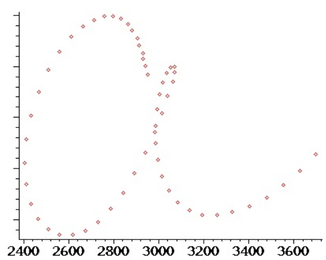
Online Stroke Modeling for Handwriting Recognition

Oleg Golubitsky   Stephen M. Watt

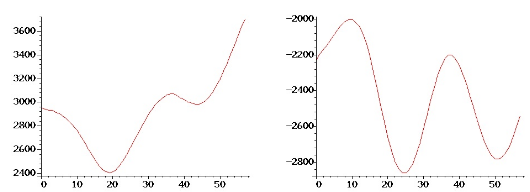
University *of* Western Ontario

We model digital ink strokes using truncated orthogonal series for the coordinate functions. We show how this allows

- compact, accurate character models,
- fast, online computation of series coefficients as the stroke is written,
- fast comparison of character models.



$\longmapsto$



$\longmapsto$

Sequence of sample points   Coord. functions approx. by linear splines

$$\longmapsto \quad \begin{cases} m_i^x = \int\limits_0^N t^i x(t)dt \\ m_i^y = \int\limits_0^N t^i y(t)dt \end{cases} \quad \longmapsto \quad \begin{cases} c_i^x = \int\limits_0^N x(t)P_i(\frac{t}{N})dt \\ c_i^y = \int\limits_0^N y(t)P_i(\frac{t}{N})dt \end{cases}$$

Moments   Legendre coefficients

$P_i(t)$ are Legendre polynomials on $[0,1]$ satisfying $\int_0^1 P_i(t)P_j(t)dt = \delta_{ij}$, and

$$x(t) = \sum_{i=0}^{\infty} c_i^x P_i\left(\frac{t}{N}\right), \qquad y(t) = \sum_{i=0}^{\infty} c_i^y P_i\left(\frac{t}{N}\right)$$

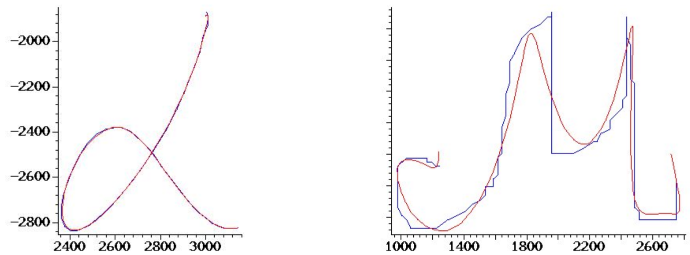
are Legendre expansions of the coordinate functions.

Truncate the Legendre expansions at degree d:

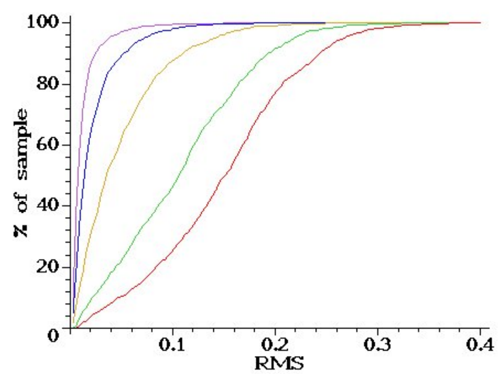
$$x(t) \approx \sum_{i=0}^{d} c_i^x P_i\left(\frac{t}{N}\right), \qquad y(t) \approx \sum_{i=0}^{d} c_i^y P_i\left(\frac{t}{N}\right)$$

Complexity: $O(d)$ online, $O(d^2)$ at pen-up.

Truncated Legendre expansions of degree $d = \mathbf{10}$ approximate the coordinate functions of character curves accurately:



Cutoffs of the root mean square error of approximations of 987 mathematical symbols by truncated Legendre expansions of degrees 3,4,6,8,10:



Coefficients of truncated Legendre series

- yield a robust representation of handwritten characters by points in a low-dimensional Euclidean vector space,
- this allows fast comparison of two characters (about 100 machine instructions) by computing the mean square distance between the curves:

$$\rho^2(C, \bar{C}) = \int_0^1 \left[x\left(\frac{t}{N}\right) - \bar{x}\left(\frac{t}{\bar{N}}\right) \right]^2 + \left[y\left(\frac{t}{N}\right) - \bar{y}\left(\frac{t}{\bar{N}}\right) \right]^2 dt \approx \sum_{i=0}^{d} [c_i^x - c_i^{\bar{x}}]^2 + [c_i^y - c_i^{\bar{y}}]^2$$