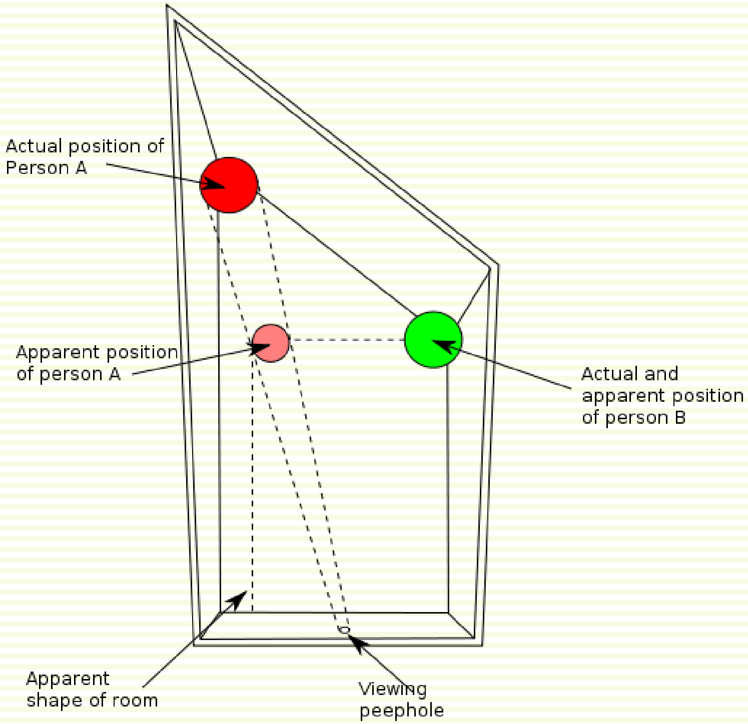
- Ducks are smaller than elephants, duck is closer

Illusions: Linear Perspective + Relative Size



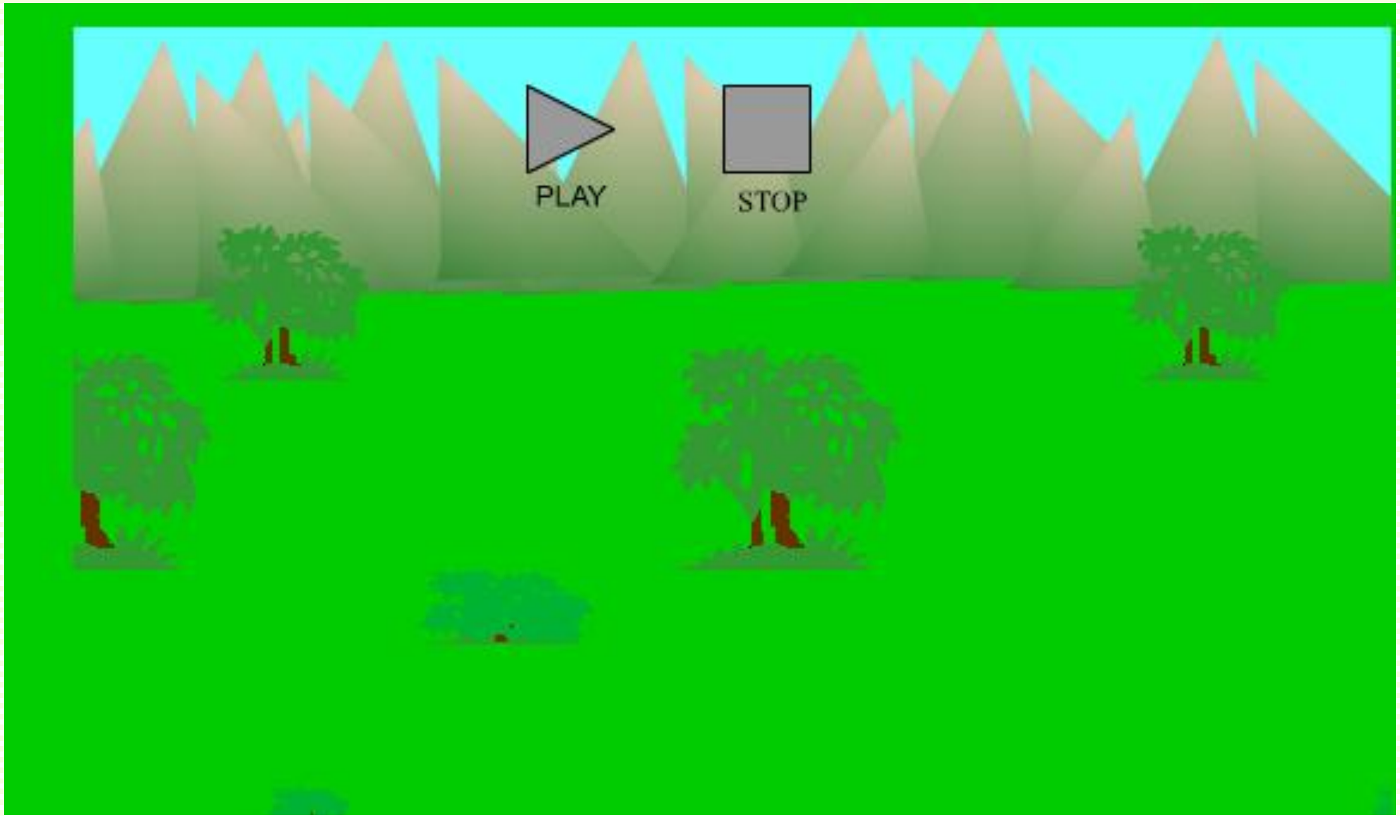
Illusions: Linear Perspective + Relative Size

Illusions: Ames Room



Cues from Multiple Image: Motion Parallax

- Closer objects appear to move more than further away objects



http://psych.hanover.edu/KRANTZ/MotionParallax/MotionParallax.html

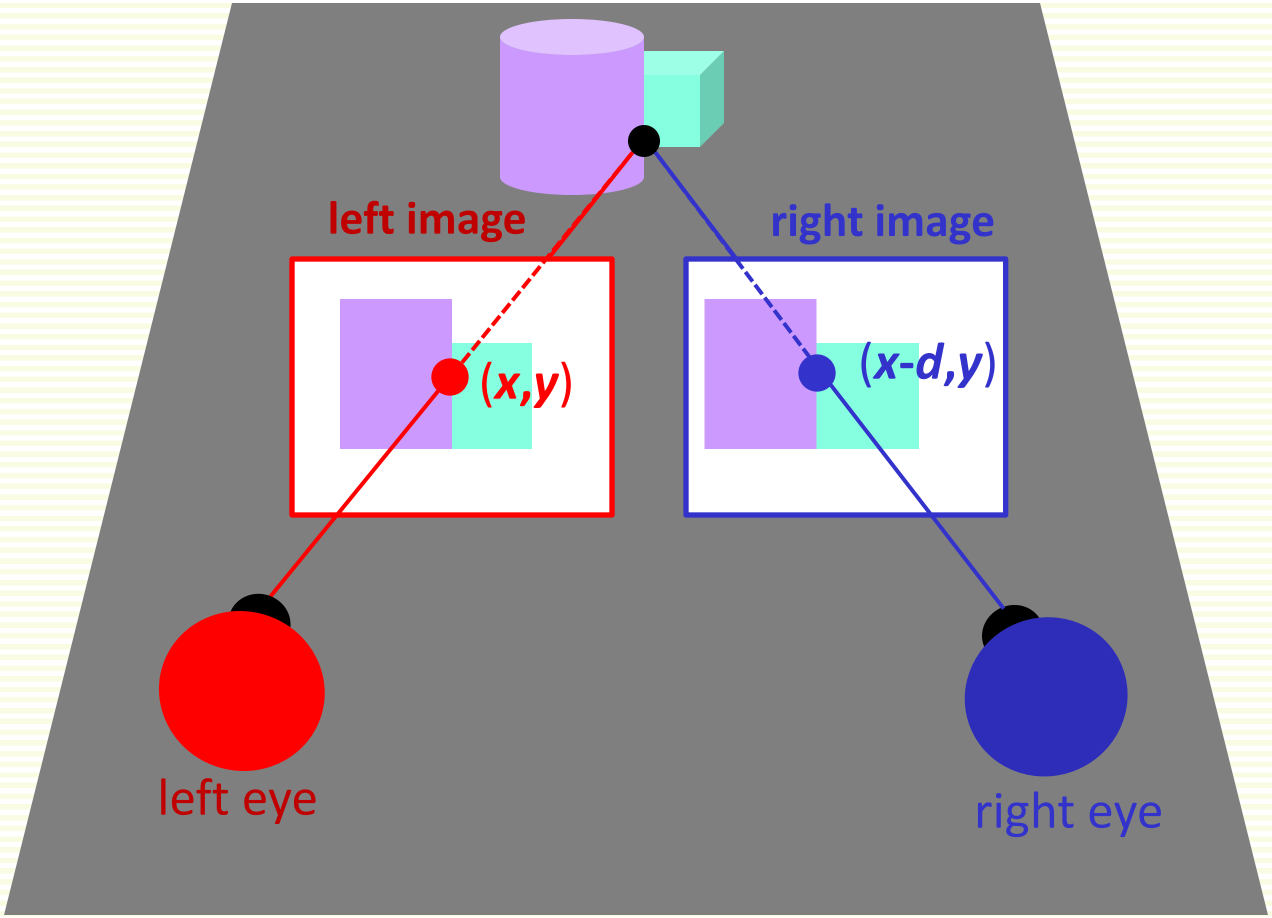
3D Shape from **X**

- **X** = shading, texture, motion, ...
- We will focus on **stereo**
    - depth perception from two **stereo images**

Why Two Eyes? Cylopes?

Why Two Eyes?

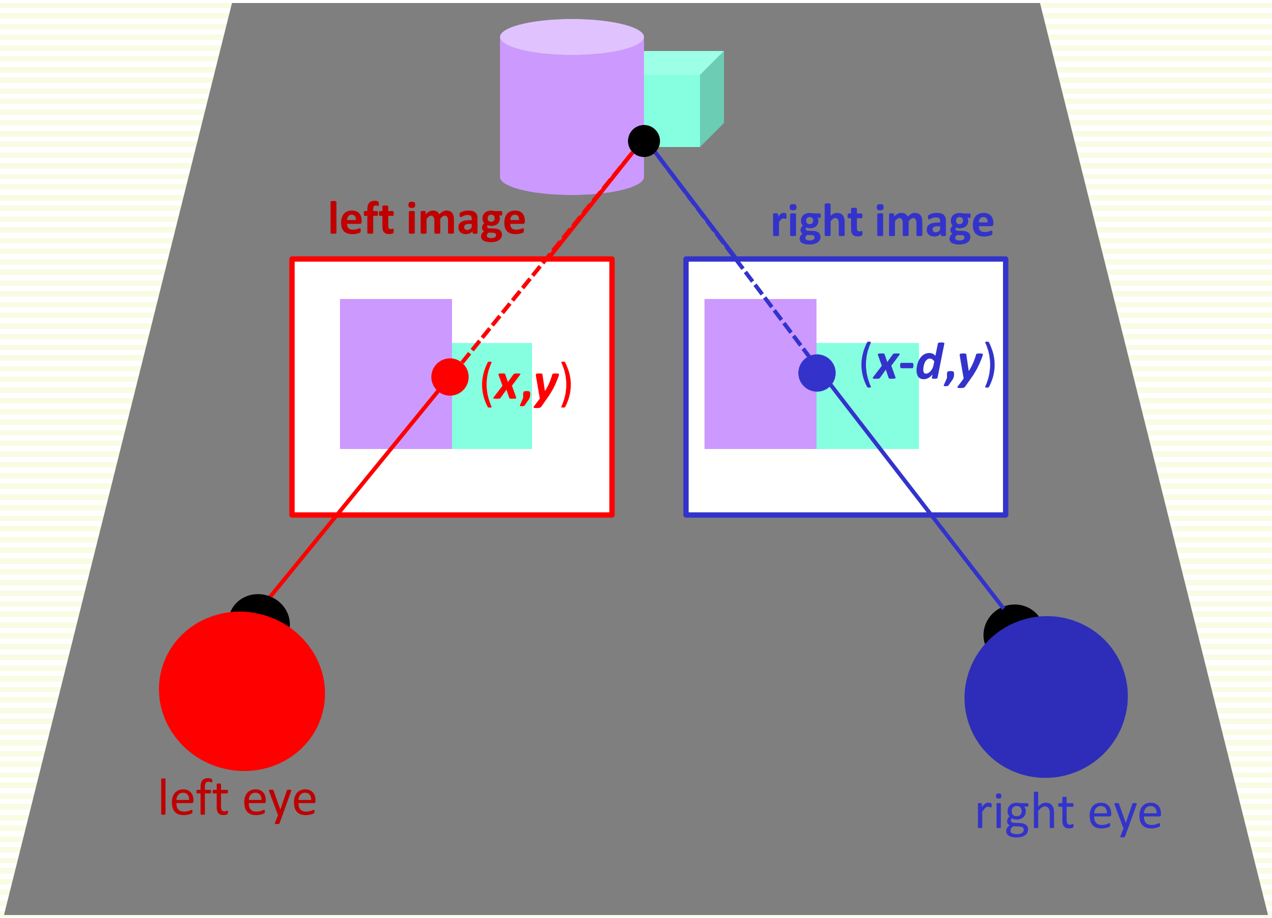
- Charles Wheatstone first explained stereopsis in 1838



3D Scene

Why Two Eyes?

- **Disparity** ***d*** is the difference in **x** coordinates of corresponding points



3D Scene

Stereoscopes

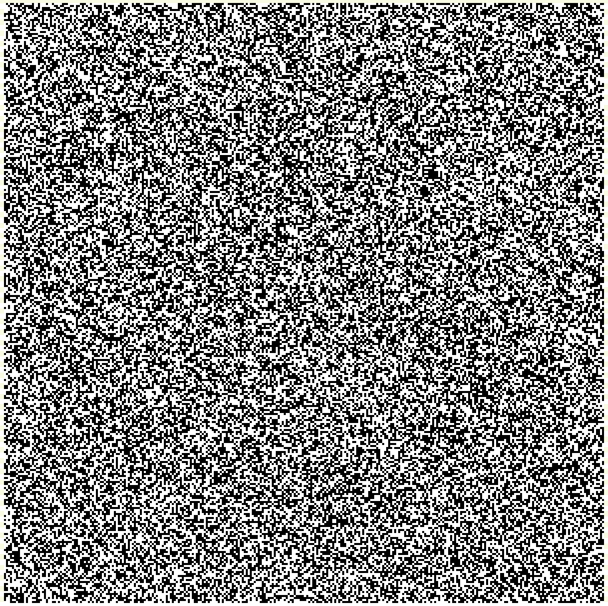
- Wheatstone invented the first stereoscope

Anaglyph Images

- Encodes left and right image into a single picture
  - left eye image is transferred to the red channel
  - right eye image to the green+blue = cyan channel
- Red filter lets through only the left image
- Cyan filter lets through only theright eye image
- Brain fuses into 3D
- Similar technology for 3D movies
- Works for most of us

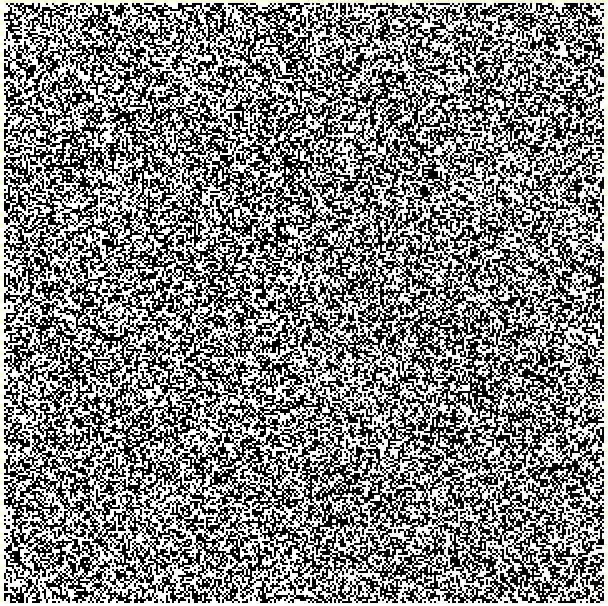
What is Needed for Stereopsis?

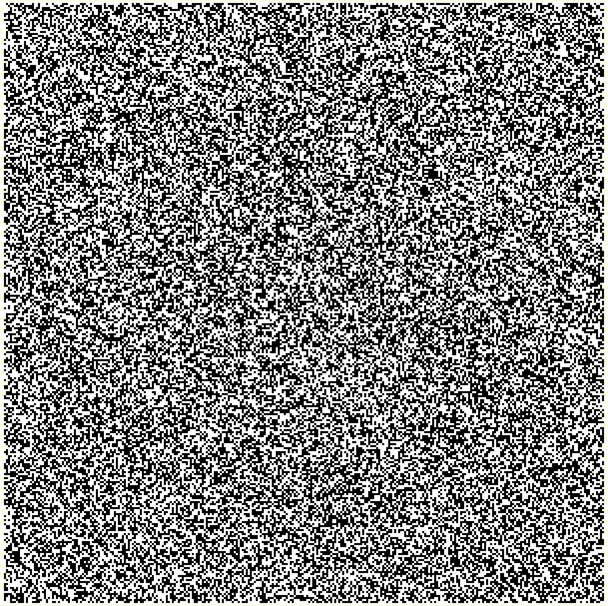
- Need monocular cues for stereopsis? Need object cues? Answered by Julesz in 1960
- Image with no monocular cues and no recognizable objects: random dots



Need Object Recognition for Stereopsis?

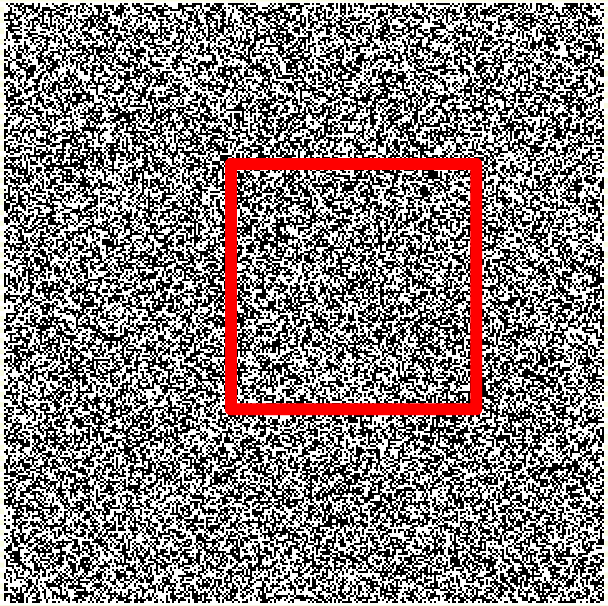
- Answered by Julesz in 1960
- Make a copy of it

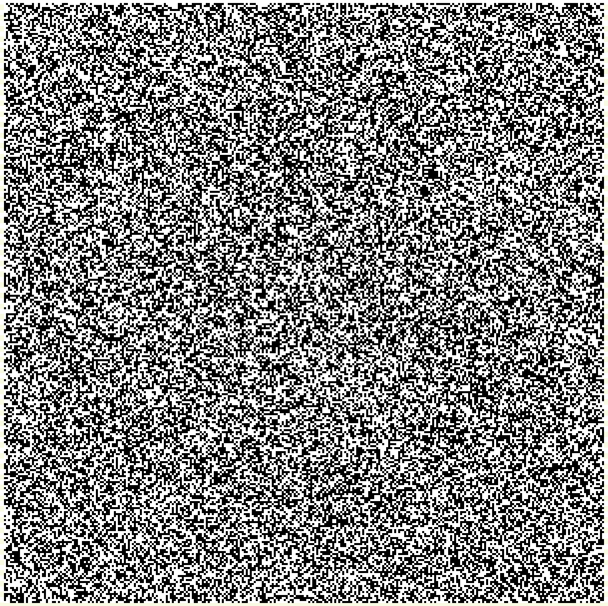




Need Object Recognition for Stereopsis?

