On the Approximate GCD in Initial Value Problems

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Abstract

The computation of approximate greatest common divisors of polynomials has been considered by a number of authors under various different assumptions. Approximate GCDs have a number of applications, and have been used as an approach to ill-conditioned algebraic equations [Noda and Sasaki, J Comp and App Math 1991].

This paper examines the application of the approximate GCD to initial value problems. We consider initial value problems of the form \[ \sum_{i=0}^{n} a_i D^i y(t) = f(t) \] where \( a_i \) and \( y^{(i)}(0) \) are given constants. Under appropriate conditions, equations such as this may be solved by integral transform methods.

From the Laplace transform of the initial value problem, we see that \( \mathcal{L}[y(t)](s) \) is a rational function whenever \( \mathcal{L}[f(t)](s) \) is a rational function. Depending on \( a_i \) and \( y^{(i)}(0) \), there may be a non-trivial approximate GCD between the numerator and denominator of \( \mathcal{L}[y(t)](s) \). This paper examines the consequences of this fact and shows how the approximate GCD may be used to remove spurious singularities.