In Honour of Keith Geddes on his 60th Birthday

This issue celebrates the scientific career of Professor Keith Geddes, one of the key figures in symbolic computation and one of the creators of the MAPLE computer algebra system. This celebration began two years ago, in 2008, a year that marked several numerically significant events in Keith Geddes' life and career: He was born 60 years prior in Saskatchewan; he began his research 40 years prior as a graduate student at the University of Toronto; he co-founded Maplesoft 20 years prior in Waterloo; finally, 2008 was the year Keith chose to retire from the University of Waterloo. Thus the year marked the $20k$ year anniversaries of significant events for $k = 0, 1, 2, 3$. To commemorate the occasion, the *Milestones in Computer Algebra* conference was held 1-3 May 2008 in Stonehaven Bay, Tobago. Colleagues, students and friends celebrated Professor Geddes' achievements in fundamental research, in technology transfer, and in the training of the next generation of scientists, mathematicians and engineers. The articles presented here are refined versions of papers from that meeting. Before presenting these articles, we give here a short outline of some of the elements of Keith Geddes' work.

Keith received his PhD in 1973 from the University of Toronto under the supervision of John C. Mason, following which he accepted a position at the University of Waterloo, which he held until retirement. As a professor, Keith has supervised or co-supervised eight PhD students and forty eight Masters students. His research has spanned the areas of numerical approximation, algebraic algorithms for symbolic computation, hybrid symbolic-numeric computation and the design and implementation of computer algebra systems. Keith is the principal author of the textbook *Algorithms for Computer Algebra* [27], which served a generation as the standard reference in the field. Keith has actively supported the computer algebra community through the ACM Special Interest Group on Symbolic and Algebraic Manipulation (SIGSAM), which he chaired from 1991 to 1993, and numerous conference program committees. Keith served as the Scientific Director for the Ontario Research Centre for Computer Algebra (ORCCA) from its inception in 1999, until his retirement. Finally, Keith is perhaps best known as co-founder of the MAPLE computer algebra system. Through his teaching, research, service and software, the work of Keith Geddes has touched literally millions of individuals.

Research Highlights

Keith's 1973 PhD thesis and subsequent research work in the 1970s lay in the area of numerical approximation of functions. Two research papers [7; 9] deal with approximation by rational functions, specifically Padé and Chebyshev-Padé approximants. Two other papers [1; 5] study the topic of near-minimax polynomial approximation in regions of the complex plane. Following this, Keith's research focus moved toward computer algebra, where he investigated a number of problems, including GCDs, limits, integration and symbolic-numeric computation. Since 1980, this research on algorithms for computer algebra has been intertwined with design and development of the MAPLE computer algebra system, about which we say more later. We summarize some of Keith's many contributions here; a bibliography of his publications appears following this foreword.

Keith's work in the area of symbolic-numeric computation, e.g. [17; 26; 49], has spanned three decades. It exploits the increased problem-solving power that can be realized by a hybrid approach combining traditional numerical computation with automated symbolic analysis. One paper [44], joint with Masters student Wei Wei Zheng, considers numerical computations where a very high precision result is desired. It shows that, rather than performing the complete computation in high precision, it can be significantly more efficient to exploit the speed of hardware precision and to use an iterative refinement scheme to build up the desired high precision result. Another paper [50] develops a new hybrid symbolic-numeric method for the fast and accurate evaluation of multiple integrals. The method is shown to be effective both in high dimensions and with high accuracy. This work exploits the theory of tensor product series developed with Frederick Chapman in his 2003 PhD thesis.

During this period Keith also investigated the computation of GCDs and limits. Two papers [18; 19] present a probabilistic approach to polynomial GCD computation. The significance of these algorithms is that their practical implementations achieve an important speedup compared to traditional algorithms. PhD student Trevor Smedley wrote a thesis on the latter topic. The problem of computing limits of mathematical functions is the topic of another paper [20] that presents a new algorithm based on hierarchical series.

Keith's work on the symbolic computation of integrals has focused on practical problems in computing closed-form solutions of indefinite and definite integrals in computer algebra systems. One paper [22], joint with Masters student L.Y. Stefanus, studies the implementation and extension of the parallel Risch-Norman algorithm. Other papers, e.g. [24; 42; 48], deal with various other aspects of the integration problem.