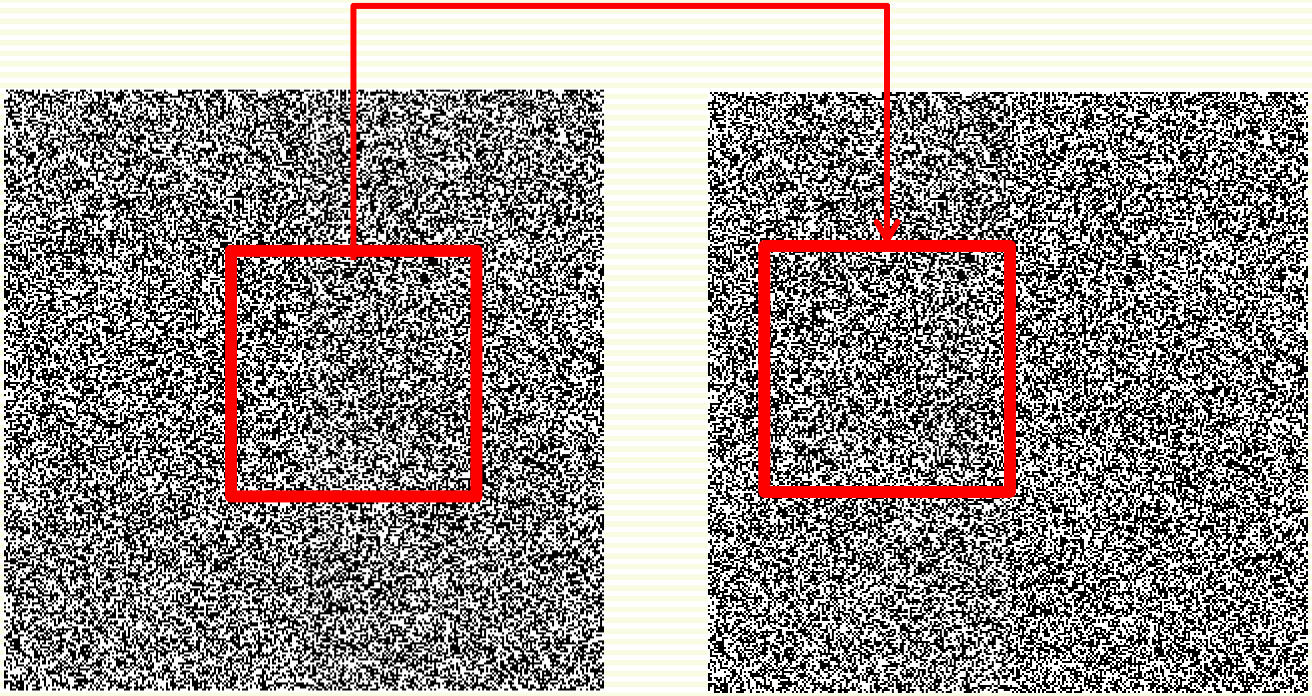
- Answered by Julesz in 1960
- Select a square





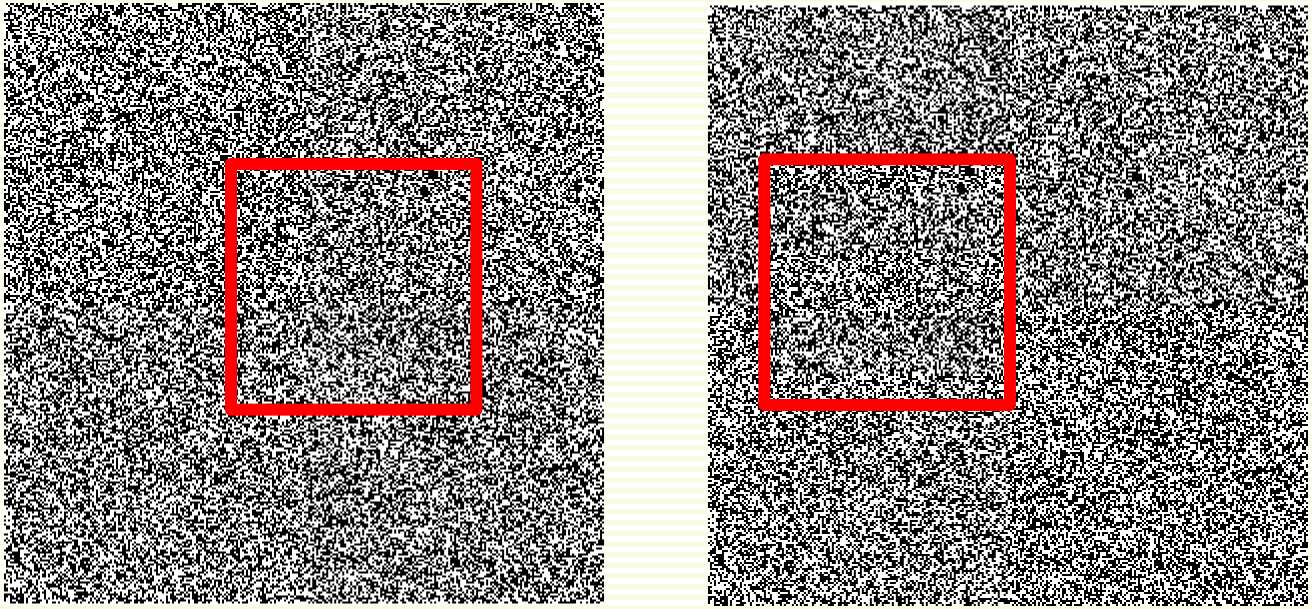
Need Object Recognition for Stereopsis?

- Answered by Julesz in 1960
- Copy square the right image, shifting by **d** to the left
  - random dot stereogram



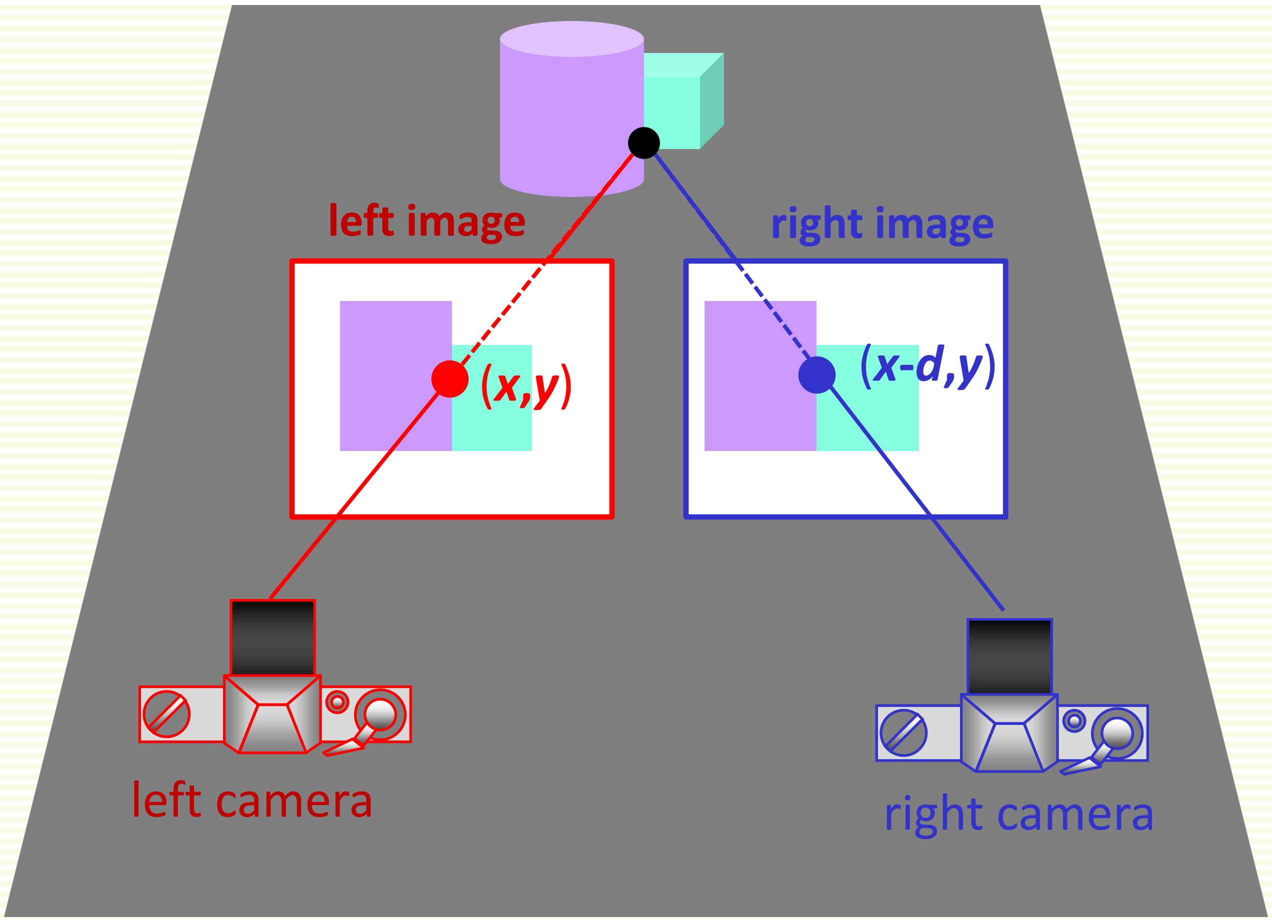
Need Object Recognition for Stereopsis?

- Answered by Julesz in 1960
- Random dot stereogram
- Humans perceive square floating in front of background



3D Shape from Stereo

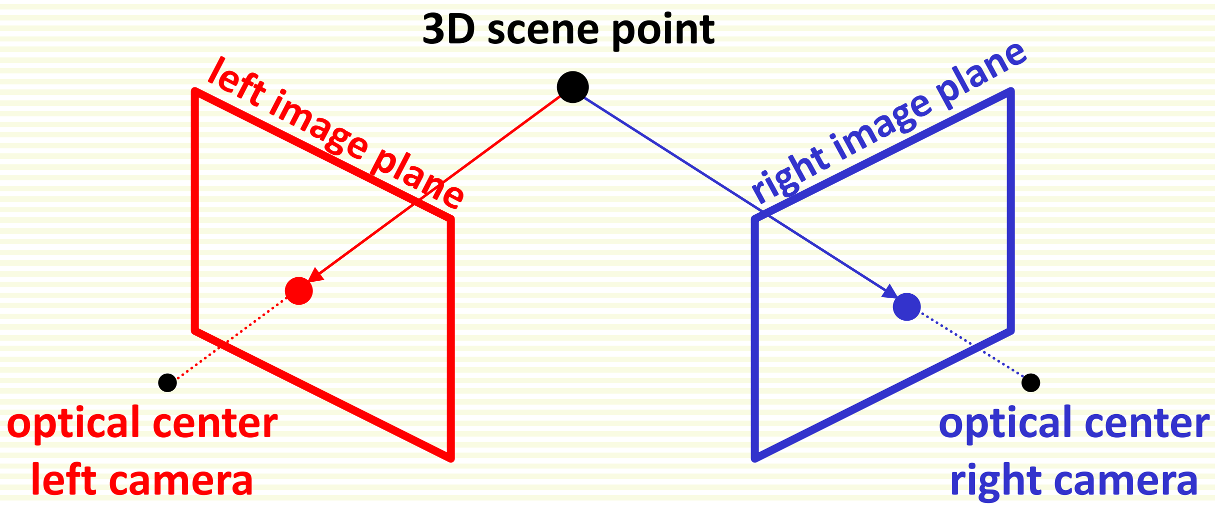
- Use two cameras instead of two eyes



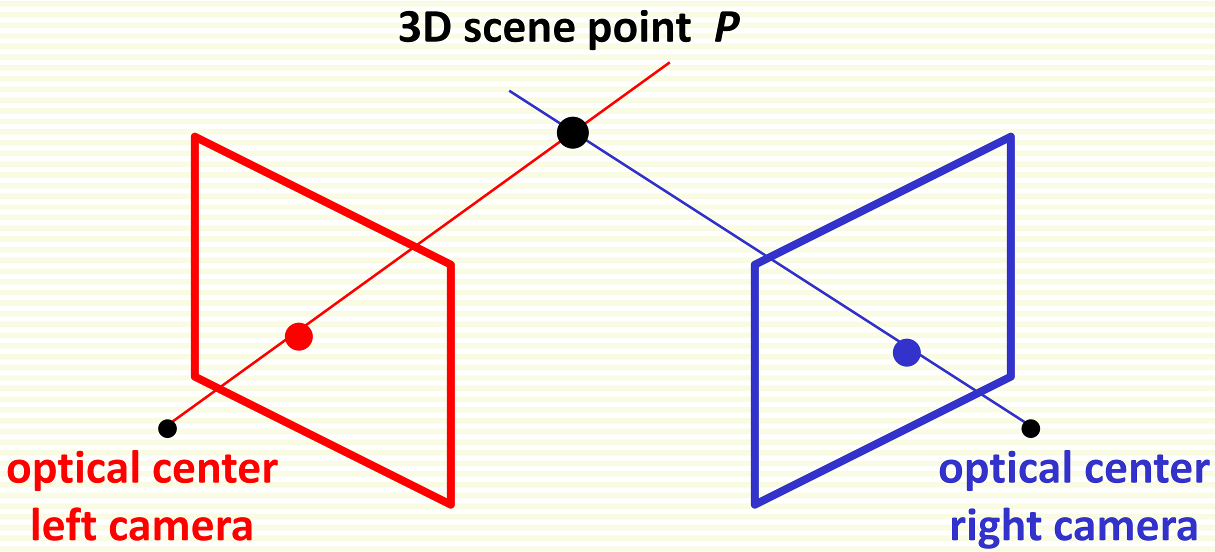
3D Scene

Stereo System

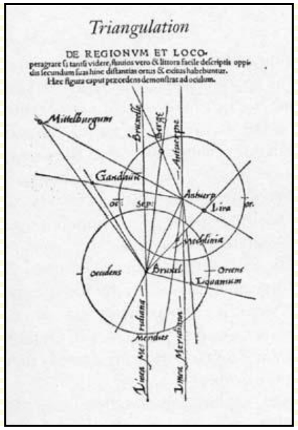
- Unlike eyes, usually stereo cameras are not on the same plane
  - better numerical stability



Stereo System: Triangulation



- Depth by triangulation
  - given two corresponding points in the left and right image
  - cast the rays through the optical camera centers
  - ray intersection is the corresponding 3D world point ***P***
  - depth of ***P*** is based on camera positions and parameters
- Triangulation ideas can be traced to ancient Greece



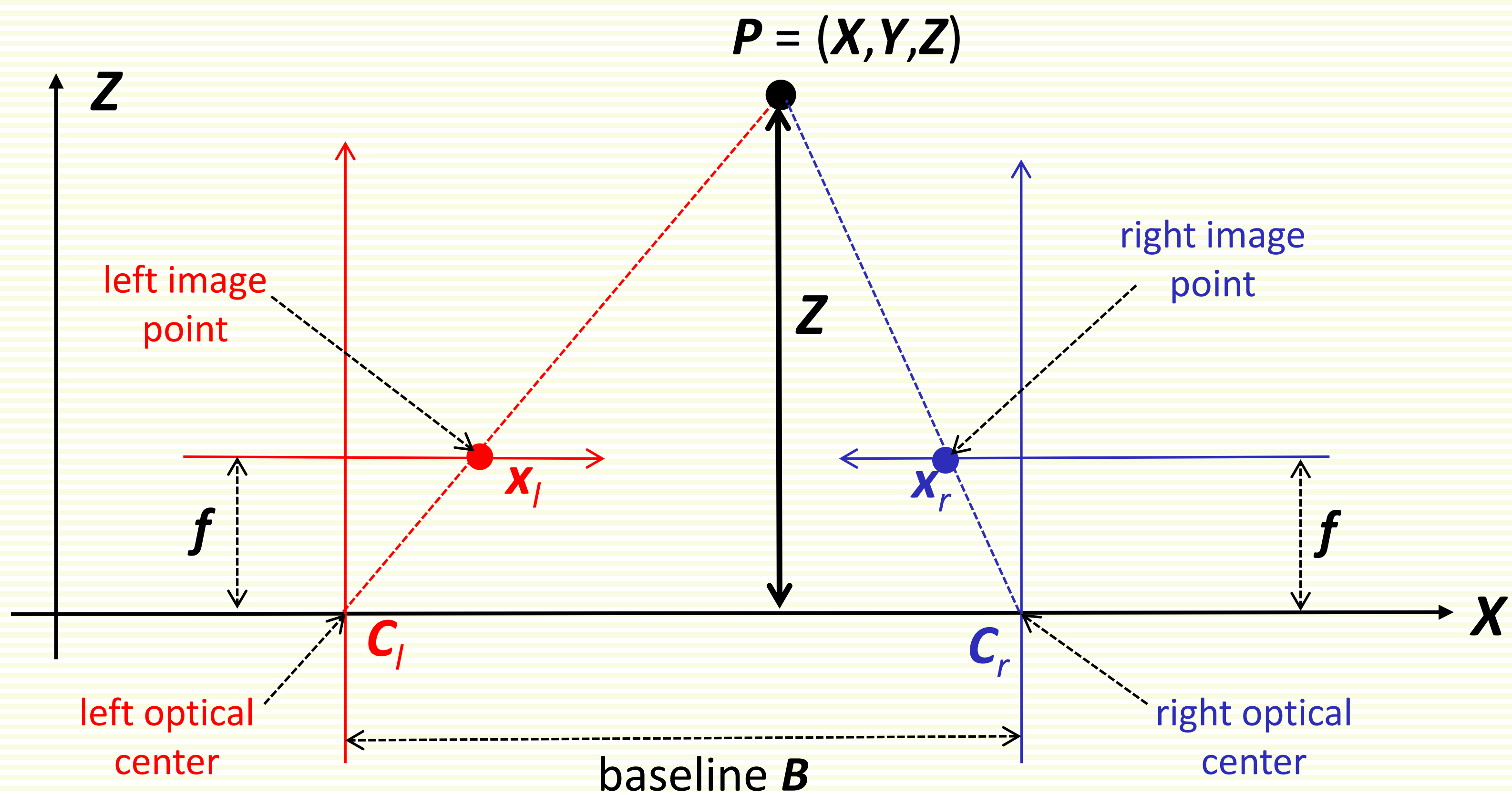
document from 1533

What is needed for Triangulation

1. Distance between cameras, camera focal length
   - Solved through **camera calibration**, essentially a solved problem
   - We will not talk about it
   - Code available on the web
     - OpenCV http://www.intel.com/research/mrl/research/opencv/
     - Matlab, J. Bouget http://www.vision.caltech.edu/bouguetj/calib_doc/index.html
     - Zhengyou Zhang http://research.microsoft.com/~zhang/Calib/
2. Pairs of corresponding pixels in left and right images
   - Called **stereo correspondence problem**, still much researched

