

On the Functional Decomposition of Multivariate Laurent Polynomials

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Determining whether a univariate polynomial may be written as the functional composition of two others of lower degree is a question that has been studied since at least the work of Ritt [1]. Algorithms by Barton and Zippel [2] and then by Kozen and Landau [3] have been incorporated in many computer algebra systems. Generalizations have been studied for functional decomposition of rational functions [4], algebraic functions [5], multivariate polynomials [6] and univariate Laurent polynomials [7].

We explore the functional decomposition problem for multivariate Laurent polynomials, considering the case $f = g \circ h$ where g is univariate and h may be multivariate. We present an algorithm to find such a decomposition if it exists.

The algorithm proceeds as follows: First, a variable weighting is chosen to make the weighted degree zero term in f constant. The positive degree and negative degree parts of h are then reconstructed separately, in a manner similar to that of Kozen and Landau, but by treating the homogeneous collections of terms by grade rather than individual monomials. Then terms of the univariate polynomial g are reconstructed degree by degree using a generic univariate projection of h .

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