More recently, Keith as been interested in the symbolic computation of summations and identities. Two papers [43; 47] are joint with PhD student Ha Le, Sergei Abramov of Moscow State University, and Jacques Carette, then of Maplesoft. This work develops some new algorithms and a library of MAPLE routines for computing closed-form solutions of summation problems. Another paper [51], joint with former PhD student Frederick Chapman, presents an algorithm for generating and proving a variety of mathematical function identities.

Algorithms for Computer Algebra

Keith Geddes' textbook, *Algorithms for Computer Algebra*, with Stephen Czapor and George Labahn, appeared in the early 1990s and served as the first standard reference in the field. This work compiled and fleshed out a body of essential computer algebra knowledge, much of which had previously been available only through a myriad of conference and journal papers, unpublished theses and word of mouth. Even today, it remains the only general text to explain certain important topics in detail, such as multivariate Hensel lifting and the Risch algorithm for indefinite integration. One of the unique benefits of this text is that its chapters were refined over a decade by use in a heavily subscribed computer algebra course and through implementation by the authors of all of the algorithms in a widely used computer algebra system.

Maple

Keith Geddes and Gaston Gonnet, now at ETH Zürich, created the Maple research project in the Symbolic Computation Group at the University of Waterloo and are co-founders of the software company Maplesoft. While both contributed to all aspects of the Maple system, Keith Geddes was the principal architect of the core library of fundamental mathematical algorithms.

Maple was conceived in December of 1980 and was used immediately as a research and teaching tool. Two papers [10; 13], published in 1983 and 1984, describe the basic design decisions taken for the system and present performance measurements. A detailed history of Maple is available elsewhere (`http://www.scg.uwaterloo.ca/history`). We find it remarkable that the set of decisions taken three decades ago remain a sound basis for a major computer algebra system, and consequently a highly successful and well-respected mathematical software business. The current version of Maple is described in the reference works [52; 53].

It is hard to find a part of the Maple library where Keith has not had a direct influence. His contributions to the basic mathematics engine of Maple are used by every Maple user every day. Without his work on efficient basic routines for polynomial algebra, including work on the Hensel-based GCD and factorization of multivariate polynomials, Maple would not be usable at all.

Although his textbook and published research articles would make a successful career for anyone, Keith Geddes would argue that it is his code in Maple that represents his most significant contribution to research and practical applications.

The Milestones in Computer Algebra Conference

To celebrate the Keith's contributions to computer algebra, some four dozen scientific colleagues from three continents came together for the *Milestones in Computer Algebra* conference 1-3 May 2008. The participants included eight distinguished invited speakers, two dozen colleagues who contributed scientific papers and posters, and others who come to pay their respects. In addition, a great many colleagues sent their wishes, but could not attend in person. Each of the invited talks touched on Keith's work in some important way, from providing historical context to presenting related current research. These were presented over three days, in the sequence:

- Joel Moses, Macsyma*: A Personal History*
- Michael Monagan. *How Fast Can We Multiply and Divide Sparse Polynomials?*
- Gaston Gonnet, *Maple as a Prototyping Language*
- Peter Paule, *Integrals, Sums and Computer Algebra*
- B. David Saunders, *Linear Algebra*
- David Stoutemyer, *Ten Commandments for Good Default Expression Simplification*
- Jan Verschelde, *Tropical Algebraic Geometry in Maple*

In addition, Jim Cooper, the President and CEO of Maplesoft, recounted his experiences with Maple, the company and our field in his post-banquet presentation, *20 Years and 6 Definitions of Zero Later...*.

This Issue

This issue presents refined versions of some of the contributions to the *Milestones in Computer Algebra* conference. From the above discussion, we can appreciate the breadth of the contribution of Keith Geddes to computer algebra and are pleased to say that the articles in this issue reflect that legacy.