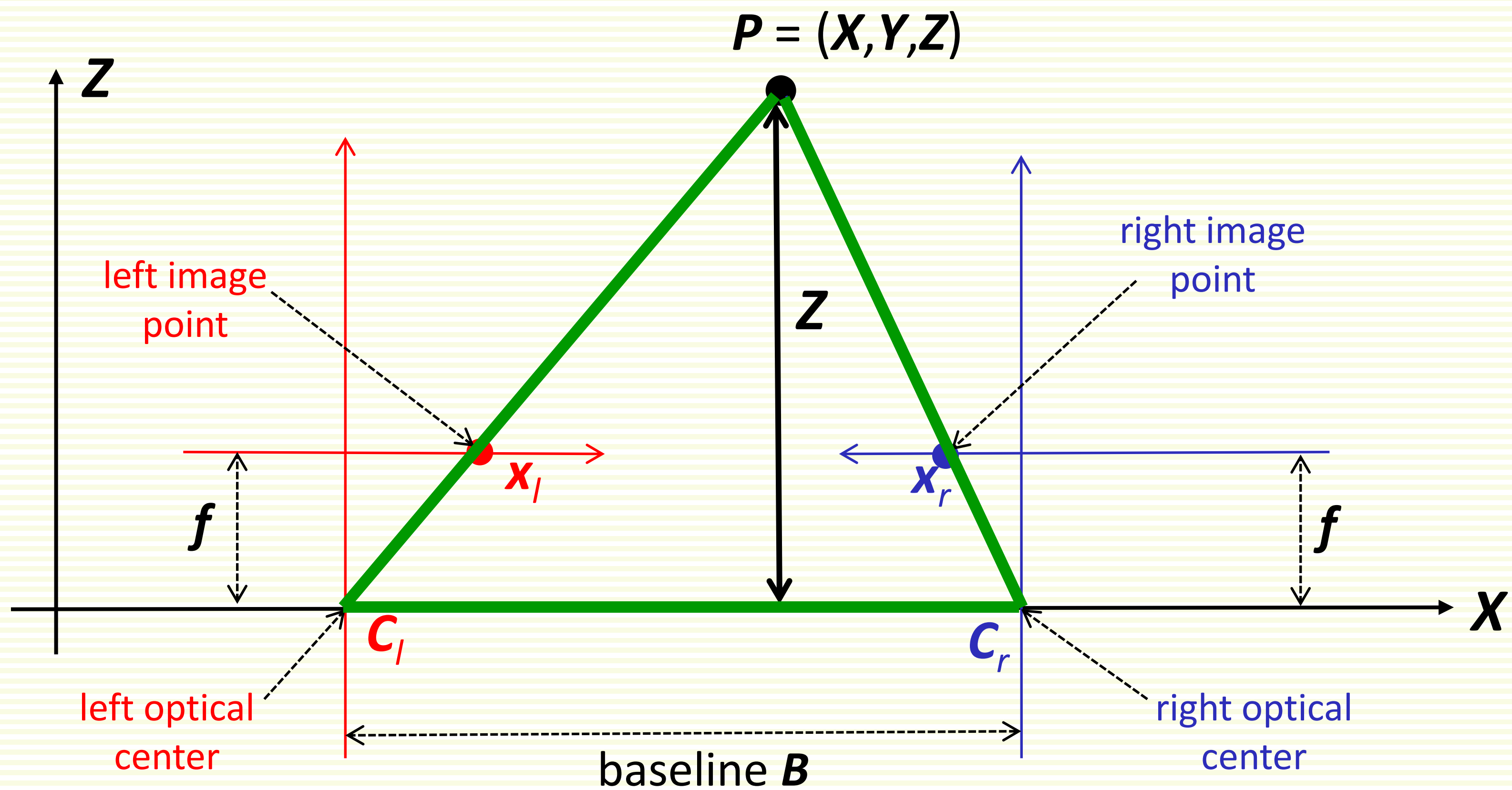
Formula: Depth from Disparity

- Top down view on geometry (slice through ***XZ*** plane)
  - from camera calibration, know the distance between camera optical centers called **baseline *B***, and camera focal length ***f***



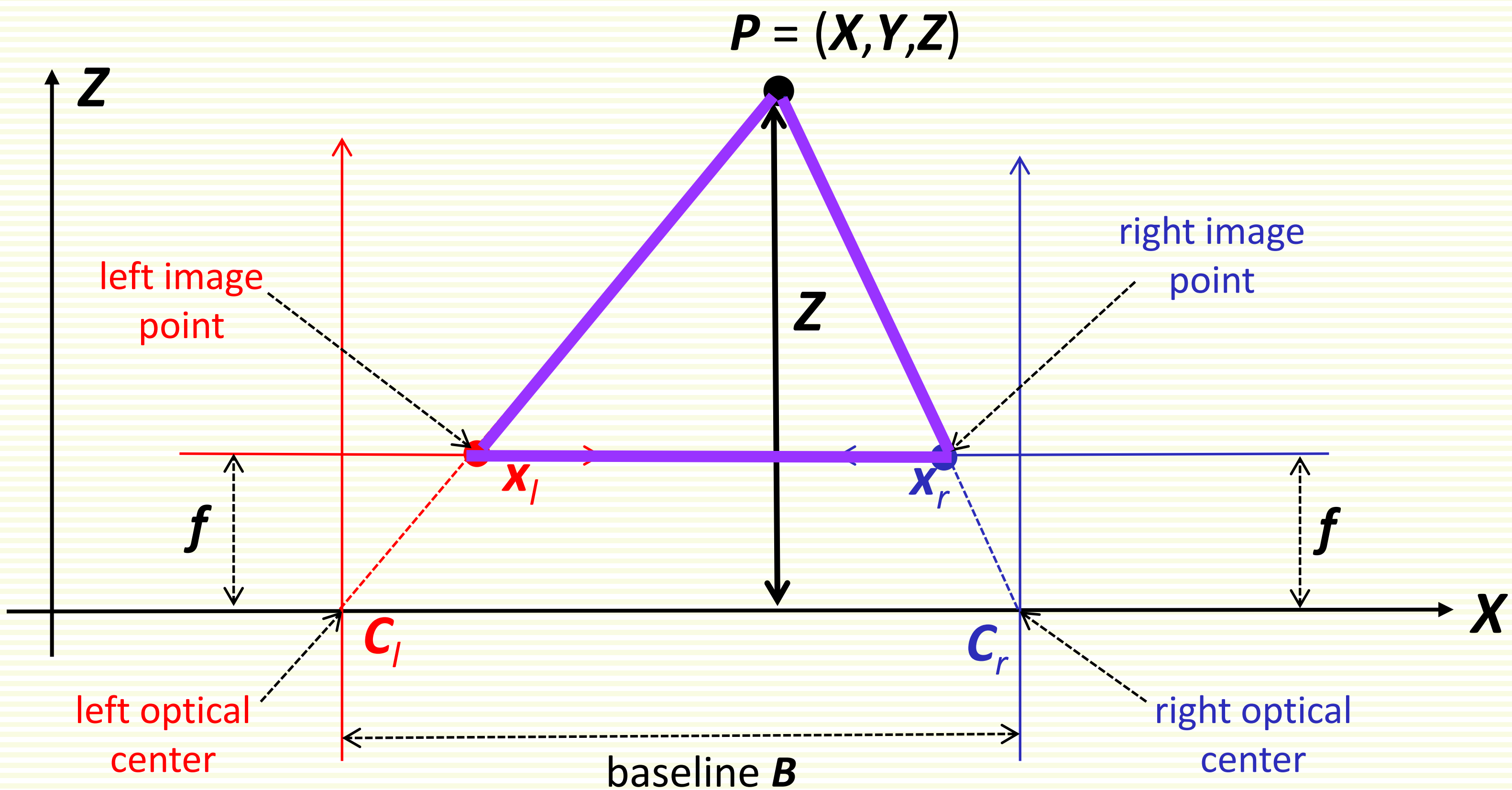
Formula: Depth from Disparity

- Height to base ratio of triangle $C_l P C_r$: $\dfrac{Z}{B}$



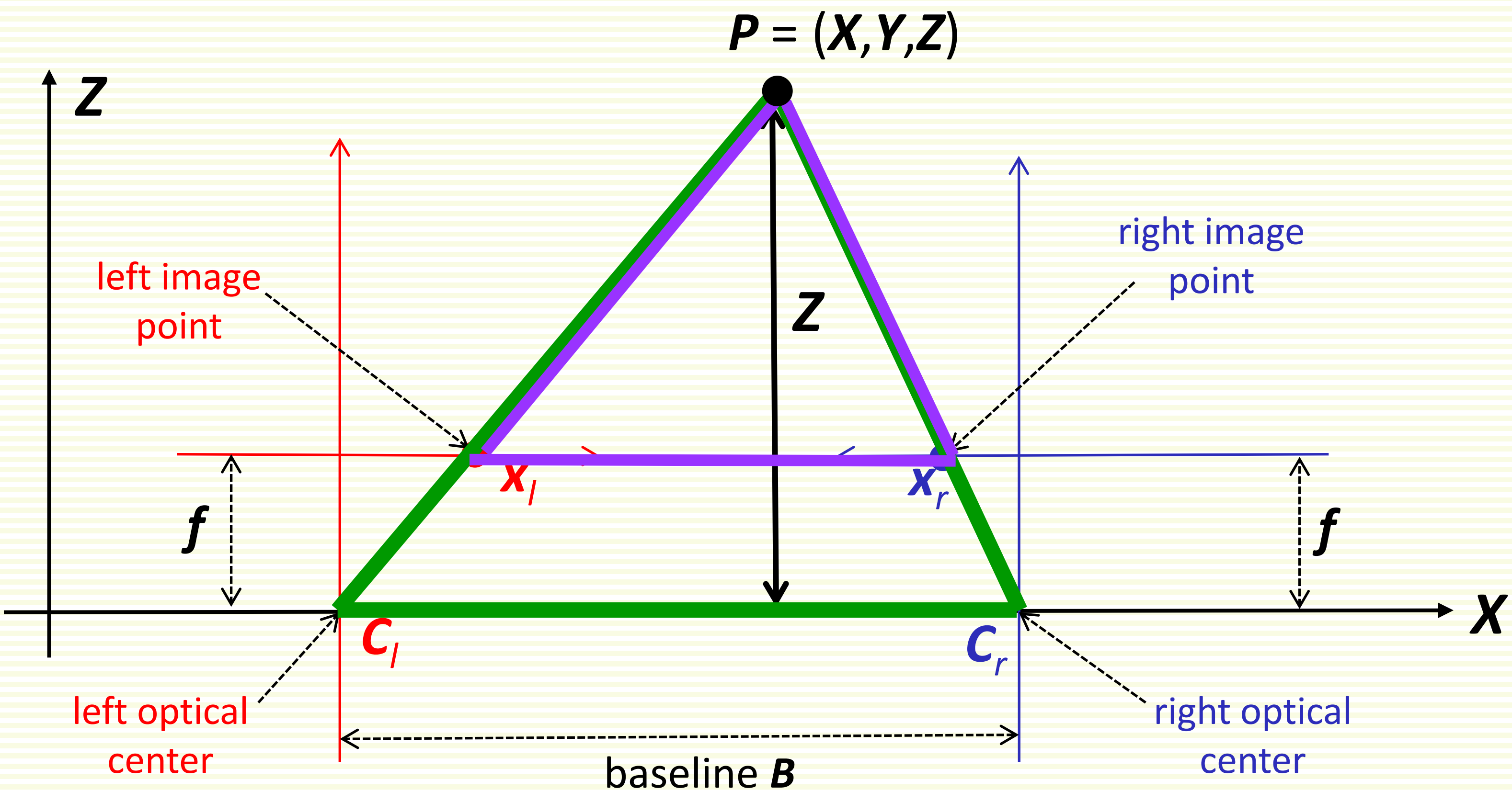
Formula: Depth from Disparity

- Height to base ratio of triangle $x_l P x_r$: $\dfrac{Z - f}{B - x_l + x_r}$
- x_l is positive, x_r is negative



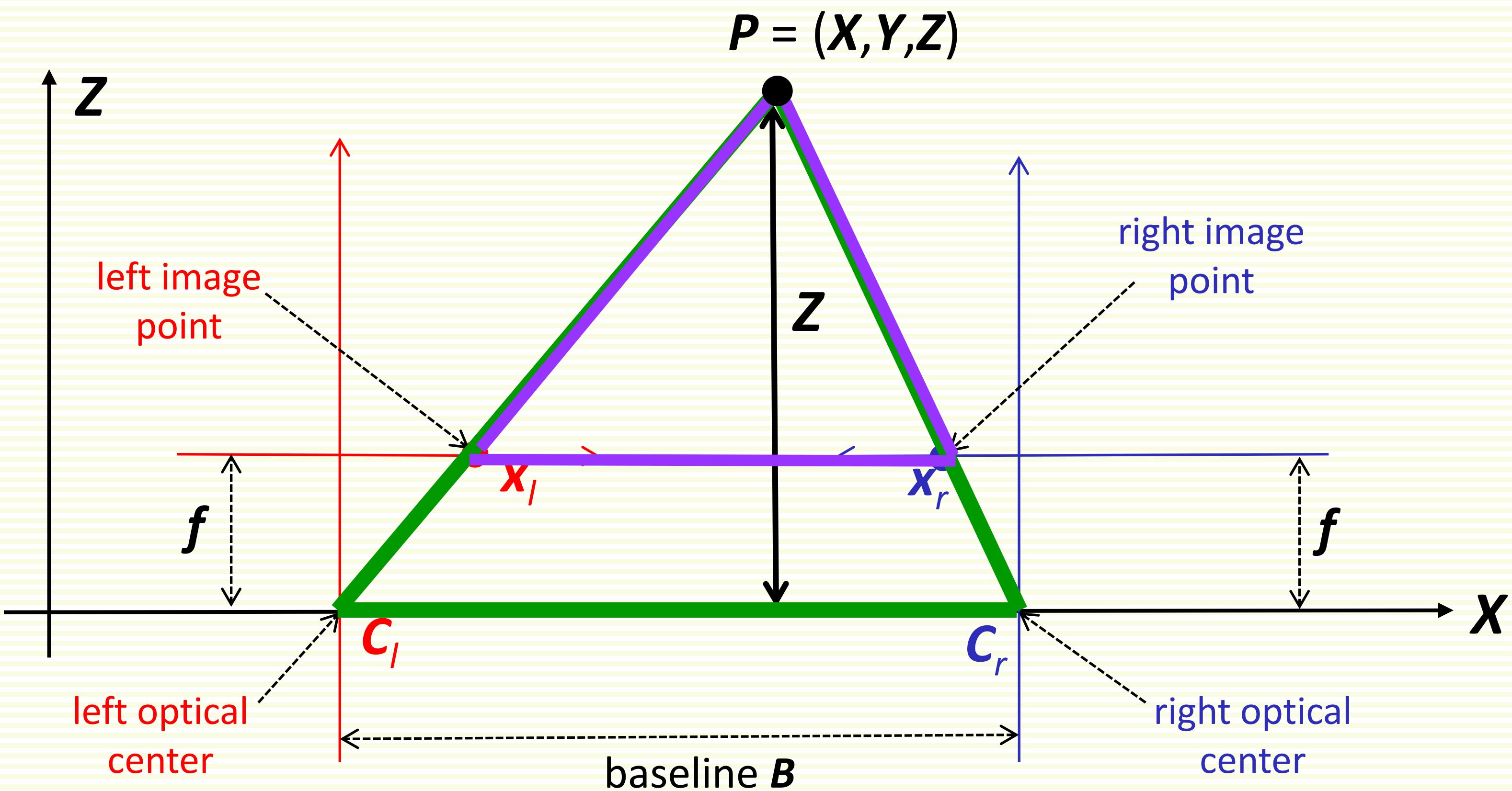
Formula: Depth from Disparity

- $C_l P C_r$ and $\Delta\, x_l P x_r$ are similar: $$\frac{Z}{B} = \frac{Z-f}{B - x_l + x_r}$$



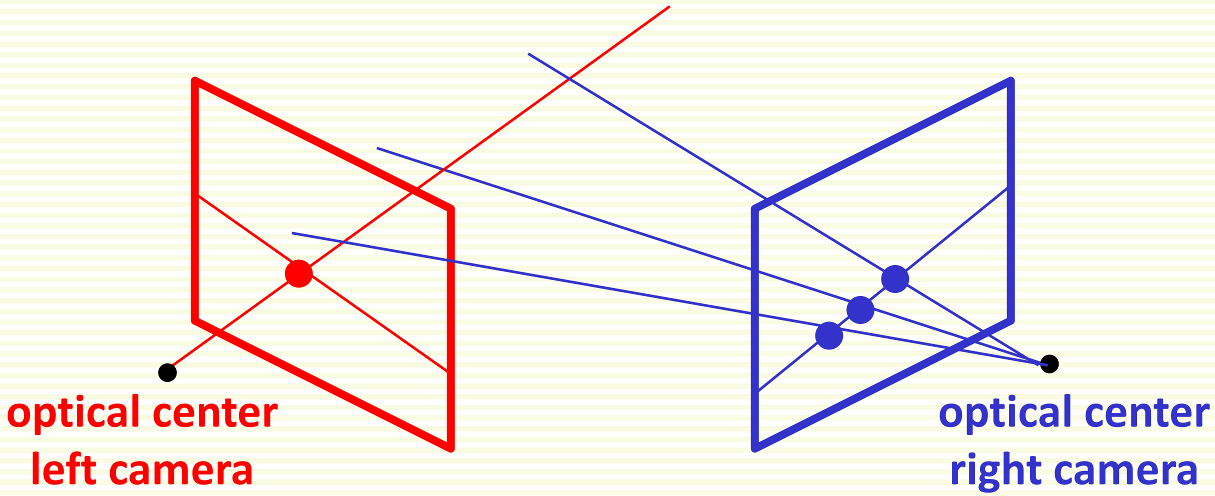
Formula: Depth from Disparity

- Rewriting: $Z = \dfrac{B \cdot f}{x_l - x_r}$
- x_l - x_r is the **disparity**



Stereo Correspondence: Epipolar Lines

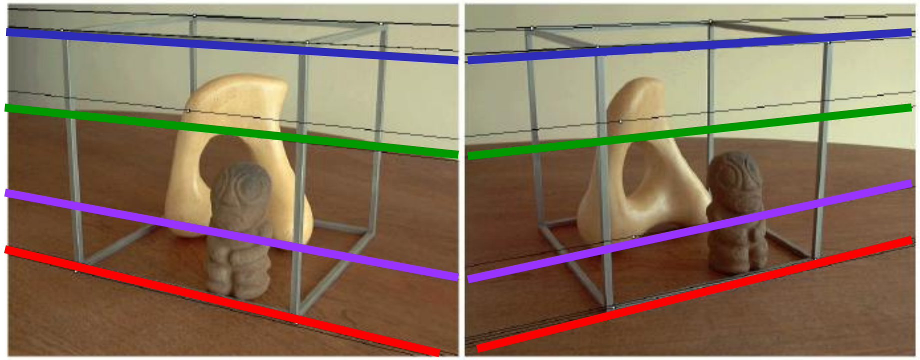
- Which pairs of pixels correspond to the same scene element ?



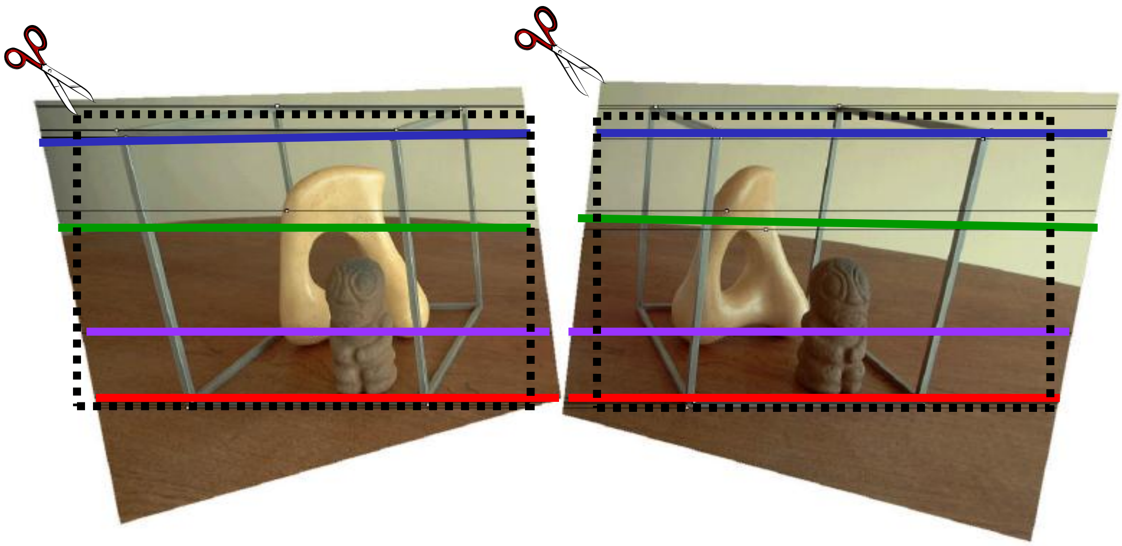
- Epipolar constraint
  - Given a left image pixel, the corresponding pixel in the right image must lie on a line called the **epipolar** line
  - reduces correspondence to 1D search along **conjugate** epipolar lines
  - demo: http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html

Stereo Rectification

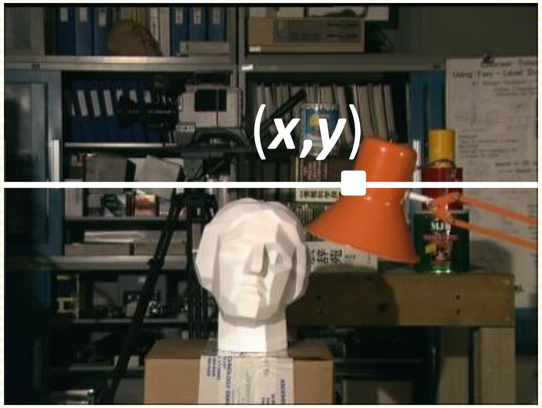
- Epipolar lines can be computed from camera calibration



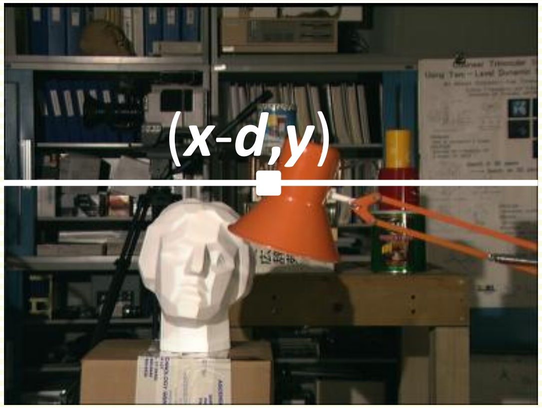
- Usually they are not horizontal
- Can **rectify** stereo pair to make epipolar lines horizontal



Stereo Correspondence



left image

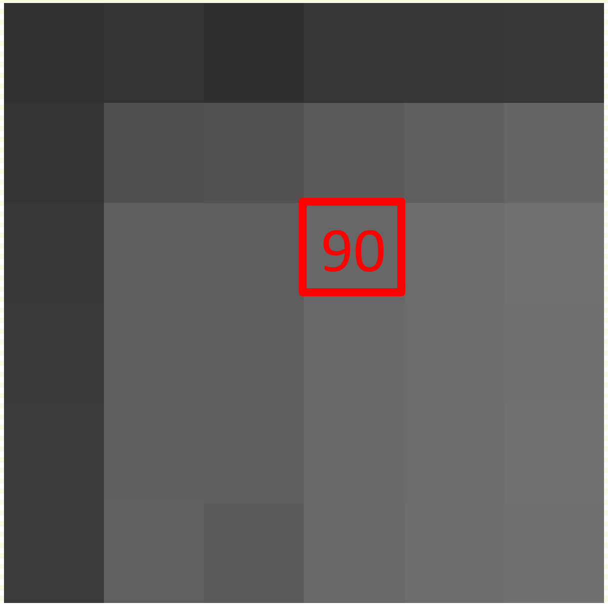


right image

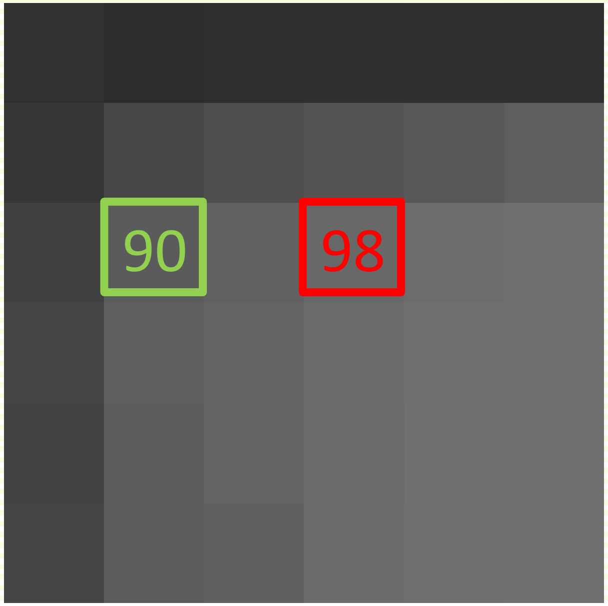
- From now on assume stereo pair is rectified
- How to solve the correspondence problem?
- Corresponding pixels should be similar in intensity
  - or color, or something else

Difficulties in Stereo Correspondence

- Image noise
  - corresponding pixels have similar, but not exactly the same intensities



left image patch

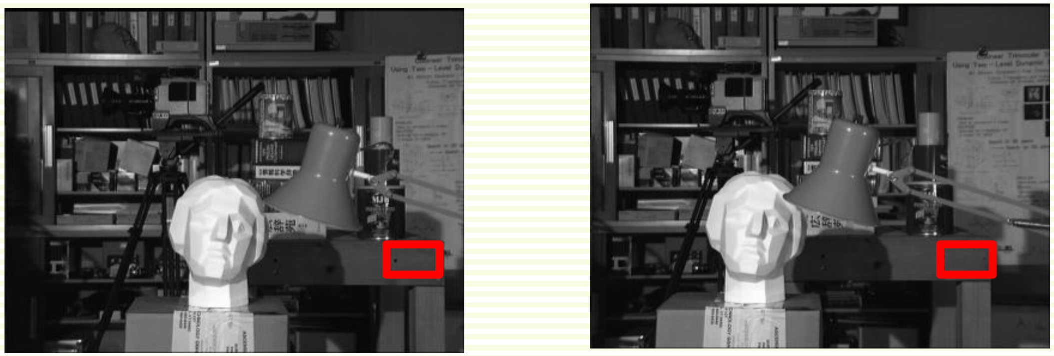


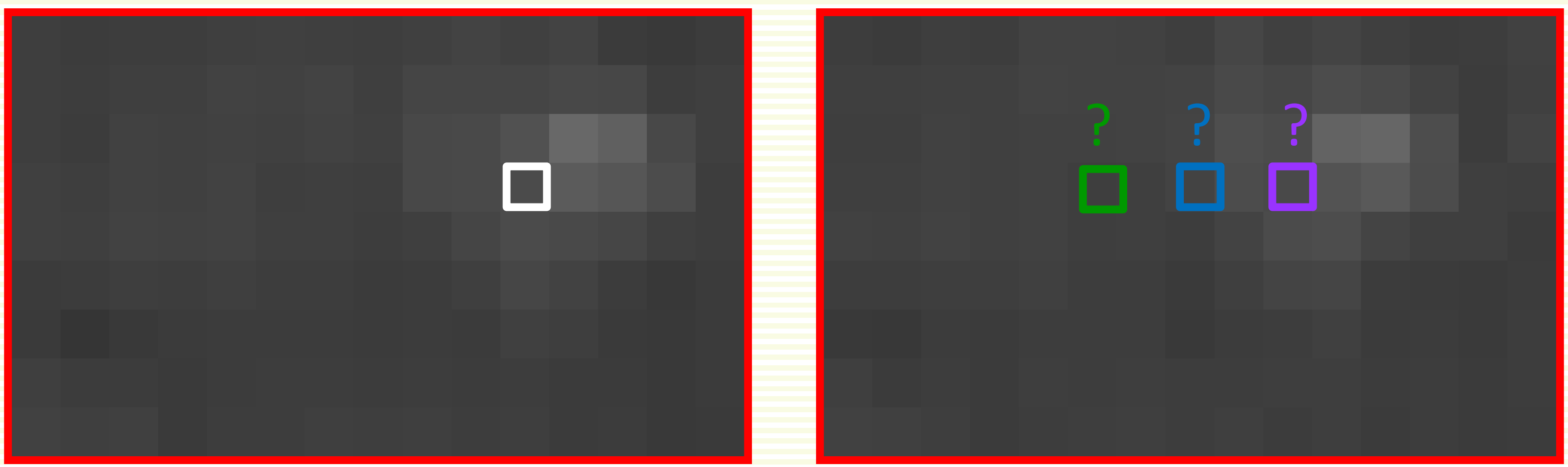
right image patch

- Matching each pixel individually is unreliable

Difficulties in Stereo Correspondence

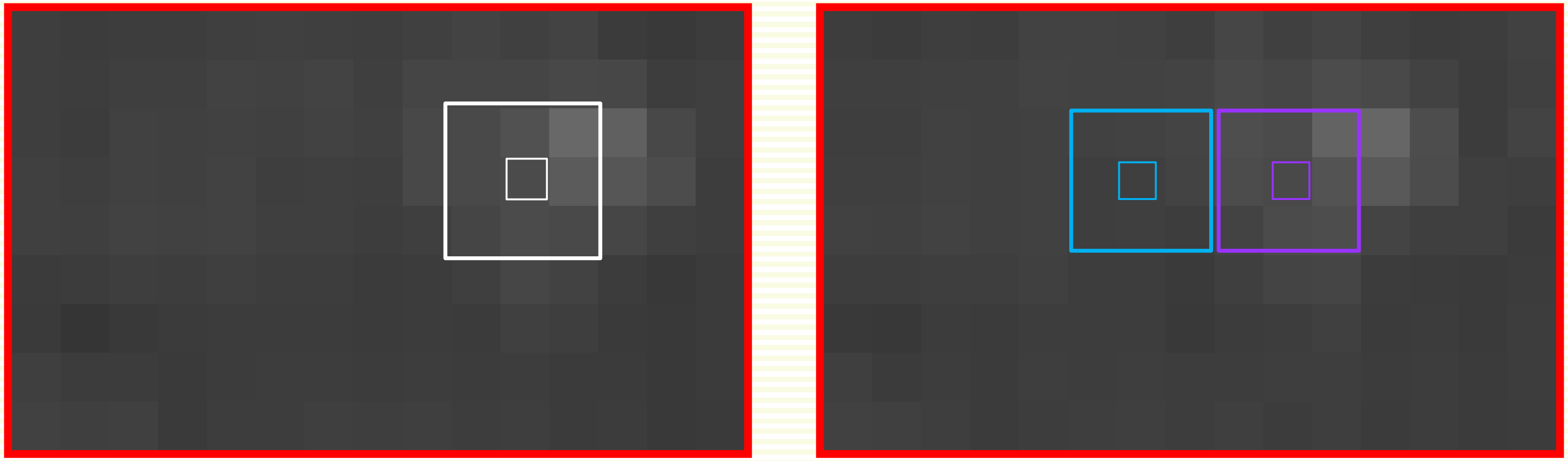
- regions with (almost) constant intensity





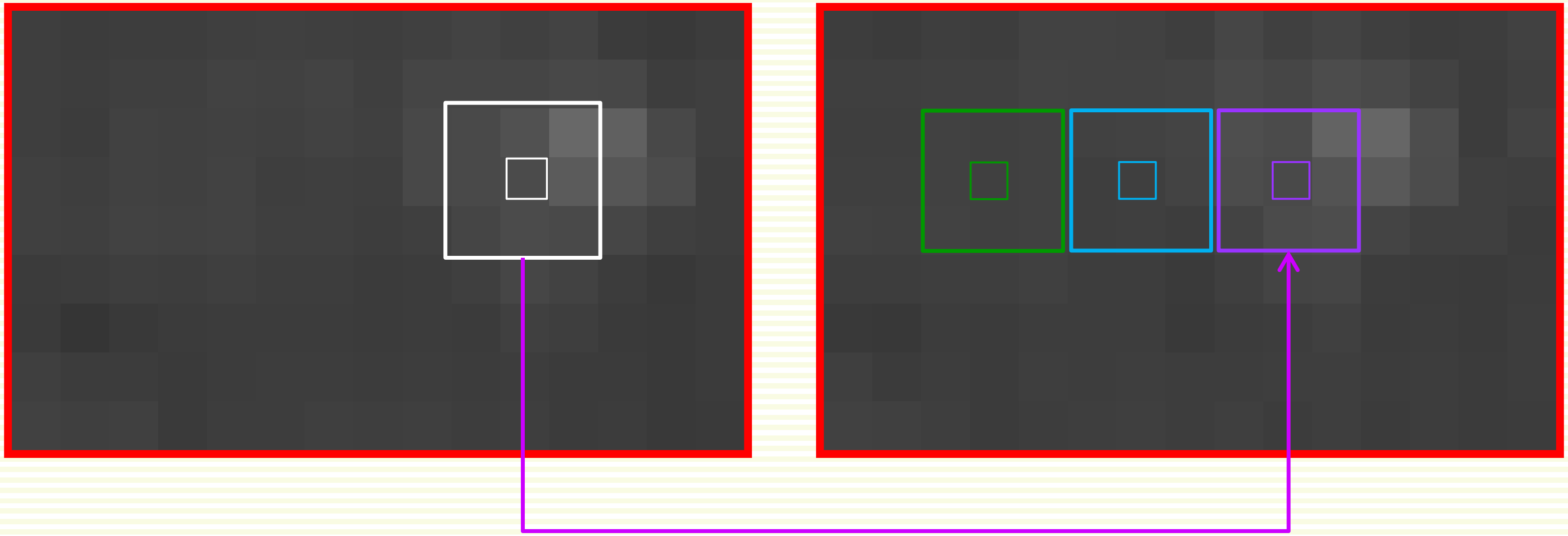
- Matching each pixel individually is unreliable

Window Matching Correspondence



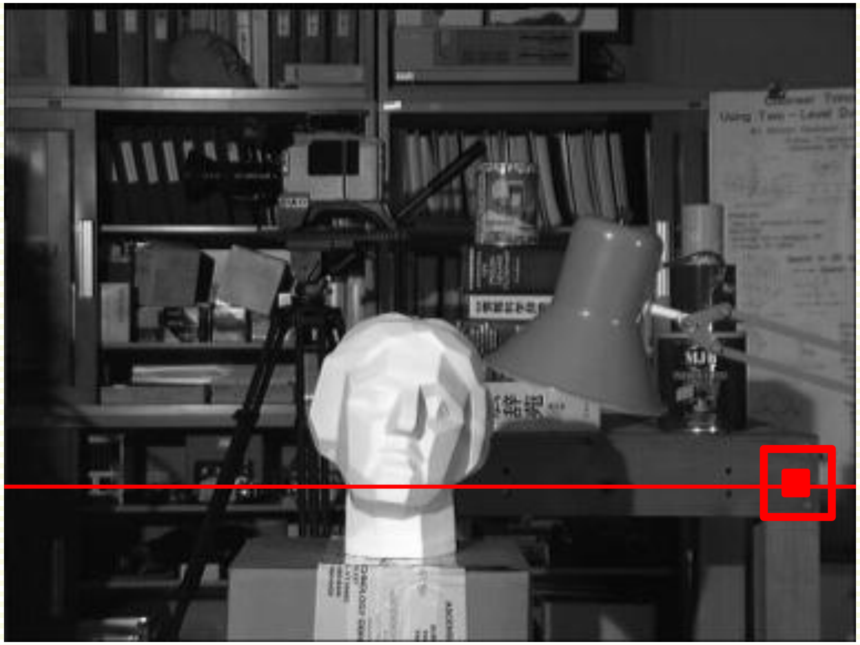
- Use a window (patch) of pixels
  - more likely to have enough intensity variation to form a distinguishable pattern
  - also more robust to noise

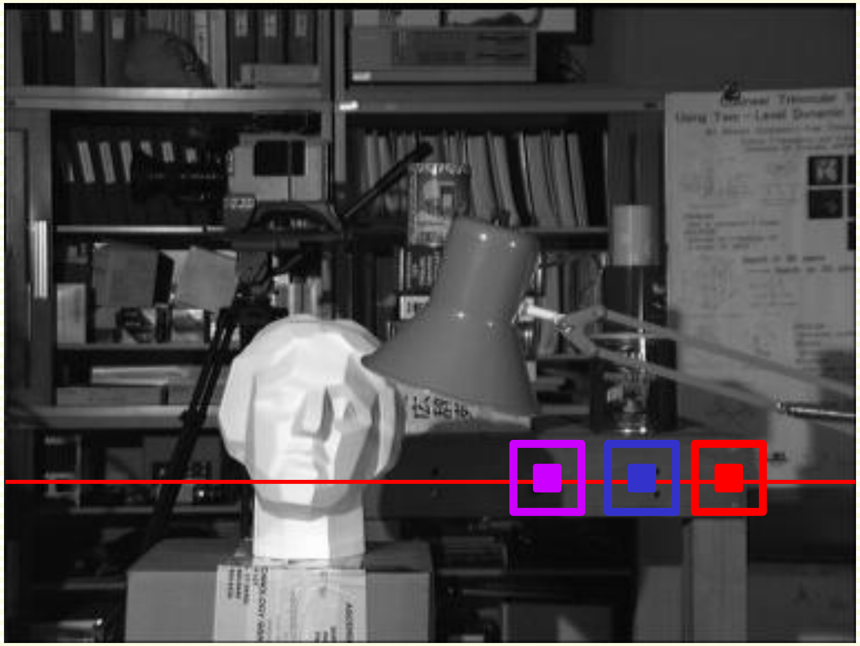
Window Matching Correspondence



- Use a window (patch) of pixels
  - more likely to have enough intensity variation to form a distinguishable pattern
  - also more robust to noise