We begin with the paper "High-Precision Numerical Integration: Progress and Challenges" by Bailey and Borwein, who couple high-precision numerical integration with integer relation detection methods to dramatically extend our capability for numerical evaluation of integrals. The interplay of symbolic and numeric techniques for numerical problems is continued with the paper of Adrovic and Verschelde on "Tropical Algebraic Geometry in Maple: a preprocessing algorithm for finding common factors to multivariate polynomials with approximate coefficients", which employs techniques from tropical algebraic geometry to tackle the important problem of computing common factors of polynomials with approximate coefficients.

Villard's paper "Kaltofen's Division-Free Determinant Algorithm Differentiated for Matrix Adjoint Computation" gives a fast algorithm for the adjoint of a matrix without using division that hence works over any commutative ring.

One of the most important considerations in a computer algebra system is that of data structures and the tradeoff between compactness and efficiency. Roche employs amortized analysis techniques in his paper "Chunky and Equal-Spaced Polynomial Multiplication" to provide gradient between sparse and dense representations. Monagan and Pearce, in "Sparse Polynomial Division Using a Heap", give a heap-based algorithm for multiplication and division of sparse polynomials with integer coefficients. They demonstrate impressive improvements in the implementation of their techniques. The problems of efficient polynomial multiplication and dot product are explored by Dumas, Fousse, and Salvy in their paper "Simultaneous Modular Reduction and Kronecker Substitution for Small Finite Fields". This paper provides precise analyses and implementation, as well as some important applications. Li, Moreno Maza, Rasheed and Schost, in their paper "The MODPN Library: bringing fast polynomial arithmetic into MAPLE", make a study of the implementation and integration of asymptotically fast arithmetic for triangular sets in MAPLE. Of particular interest is the interplay of low- and high-level software within a sophisticated algebraic library.

David Stoutemyer provides "Ten Commandments for Good Default Expression Simplification", which elaborates on principles of expression simplification and in particular some benefits of the recursive partially-factored semi-fraction form.

Finally, we include in a new "Historical Perspective" category for the *Journal*, a contribution by Joel Moses, one of the founders of the field. His article, "MACSYMA: A Personal History", describes the history of the MACSYMA system and some consequences for modern computer algebra. This brings us full circle to the work of Keith Geddes, as this is the context in which MAPLE was born.

We close on a personal note. We take this occasion to express our deep appreciation for having Keith Geddes as a mentor, a colleague and a friend.

August 8, 2010

Mark W. Giesbrecht
Stephen M. Watt

PhD Students of Keith Geddes

M. Olorunsola Afolabi (1979), Stephen M. Watt (1986), Stephen R. Czapor (1988), Trevor J. Smedley (1989), Masoud Kavian (1994 incomplete, deceased), Ha Quang Le (2003), Frederick W. Chapman (2003).

Masters Students of Keith Geddes

S. Saxena (1976), H.J. Harrison (1976), S.C. Chan (1976), M.O. Afolabi (1976), V.F. Santos (1977), S. Nahar (1977), T.Y. Hou (1977), S.Y. Fan (1977), D.J. Allenger (1977), W.C.S. Wong (1978), C.K. Wong (1978), E.O. Omojokun (1978), E. Mohammed (1978), L.J. Makela (1978), C.A. Edwards (1978), K.F. O'Connor (1979), J.C. Li (1979), W.F.D. Leung (1979), K.C. Lam (1979), Y.P. Haw (1979), F.A. Akinniyi (1979), K.C. Ng (1980), H.Z. Yang (1982), P.C. McGeer (1982), B. Farrahi (1982), K.A. Friend (1983), R.K. Bell (1983), J.B. Chiasson (1984), G. Zou (1985), T.C. Chen (1985), T.J. Smedley (1987), L.Y. Stefanus (1988), B.A. Filliman (1988), S. Suarga (1992), K.W.W. Ma (1994), M. Kavian (1994), H.F.J. Chan (1996), T.J. Holly (1998), F.W. Chapman (1998), Ka-Wai Chan (1999), Wei Wei Zheng (2002), J. Taylor (2003), O. A. Carvajal (2004), Sauw Ming Liong (2004), T. Robinson (2005), Jingchi (Evan) Chen (2007), Xiang (Sophy) Wang (2008), J. Peasgood (2009).

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