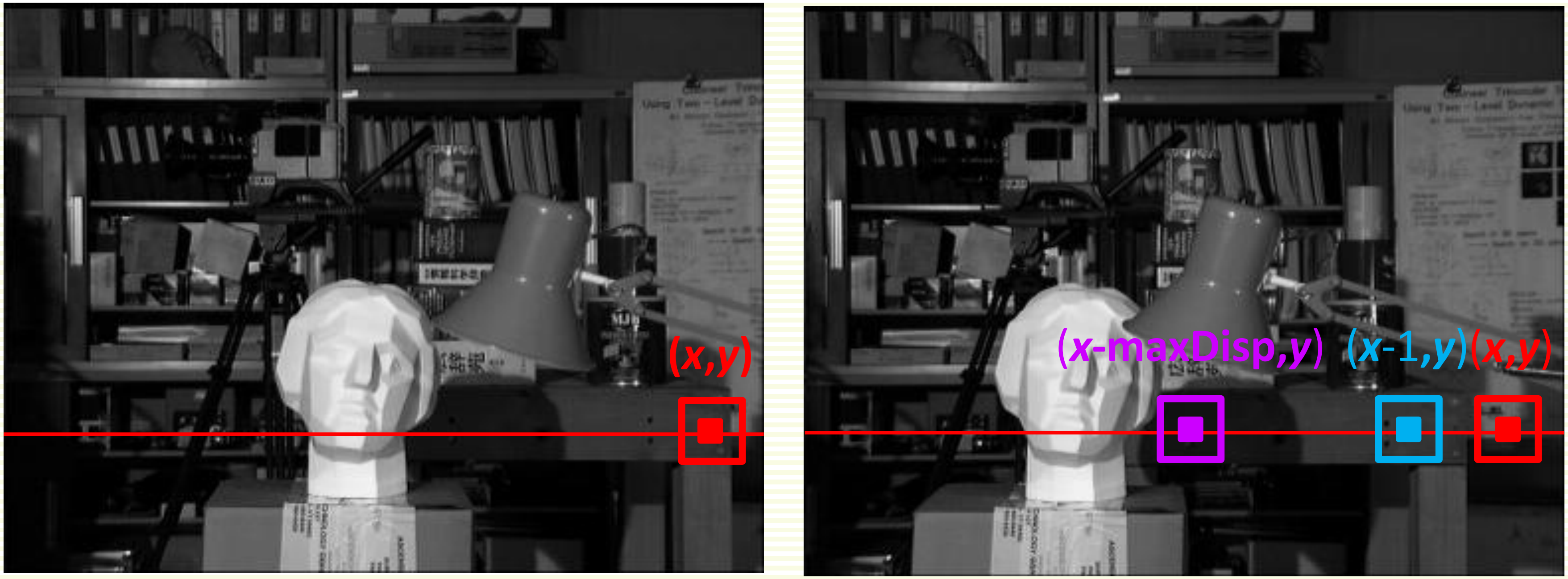
Window Matching: Basic Algorithm





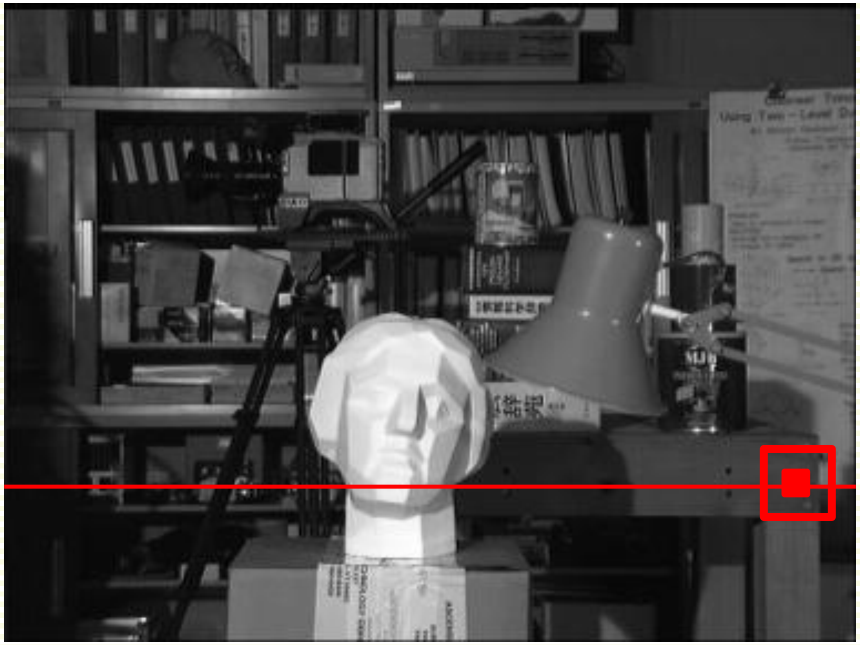
- for each epipolar line
  - for each pixel ***p*** on the left line
    - compare window around ***p*** with same window shifted to many right window locations on corresponding epipolar line
    - pick location corresponding to the best matching window

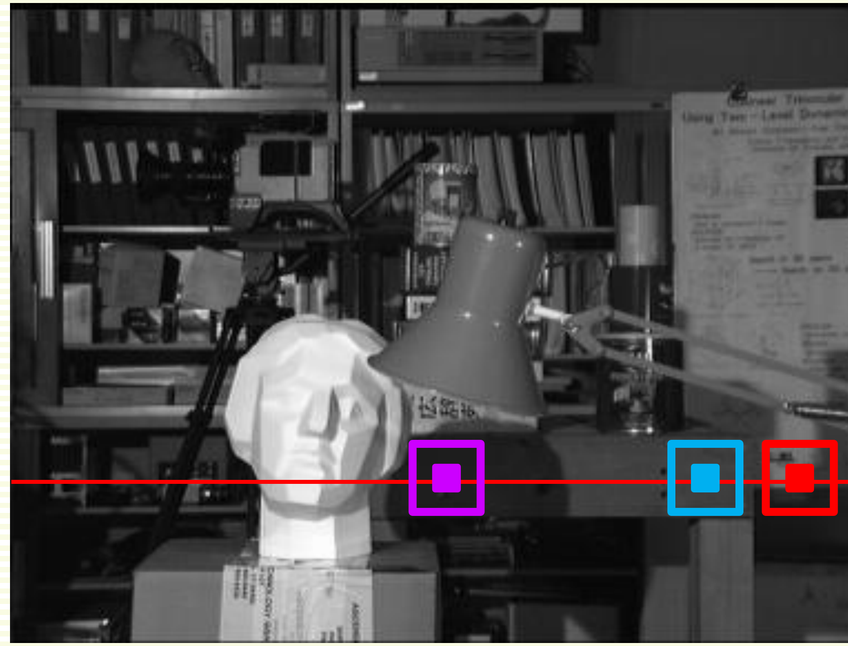
Which Locations to Try?



- Disparity cannot be negative
- Maximum possible disparity is limited by the camera setup
  - assume we know **maxDisp**
- Disparity can range from **0** to **maxDisp**
  - consider only (***x***,***y***), (***x***-1,***y***),…(***x-maxDisp***,***y***) in the right image

Window Matching Cost





- How to define the best matching window?
- Define window cost
  - sum of squared differences (SSD)
  - or sum of absolute differences (SAD)
  - many other possibilities
- Pick window of best (smallest) cost

SSD Window Cost

left image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
2	7	46	46	46	6	7
5	9	46	46	44	9	7
4	7	47	47	47	2	4
4	7	56	56	46	6	7
3	4	4	1	4	3	2

right image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
46	46	46	3	6	6	7
48	46	44	6	4	9	7
47	47	47	7	4	2	4
58	56	46	5	6	6	7
3	4	4	1	4	3	2

$$
\begin{aligned}
&(46-44)^2 + (46-6)^2 + (44-4)^2 + \\
&(47-47)^2 + (47-7)^2 + (47-4)^2 + \\
&(56-46)^2 + (56-5)^2 + (46-6)^2 = 12454
\end{aligned}
$$

Algorithm with SSD Window Cost

left image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
2	7	46	46	46	6	7
5	9	46	46	44	9	7
4	7	47	47	47	2	4
4	7	56	56	46	6	7
3	4	4	1	4	3	2

right image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
46	46	46	3	6	6	7
48	46	44	6	4	9	7
47	47	47	7	4	2	4
58	56	46	5	6	6	7
3	4	4	1	4	3	2

$$
\begin{aligned}
&(46-44)^2+(46-6)^2+(44-4)^2+\\
&(47-47)^2+(47-7)^2+(47-4)^2+\\
&(56-46)^2+(56-5)^2+(46-6)^2=12454
\end{aligned}
$$

- This shift corresponds to disparity 0

Algorithm with SSD Window Cost

left image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
2	7	46	46	46	6	7
5	9	46	46	44	9	7
4	7	47	47	47	2	4
4	7	56	56	46	6	7
3	4	4	1	4	3	2

right image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
46	46	46	3	6	6	7
48	46	44	6	4	9	7
47	47	47	7	4	2	4
58	56	46	5	6	6	7
3	4	4	1	4	3	2

$$
\begin{aligned}
&(46-46)^2+(46-44)^2+(44-6)^2+\\
&(47-47)^2+(47-7)^2+(47-7)^2+\\
&(56-56)^2+(56-46)^2+(46-5)^2=6425
\end{aligned}
$$

- This shift corresponds to disparity 1

Algorithm with SSD Window Cost

left image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
2	7	46	46	46	6	7
5	9	46	46	44	9	7
4	7	47	47	47	2	4
4	7	56	56	46	6	7
3	4	4	1	4	3	2

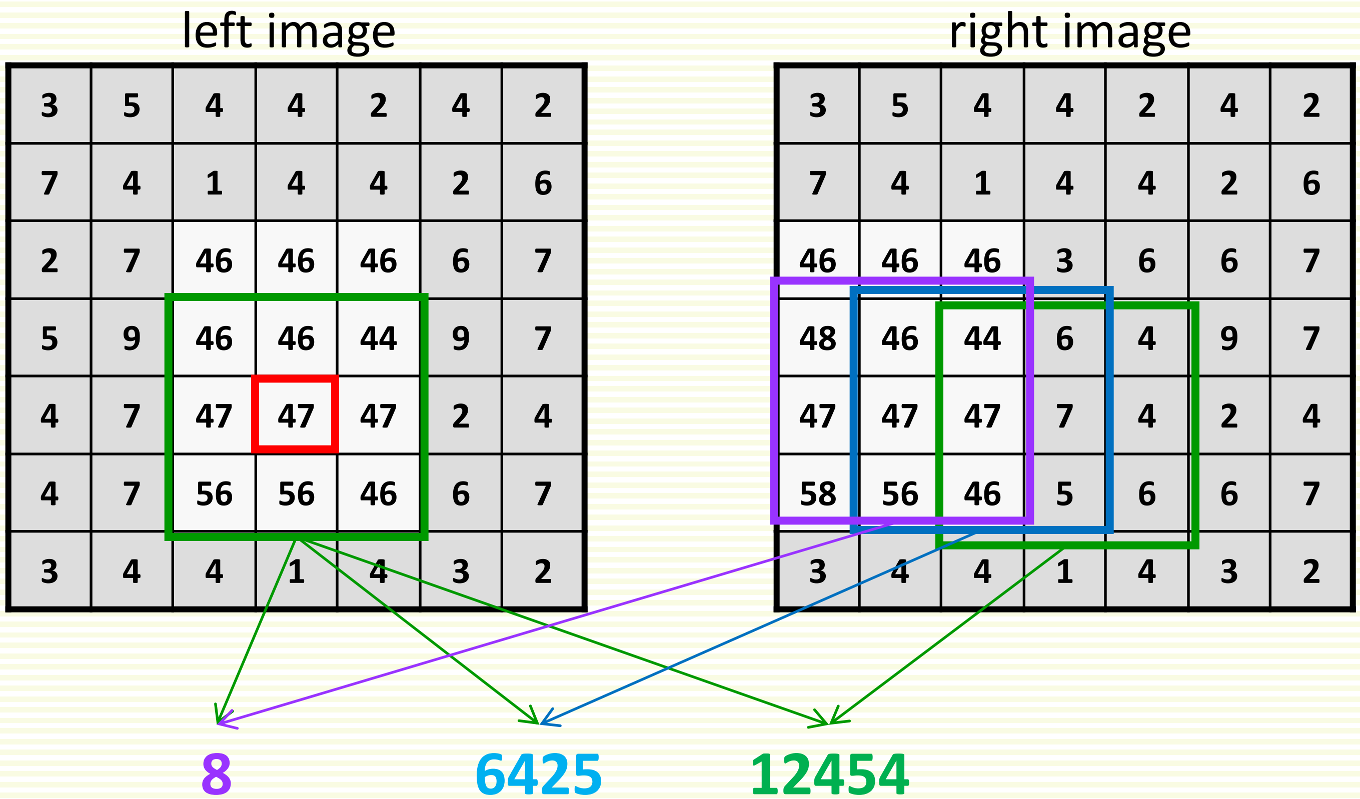
right image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
46	46	46	3	6	6	7
48	46	44	6	4	9	7
47	47	47	7	4	2	4
58	56	46	5	6	6	7
3	4	4	1	4	3	2

$$
\begin{aligned}
&(46-48)^2+(46-46)^2+(44-44)^2+\\
&(47-47)^2+(47-47)^2+(47-47)^2+\\
&(56-58)^2+(56-56)^2+(46-46)^2=8
\end{aligned}
$$

- This shift corresponds to disparity 2

Algorithm with SSD Window Cost



left image

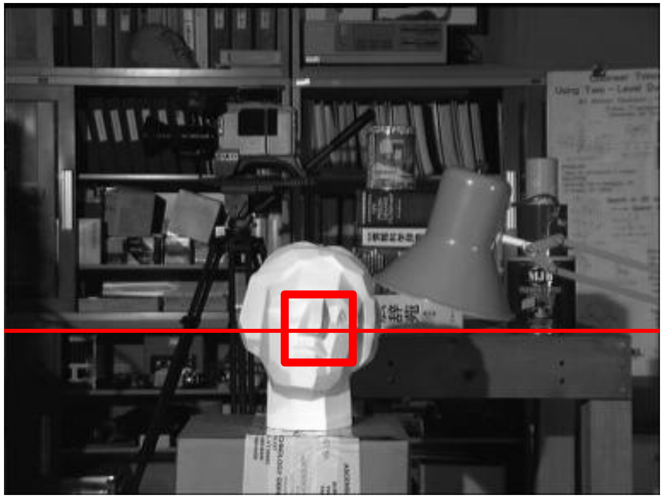
3	5	4	4	2	4	2
7	4	1	4	4	2	6
2	7	46	46	46	6	7
5	9	46	46	44	9	7
4	7	47	47	47	2	4
4	7	56	56	46	6	7
3	4	4	1	4	3	2

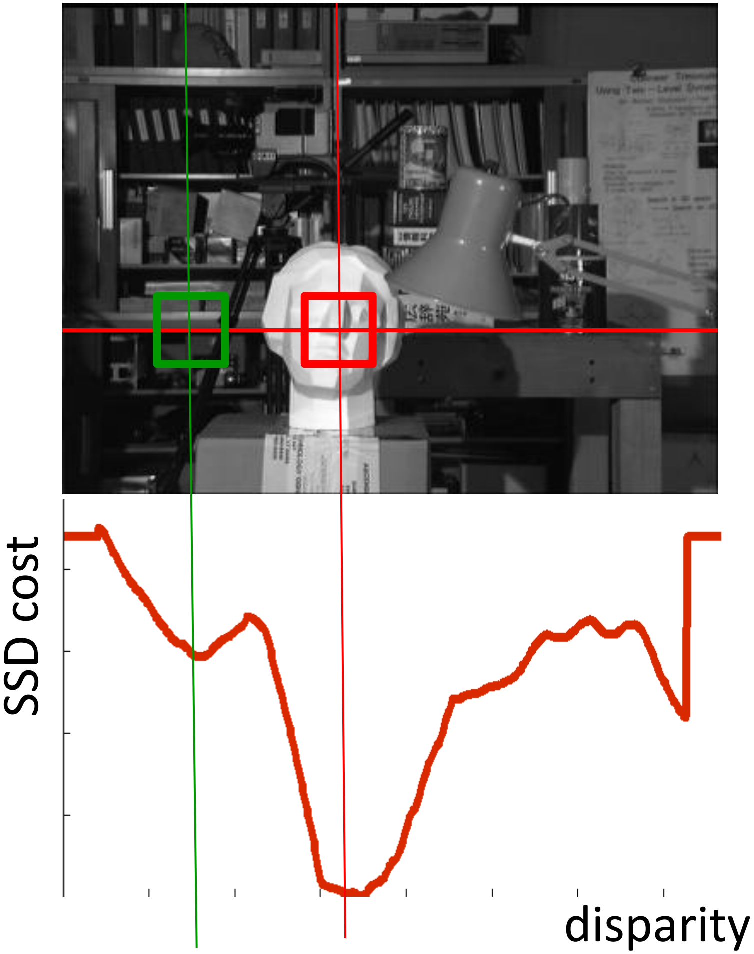
right image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
46	46	46	3	6	6	7
48	46	44	6	4	9	7
47	47	47	7	4	2	4
58	56	46	5	6	6	7
3	4	4	1	4	3	2

- Best SSD window cost is **8** at disparity **2**
- Red pixel is assigned disparity 2
- Repeat this for all image pixels

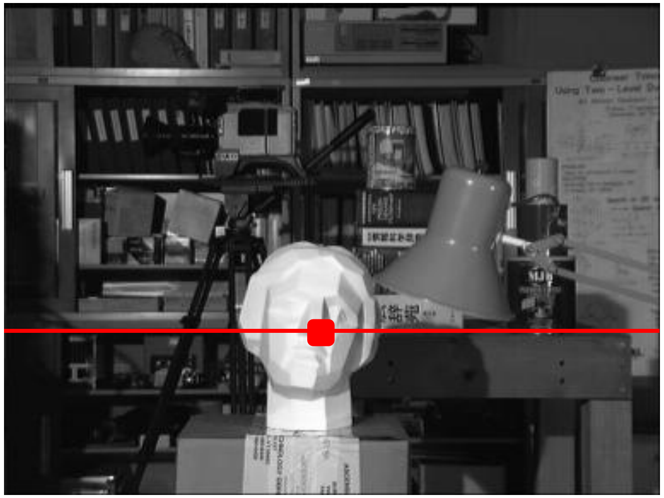
Correspondence with SSD Matching

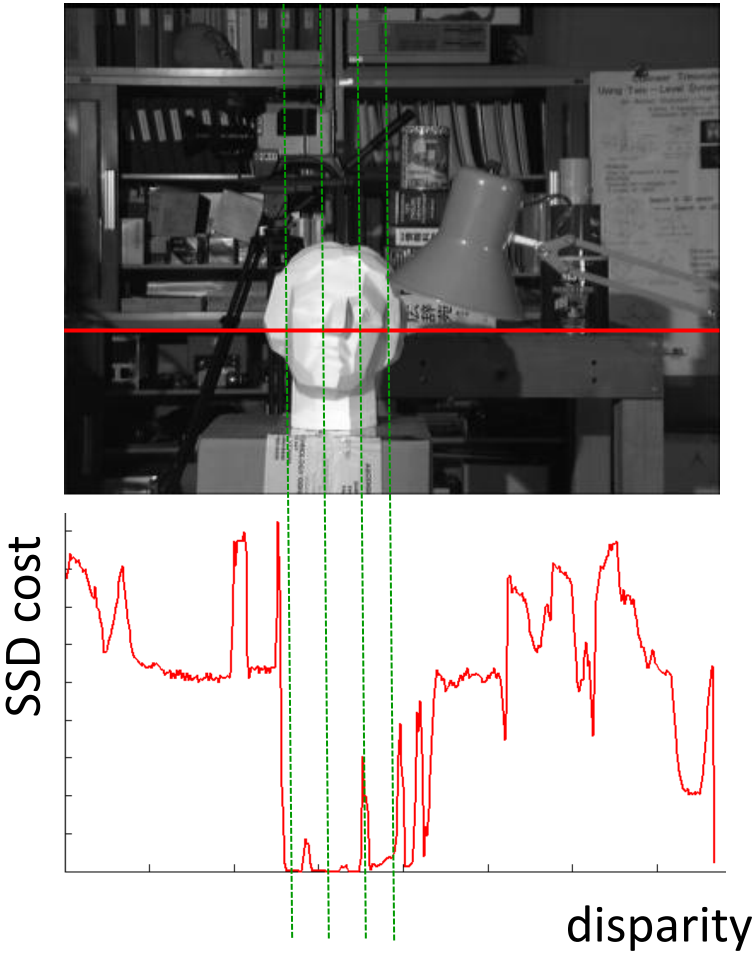




- Unique best cost location

Compare to One Pixel "Window"





- No unique best cost location

SAD Window Cost

- SSD is fragile to outliers

1	1	10
1	1	10
1	1	19

1	1	10
1	1	10
1	1	99

SSD cost = $80^2 = 6400$

1	1	10
1	1	10
1	1	19

31	31	31
31	31	31
31	31	29

SSD cost = 6384 best

- SAD (Sum of Absolute Differences) is more robust

1	1	10
1	1	10
1	1	19

1	1	10
1	1	10
1	1	99

SAD cost = 80 best

1	1	10
1	1	10
1	1	19

31	31	31
31	31	31
31	31	29

SAD cost = 232

Window Matching Efficency

- Suppose
  - image has $\boldsymbol{n}$ pixels
  - matching window is **11** by **11**
- Need $\mathbf{11 \cdot 11 = 121}$ additions and multiplications to compute one window cost
- Multiply that by number of locations to check (**maxDisp**+1)
- Multiply that by $\boldsymbol{n}$ image pixels
- $\cancel{\mathbf{121}} \cdot \boldsymbol{n} \cdot (\mathbf{maxDisp}+1)$
- Tooooo slooooow
  - gets worse for larger windows
- Can get cost down to $\boldsymbol{n} \cdot (\mathbf{maxDisp}+1)$ with integral images

Speedups: Integral Image

- Given image $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})$, the **integral** image $\boldsymbol{I}(\boldsymbol{x},\boldsymbol{y})$ is the sum of values in $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})$ to the left and above $(\boldsymbol{x},\boldsymbol{y})$, including $(\boldsymbol{x},\boldsymbol{y})$

0	0	0	5	5
0	0	5	5	5
0	5	5	5	10
5	5	5	10	0
5	5	10	0	0

$\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})$

0	0	0	5	10
0	0	5	15	25
0	5	15	30	50
5	15	30	55	75
10	25	50	75	95

$\boldsymbol{I}(\boldsymbol{x},\boldsymbol{y})$

- Example: $\boldsymbol{I}(2,2) = 0 + 0 + 0 + 0 + 0 + 5 + 0 + 5 + 5 = 15$

Speedups: Integral Image

- Given image ***f***(***x***,***y***), the **integral** image ***I***(***x***,***y***) is the sum of values in ***f***(***x***,***y***) to the left and above (***x***,***y***), including (***x***,***y***)

0	0	0	5	5
0	0	5	5	5
0	5	5	5	10
5	5	5	10	0
5	5	10	0	0

f(***x***,***y***)

0	0	0	5	10
0	0	5	15	25
0	5	15	30	50
5	15	30	55	75
10	25	50	75	95

I(***x***,***y***)

- Example: ***I***(4,1) = 0 + 0 + 0 + 5 + 5 + 0 + 0 +5 + 5 + 5 = 25

Efficiently Computing Integral Image

- Suppose computed integral image up to location (***x*,*y***)

$$I(x,y) = f(x,y)$$

0	0	0	5	5
0	0	5	5	5
0	5	5	5	10
5	5	5	10	0
5	5	10	0	0

$f(x,y)$

		+		

$I(x,y)$

Efficiently Computing Integral Image

- Suppose computed integral image up to location $(\boldsymbol{x},\boldsymbol{y})$

$$I(x,y) = f(x,y) + I(x-1,y)$$

0	0	0	5	5
0	0	5	5	5
0	5	5	5	10
5	5	5	10	0
5	5	10	0	0

$f(x,y)$

+	+			
+	+			
+	+			
+	+	+		

$I(x,y)$

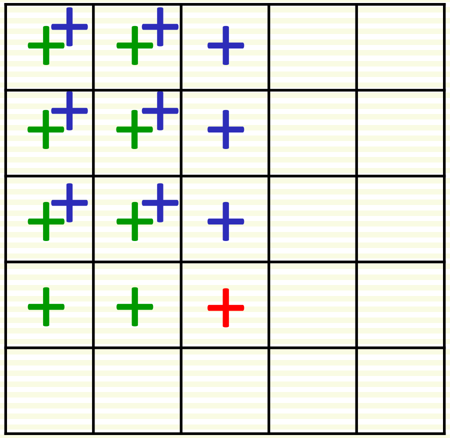
Efficiently Computing Integral Image

- Suppose computed integral image up to location ($\boldsymbol{x,y}$)

$$I(x,y) = f(x,y) + I(x-1,y) + I(x,y-1)$$

0	0	0	5	5
0	0	5	5	5
0	5	5	5	10
5	5	5	10	0
5	5	10	0	0

$f(x,y)$



$I(x,y)$

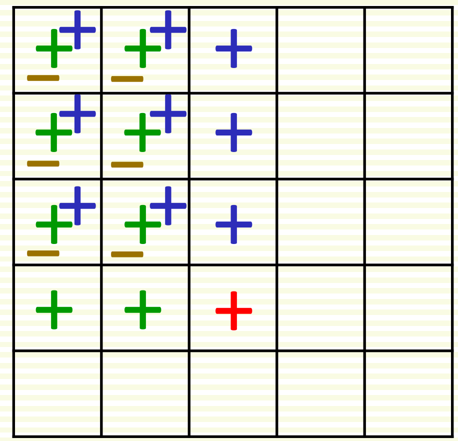
Efficiently Computing Integral Image

- Suppose computed integral image up to location $(\boldsymbol{x},\boldsymbol{y})$

$$\boldsymbol{I}(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{y}) + \boldsymbol{I}(\boldsymbol{x}-1,\boldsymbol{y}) + \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y}-1) - \boldsymbol{I}(\boldsymbol{x}-1,\boldsymbol{y}-1)$$

0	0	0	5	5
0	0	5	5	5
0	5	5	5	10
5	5	5	10	0
5	5	10	0	0

$\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})$



$\boldsymbol{I}(\boldsymbol{x},\boldsymbol{y})$

Integral Image: Order of Computation

- Convenient order of computation
  1. first row
  2. first column
  3. the rest in row-wise fashion

1	2	3	4	5
6	10	11	12	13
7	14	15	16	17
8	18	19	20	21
9	22	23	24	25

$I(x,y)$

Using Integral Image

- After computed integral image, sum over any rectangular window is computed with four operations
- Top left corner $(\boldsymbol{x}_1, \boldsymbol{y}_1)$ and bottom right corner $(\boldsymbol{x}_2, \boldsymbol{y}_2)$

$\boldsymbol{I}(\boldsymbol{x}_2, \boldsymbol{y}_2)$

0	0	0	5	5
0	0	5	5	5
0	5	5	5	10
5	5	5	10	0
5	5	10	0	0

$\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y})$

+	+	+	+	+
+	+	+	+	+
+	+	+	+	+
+	+	+	+	+

$\boldsymbol{I}(\boldsymbol{x}, \boldsymbol{y})$

Using Integral Image

- After computed integral image, sum over any rectangular window is computed with four operations
- Top left corner ($\boldsymbol{x}_1,\boldsymbol{y}_1$) and bottom right corner ($\boldsymbol{x}_2,\boldsymbol{y}_2$)

$$\boldsymbol{I}(\boldsymbol{x}_2,\boldsymbol{y}_2) - \boldsymbol{I}(\boldsymbol{x}_1-1,\boldsymbol{y}_2)$$

0	0	0	5	5
0	0	5	5	5
0	5	5	5	10
5	5	5	10	0
5	5	10	0	0

***f*(*x*,*y*)**

- +	- +	+	+	+
- +	- +	+	+	+
- +	- +	+	+	+
- +	- +	+	+	+

***I*(*x*,*y*)**

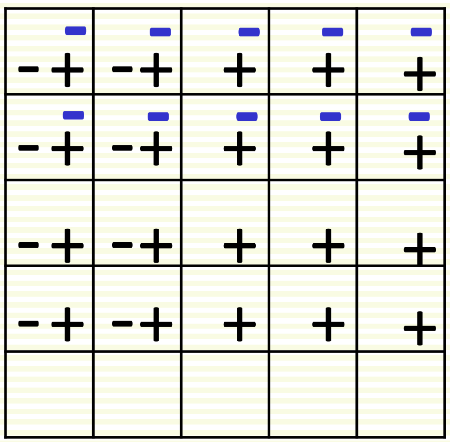
Using Integral Image

- After computed integral image, sum over any rectangular window is computed with four operations
- Top left corner $(\boldsymbol{x}_1,\boldsymbol{y}_1)$ and bottom right corner $(\boldsymbol{x}_2,\boldsymbol{y}_2)$

$$\boldsymbol{I}(\boldsymbol{x}_2,\boldsymbol{y}_2) - \boldsymbol{I}(\boldsymbol{x}_1-1,\boldsymbol{y}_2) - \boldsymbol{I}(\boldsymbol{x}_2,\boldsymbol{y}_1-1)$$

0	0	0	5	5
0	0	5	5	5
0	5	5	5	10
5	5	5	10	0
5	5	10	0	0

$\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})$



$\boldsymbol{I}(\boldsymbol{x},\boldsymbol{y})$

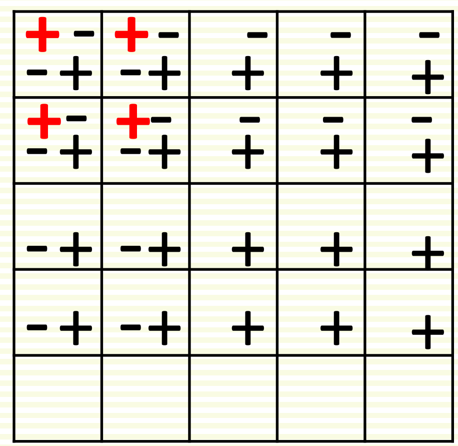
Using Integral Image

- After computed integral image, sum over any rectangular window is computed with four operations
- Top left corner $(\boldsymbol{x}_1,\boldsymbol{y}_1)$ and bottom right corner $(\boldsymbol{x}_2,\boldsymbol{y}_2)$

$$\boldsymbol{I}(\boldsymbol{x}_2,\boldsymbol{y}_2) - \boldsymbol{I}(\boldsymbol{x}_1-1,\boldsymbol{y}_2) - \boldsymbol{I}(\boldsymbol{x}_2,\boldsymbol{y}_1-1) + \boldsymbol{I}(\boldsymbol{x}_1-1,\boldsymbol{y}_1-1)$$

0	0	0	5	5
0	0	5	5	5
0	5	5	5	10
5	5	5	10	0
5	5	10	0	0

$\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})$



$\boldsymbol{I}(\boldsymbol{x},\boldsymbol{y})$

Using Integral Image

- After computed integral image, sum over any rectangular window is computed with four operations
- Top left corner ($\boldsymbol{x}_1,\boldsymbol{y}_1$) and bottom right corner ($\boldsymbol{x}_2,\boldsymbol{y}_2$)

$$\boldsymbol{I}(\boldsymbol{x}_2,\boldsymbol{y}_2) - \boldsymbol{I}(\boldsymbol{x}_1-1,\boldsymbol{y}_2) - \boldsymbol{I}(\boldsymbol{x}_2,\boldsymbol{y}_1-1) + \boldsymbol{I}(\boldsymbol{x}_1-1,\boldsymbol{y}_1-1)$$

0	0	0	5	5
0	0	5	5	5
0	5	5	5	10
5	5	5	10	0
5	5	10	0	0

$\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})$

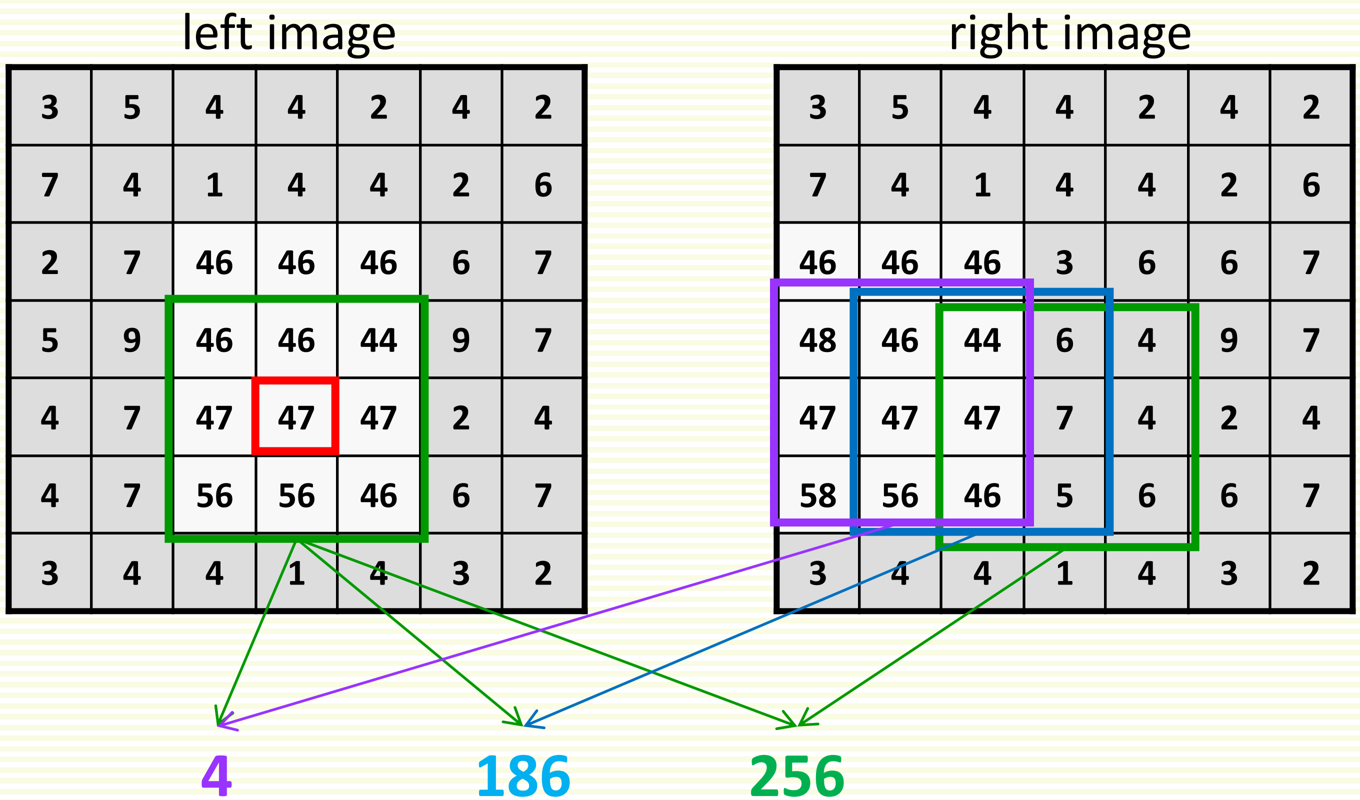
0	0	0	5	10
0	0	5	15	25
0	5	15	30	50
5	15	30	55	75
10	25	50	75	95

$\boldsymbol{I}(\boldsymbol{x},\boldsymbol{y})$

- Example: 5 + 5 +10 + 5 + 10 + 0 = 75 -15 - 25 + 0 = 35

Integral Image for Window Matching

- Assume SAD (sum of absolute differences) cost
- Need to find SAD for every pixel and every disparity in a window



Integral Image for Window Matching

- for each pixel ***p***
    - for every disparity ***d***
        - compute cost between window around ***p*** in the left image and the same window shifted by **d** in the right image
    - pick **d** corresponding to the best matching window

left image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
2	7	46	46	46	6	7
5	9	46	46	44	9	7
4	7	47	47	47	2	4
4	7	56	56	46	6	7
3	4	4	1	4	3	2

right image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
46	46	46	3	6	6	7
48	46	44	6	4	9	7
47	47	47	7	4	2	4
58	56	46	5	6	6	7
3	4	4	1	4	3	2

Integral Image for Window Matching

- For each disparity ***d*** need to compute window cost for all pixels, eventually
- For example, pick disparity ***d*** = 1

left image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
2	7	46	46	46	6	7
5	9	46	46	44	9	7
4	7	47	47	47	2	4
4	7	56	56	46	6	7
3	4	4	1	4	3	2

right image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
46	46	46	3	6	6	7
48	46	44	6	4	9	7
47	47	47	7	4	2	4
58	56	46	5	6	6	7
3	4	4	1	4	3	2

Integral Image for Window Matching

- Old inefficient algorithm:
  - for each pixel ***p*** ← swap
    - for every disparity ***d***
      - compute cost between window around ***p*** in the left image and the same window shifted by **d** in the right image
    - pick **d** corresponding to the best matching window

- New efficient algorithm:
  - for each disparity ***d***
    - for every pixel ***p*** ← use integral image
      - compute cost between window around ***p*** in the left image and the same window shifted by **d** in the right image
    - pick **d** corresponding to the best matching window

Integral Image for Window Matching

- Suppose current disparity is ***d*** = 1

left image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
2	7	46	46	46	6	7
5	9	46	46	44	9	7
4	7	47	47	47	2	4
4	7	56	56	46	6	7
3	4	4	1	4	3	2

right image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
46	46	46	3	6	6	7
48	46	44	6	4	9	7
47	47	47	7	4	2	4
58	56	46	5	6	6	7
3	4	4	1	4	3	2

- Overlay left and right image at disparity 1
- Compute AD (absolute difference) between every overlaid pair of pixels
- Compute SAD in a window for every pixel

Integral Image for Window Matching

- current disparity is d = 1

left image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
2	7	46	46	46	6	7
5	9	46	46	44	9	7
4	7	47	47	47	2	4
4	7	56	56	46	6	7
3	4	4	1	4	3	2

right image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
46	46	46	3	6	6	7
48	46	44	6	4	9	7
47	47	47	7	4	2	4
58	56	46	5	6	6	7
3	4	4	1	4	3	2

3	3	5	4	4	2	4	2
7	7	4	1	4	4	2	6
2	46	46	46	3	6	6	7
5	48	46	44	6	4	9	7
4	47	47	47	7	4	2	4
4	58	56	46	5	6	6	7
3	3	4	4	1	4	3	2

→

AD image for disparity 1

2	1	0	2	2	2
3	3	3	0	2	0
39	0	0	43	0	0
39	0	2	38	5	0
40	0	0	40	2	0
51	0	10	41	0	0
1	0	3	3	1	0

Integral Image for Window Matching

- current disparity is ***d*** = 1
- Pad AD image with zeros

left image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
2	7	46	46	46	6	7
5	9	46	46	44	9	7
4	7	47	47	47	2	4
4	7	56	56	46	6	7
3	4	4	1	4	3	2

right image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
46	46	46	3	6	6	7
48	46	44	6	4	9	7
47	47	47	7	4	2	4
58	56	46	5	6	6	7
3	4	4	1	4	3	2

3	3	5	4	4	2	4	2
7	7	4	1	4	4	2	6
2	46	46	46	3	6	6	7
5	48	46	44	6	4	9	7
4	47	47	47	7	4	2	4
4	58	56	46	5	6	6	7
3	3	4	4	1	4	3	2

→

AD image for disparity 1

0	2	1	0	2	2	2
0	3	3	3	0	2	0
0	39	0	0	43	0	0
0	39	0	2	38	5	0
0	40	0	0	40	2	0
0	51	0	10	41	0	0
0	1	0	3	3	1	0

Integral Image for Window Matching

- current disparity is ***d*** = 1

left image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
2	7	46	46	46	6	7
5	9	46	46	44	9	7
4	7	47	47	47	2	4
4	7	56	56	46	6	7
3	4	4	1	4	3	2

right image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
46	46	46	3	6	6	7
48	46	44	6	4	9	7
47	47	47	7	4	2	4
58	56	46	5	6	6	7
3	4	4	1	4	3	2

AD image for disparity 1

0	2	1	0	2	2	2
0	3	3	3	0	2	0
0	39	0	0	43	0	0
0	39	0	2	38	5	0
0	40	0	0	40	2	0
0	51	0	10	41	0	0
0	1	0	3	3	1	0

Integral Image for Window Matching

- current disparity is ***d*** = 1

left image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
2	7	46	46	46	6	7
5	9	46	46	44	9	7
4	7	47	47	47	2	4
4	7	56	56	46	6	7
3	4	4	1	4	3	2

right image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
46	46	46	3	6	6	7
48	46	44	6	4	9	7
47	47	47	7	4	2	4
58	56	46	5	6	6	7
3	4	4	1	4	3	2

AD image for disparity 1

0	2	1	0	2	2	2
0	3	3	3	0	2	0
0	39	0	0	43	0	0
0	39	0	2	38	5	0
0	40	0	0	40	2	0
0	51	0	10	41	0	0
0	1	0	3	3	1	0

Integral Image for Window Matching

- current disparity is $d = 1$

left image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
2	7	46	46	46	6	7
5	9	46	46	44	9	7
4	7	47	47	47	2	4
4	7	56	56	46	6	7
3	4	4	1	4	3	2

right image

3	5	4	4	2	4	2
7	4	1	4	4	2	6
46	46	46	3	6	6	7
48	46	44	6	4	9	7
47	47	47	7	4	2	4
58	56	46	5	6	6	7
3	4	4	1	4	3	2

AD image for disparity 1

0	2	1	0	2	2	2
0	3	3	3	0	2	0
0	39	0	0	43	0	0
0	39	0	2	38	5	0
0	40	0	0	40	2	0
0	51	0	10	41	0	0
0	1	0	3	3	1	0

Integral Image for Window Matching

- Current disparity is 1
- For each window pixel, have to compute window sums in AD image
- Apply integral image to AD image

AD image for disparity 1

0	2	1	0	2	2	2
0	3	3	3	0	2	0
0	39	0	0	43	0	0
0	39	0	2	38	5	0
0	40	0	0	40	2	0
0	51	0	10	41	0	0
0	1	0	3	3	1	0

Efficient Algorithm for Window Matching

for every pixel ***p*** **do**

  bestDisparity[***p***] = 0

  bestWindCost[***p***] = HUGE

 for disparity ***d = *** 0, 1,…, maxD **do**

  overlay images at disparity ***d***

  compute AD image for disparity ***d***

  compute Integral image from AD image

  for every pixel ***p*** **do**

   currentCost = window cost at pixel ***p***, computed from integral image

   if currentCost < bestWindCost[***p***]

    bestWindCost[***p***] = currentCost

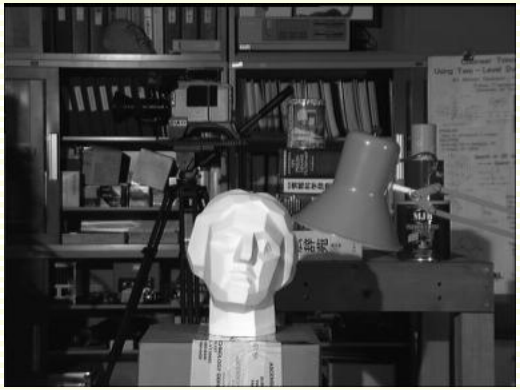
    bestDisparity[***p***] = ***d***

return bestDisparity

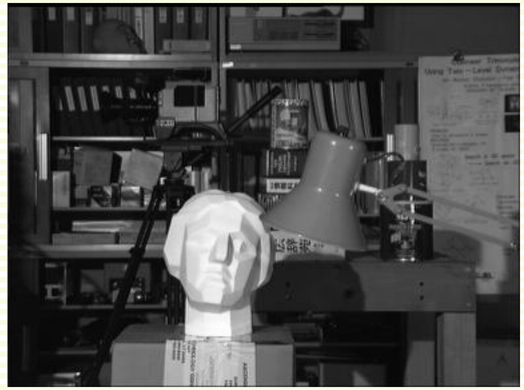
AD image for disparity 1

2	**1**	**0**	**2**	**2**	**2**
3	**3**	**3**	**0**	**4**	**0**
39	**0**	**0**	**43**	**1**	**0**
39	**0**	**2**	**38**	**2**	**0**
40	**0**	**0**	**40**	**2**	**0**
51	**0**	**10**	**41**	**0**	**0**
1	**0**	**3**	**3**	**1**	**0**

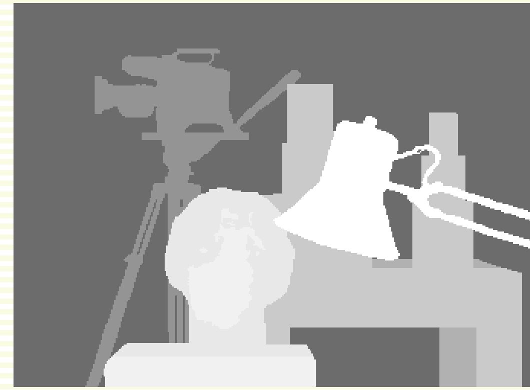
Effect of Window size



left image

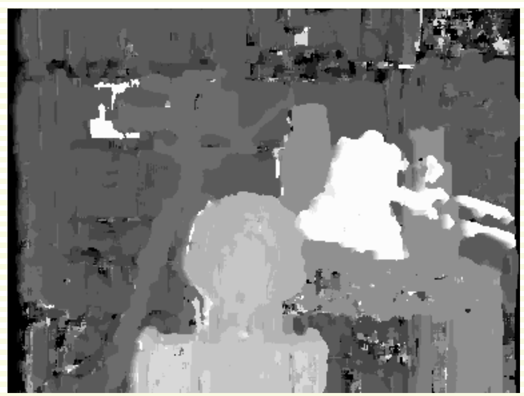


right image



true disparities
bright means larger disparity

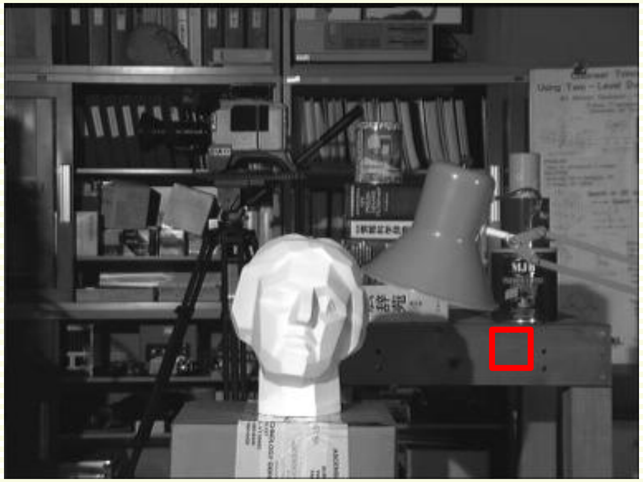
3x3 window



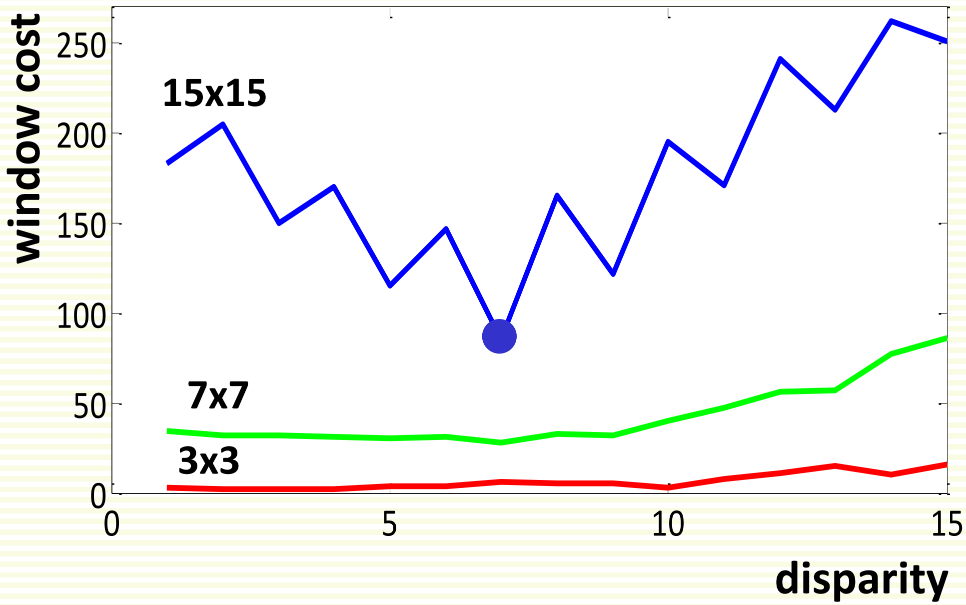
7x7 window

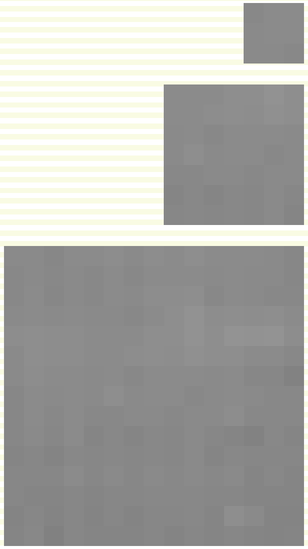
15x15 window

Effect of Window size: Low Texture Area



left image

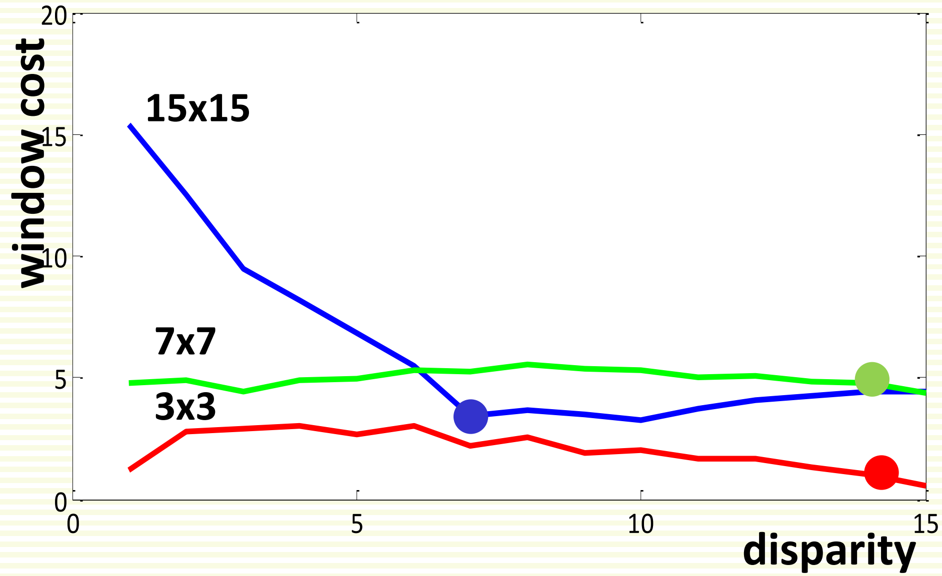




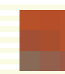
- windows of size 3x3 and 7x7 are too small to have a distinct pattern
  - no clearly best disparity
- window of size 15x15 is large enough to have a distinct pattern
  - 7 is clearly the best disparity
- window has to be **large enough**

Effect of Window size: Near Discontinuities

left image



- central pixel (the one we are matching) is the lamp



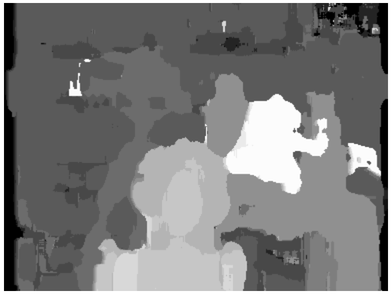
- windows of size 3x3 and 7x7 contain mostly the lamp

- window of size 15x15 contains mostly the wall
  - we match the wall instead of the lamp!

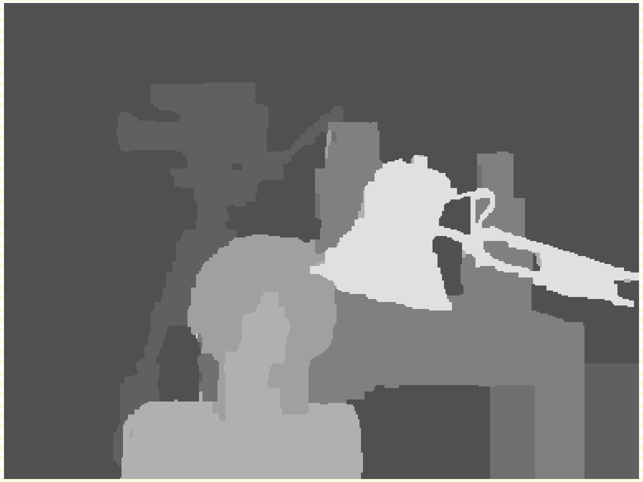
- window must be **small enough** to contain mostly the same object as the central pixel

Effect of Window size

- No single window size is ‘perfect’ for the image
- Smaller window
  - works better around object boundaries
  - noisy results in low texture areas
- Larger window
  - better results in low texture areas
  - does not preserve object boundaries well
- **Adaptive** window algorithms exist [Veksler’2001]



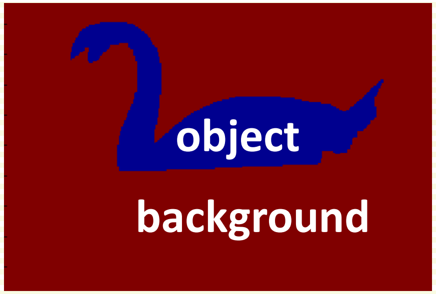
Better Stereo Algorithms



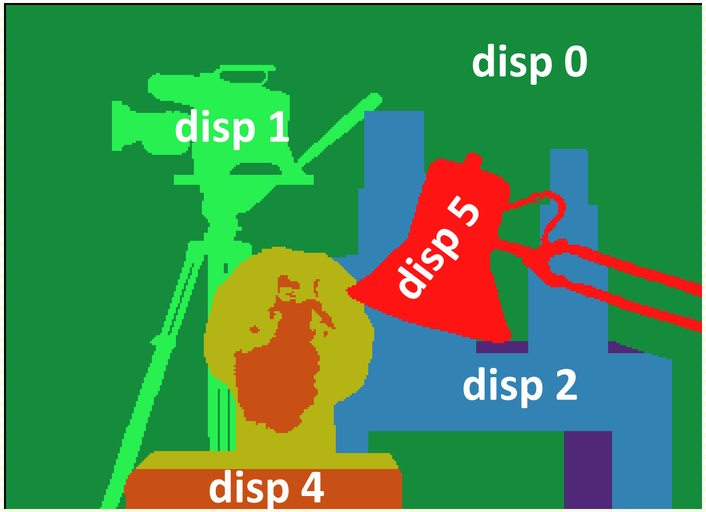
State of the art method
[Boykov, Veksler, Zabih, 2001]

ground truth

- Formulate stereo as energy minimization
- Recall binary object/background segmentation problem



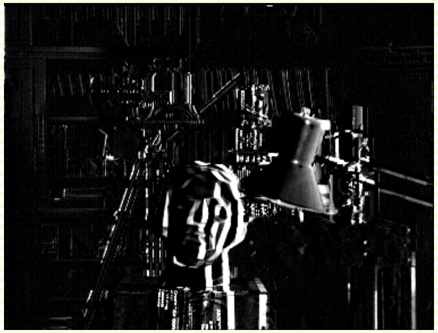
Better Stereo Algorithms



ground truth

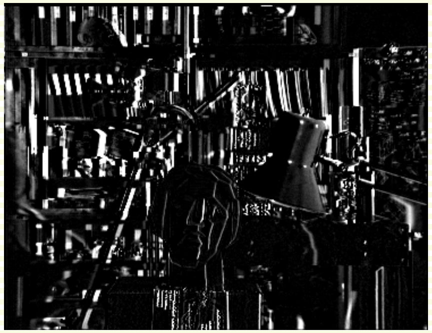
- Stereo is multi-label segmentation problem
  - region 0 = label 0 "likes" disparity 0
  - region 1 = label 1 "likes" disparity 1
  - …
  - region maxDisp = label maxDisp "likes" disparity maxDisp

Stereo with Graph Cuts

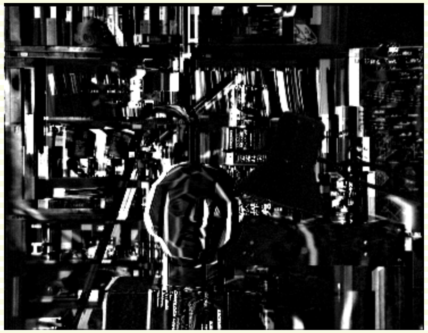


AD 5
data term for label 5

AD 8
data term for label 8



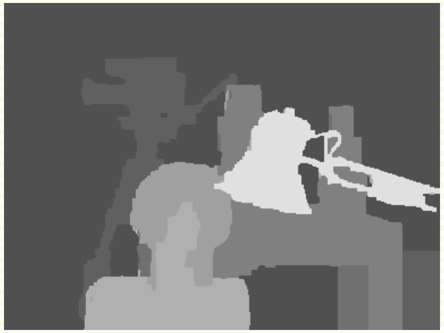
AD 10
data term for label 10



AD 14
data term for label 14

- Energy Function
  - Data Term: assign each pixel disparity label it likes
  - Smoothness Term: count number of label (disparity) discontinuities

- Solved with Graph Cuts: iteratively cuts out regions corresponding to disparities
- NP-hard with more than 2 labels, but computes a good approximation



Stereo with Graph Cuts

- Start with everything as label (disparity) 0

Stereo with Graph Cuts

- “Cut out” label (disparity) 1

Stereo with Graph Cuts

- “Cut out” label (disparity) 2

Stereo with Graph Cuts

- “Cut out” label (disparity) 3

Stereo with Graph Cuts

- "Cut out" label (disparity) 4

Stereo with Graph Cuts

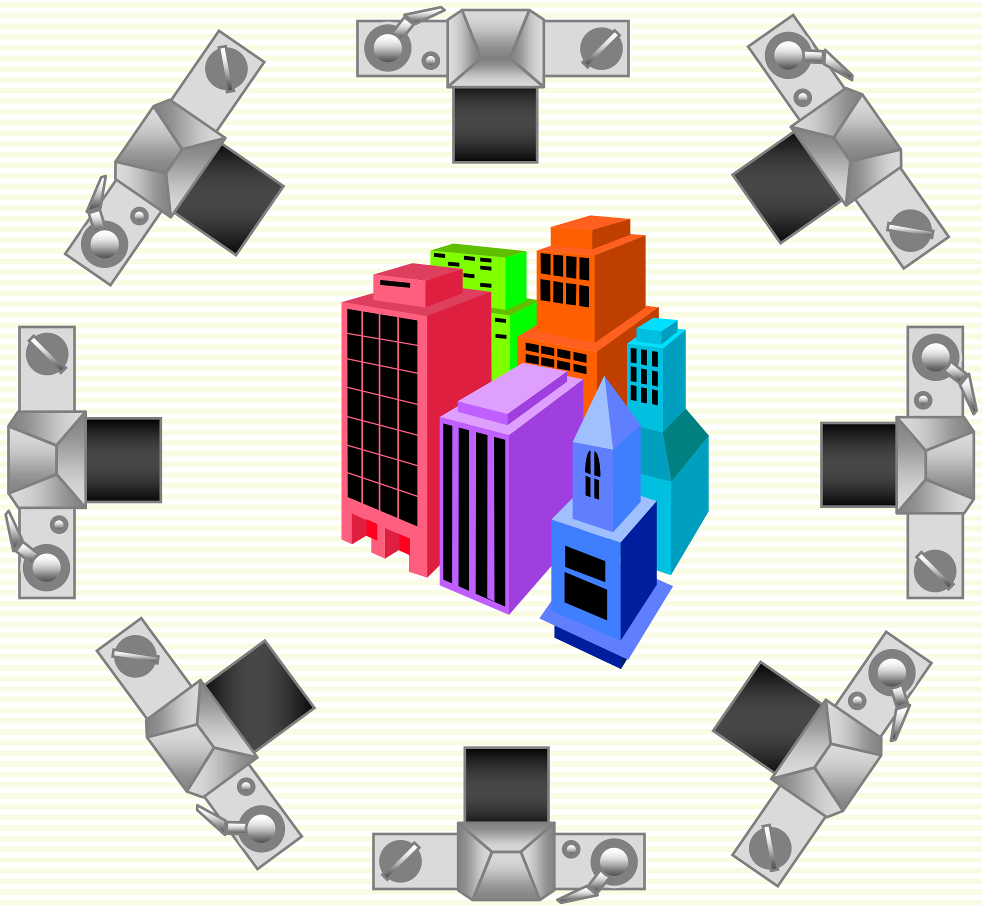
- “Cut out” label (disparity) 5

Stereo with Graph Cuts

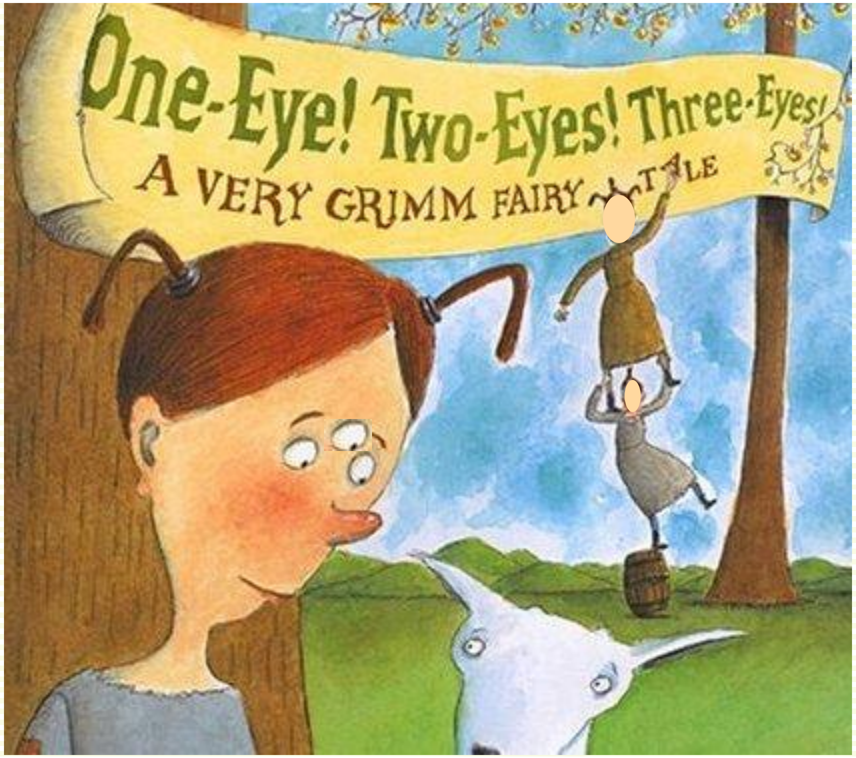
- "Cut out" label (disparity) 6

Multiple Artificial Eyes

- Two eyes better than one → three eyes better than two → four eyes better than three → … → the more, the better

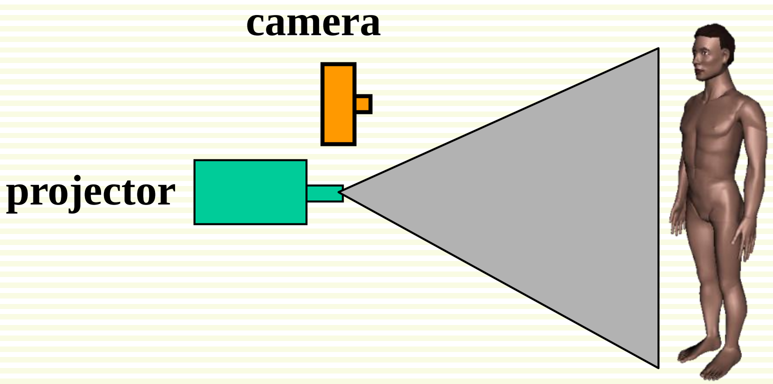


Common Folk New that Already



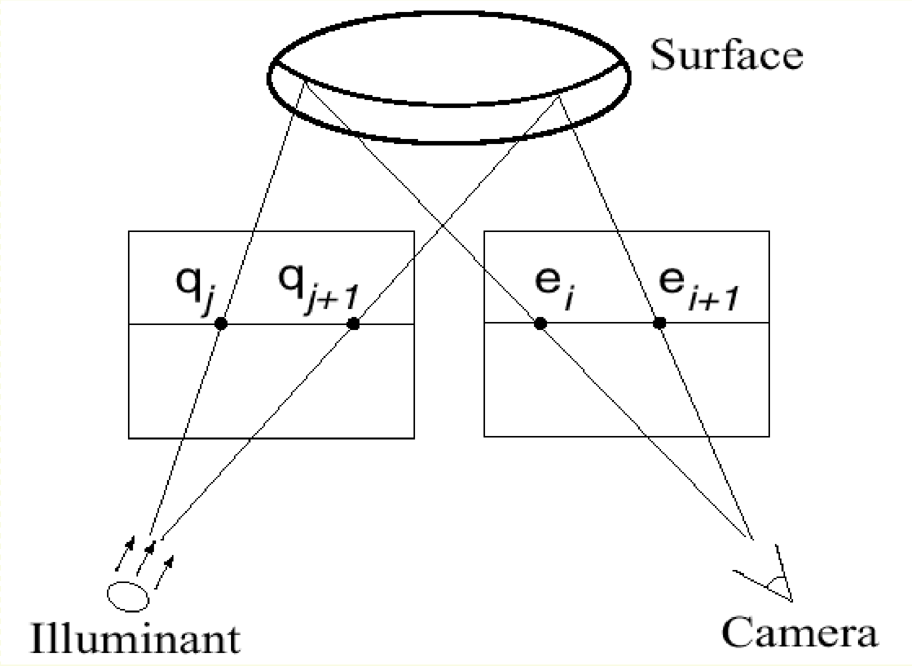
Stereo with Structured Light

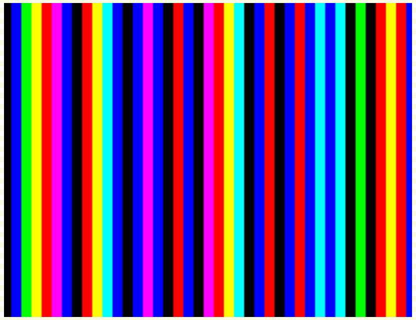
- Project “structured” light patterns onto the object
  - Simplifies correspondence problem
  - Need one camera and one projector

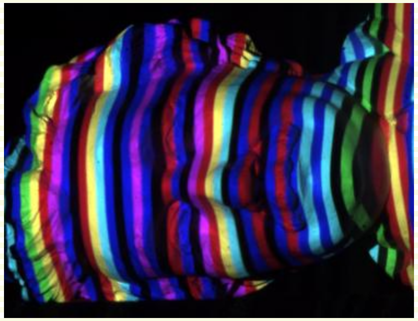


Stereo with Structured Light

- Triangulate between camera and projector



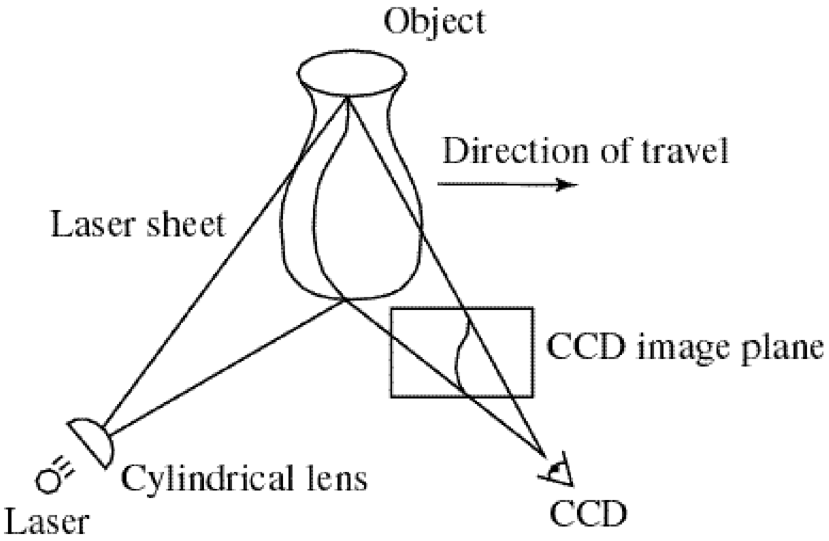


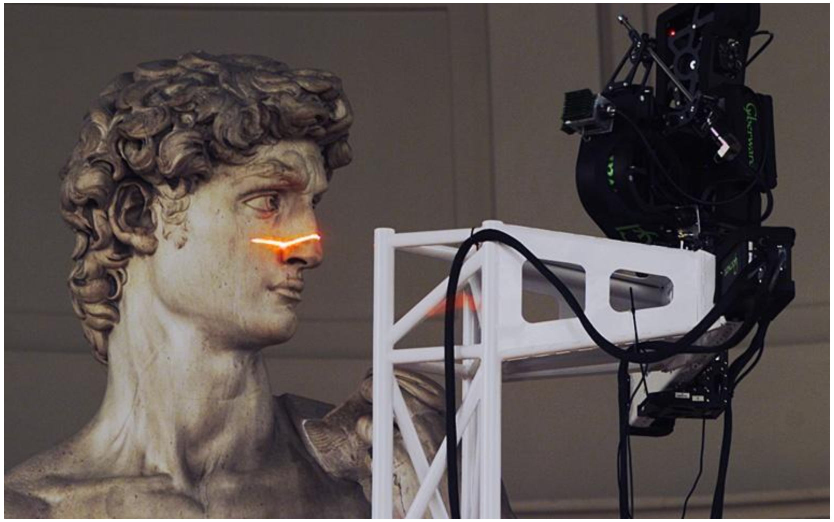


Kinect: Structured Infrared Light

http://bbzippo.wordpress.com/2010/11/28/kinect-in-infrared/

Laser Scanning





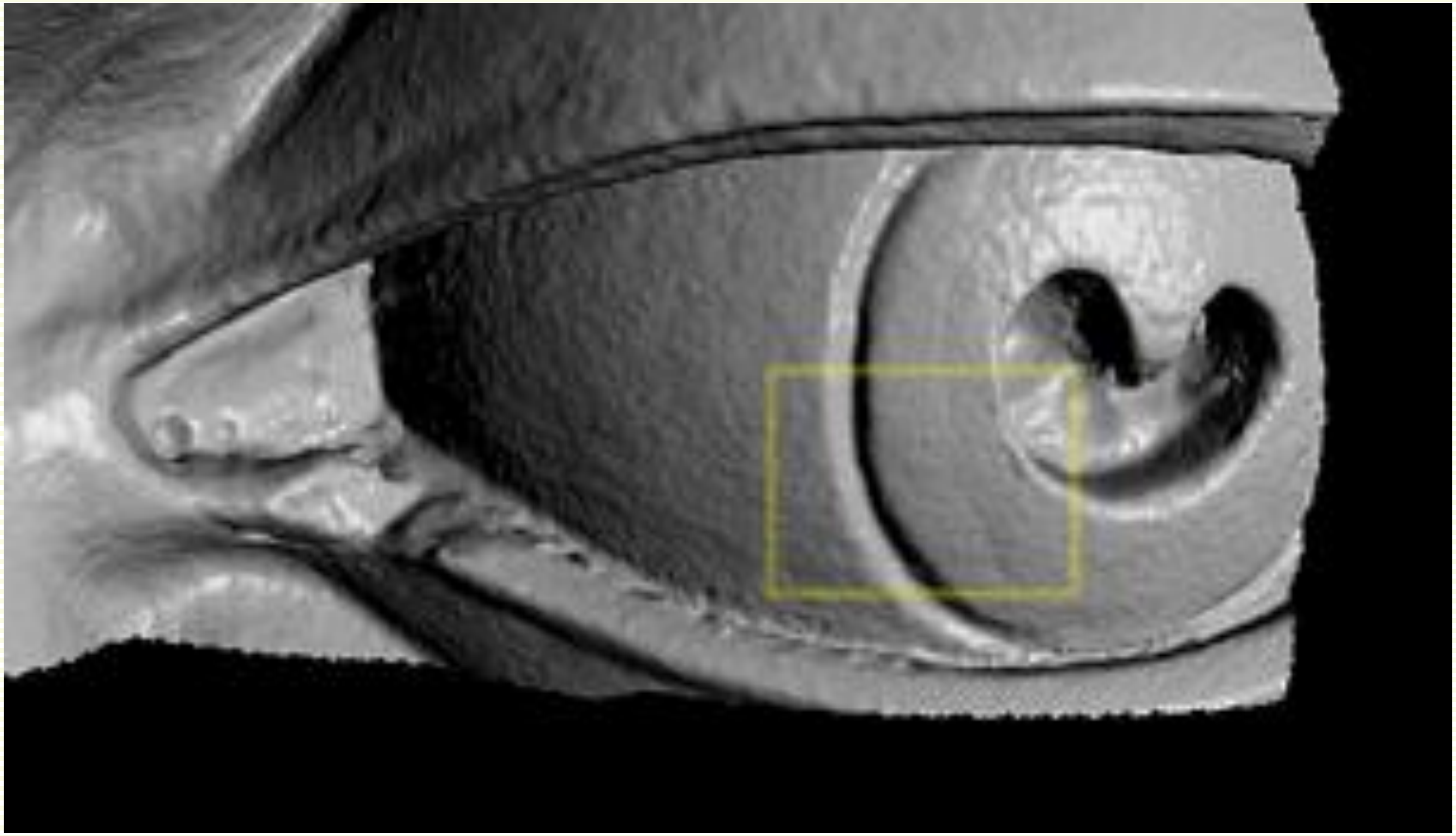
Digital Michelangelo Project
Levoy et al.
http://graphics.stanford.edu/projects/mich/

- Optical triangulation
  - Project a single stripe of laser light
  - Scan it across the surface of the object
  - This is a very precise version of structured light scanning

Laser Scanned Models

***The Digital Michelangelo Project*, Levoy et al.**

Laser Scanned Models



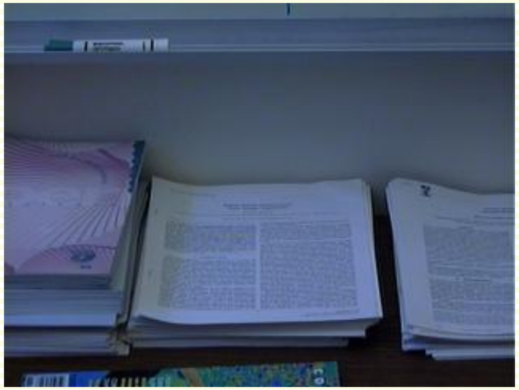
The Digital Michelangelo Project, **Levoy et al.**

Numerous Applications

- Autonomous navigation

Nomad robot searches for meteorites in Antartica
http://www.frc.ri.cmu.edu/projects/meteorobot/index.html

Novel View Synthesis


input image (1 of 2)


depth map
[Szeliski & Kang '95]


3D rendering

Applications: Video View Interpolation

http://research.microsoft.com/users/larryz/videoviewinterpolation.htm

Stereo Correspondence

- Steps:
  - Calibrate cameras
  - Rectify images
  - Stereo correspondence
  - Apply depth/disparity formula
- Stereo correspondence is still heavily researched
- The simple window matching algorithm we studied is heavily used in practice due to speed and simplicity
- Popular Benchmark http://www.middlebury.edu/stereo