Inverse Preference Elicitation
for Sequential Decision Making

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Evidence-Based Medicine

- “It’s about integrating individual clinical expertise and the best external evidence.”

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How do we provide the “best external evidence?”

- One approach: Collect and analyze data, recommend treatments
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  - Input: Patient state, Output: Salient information about available treatments that reflects the evidence in the data.
How do we provide the “best external evidence?”

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- Approach: Modify methods and algorithms that recommend a single treatment to produce richer information about available treatments
Motivation

Example: Reinforcement Learning for Chronic Disease Management

- Chronic diseases are *managed*, not cured
  - Major depressive disorder
  - Schizophrenia
  - ...

- Treatment decisions should be:
  - Personalized
    - Current treatment is chosen based on current patient state
  - Non-myopic
    - Current treatment is chosen conditioned on future treatment strategy

- Reinforcement Learning (RL) methods can be used to learn a personalized, non-myopic treatment policy from data...
  - ...but almost all current RL methods recommend a single treatment.
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Preferences in Schizophrenia Treatment

• Many treatments available for managing schizophrenia (dozens)

• Consider two important objectives or rewards:
  • Symptom reduction, weight control

• No treatment is best by both measures

• Different doctors and patients have very different preferences about relative importance of rewards, and preference information is absent from large schizophrenia datasets

• Recommending a single treatment based on available data is not appropriate.
The Inverse Preference Elicitation Project

- How can we provide salient information about available treatments that is non-myopic and that accommodates these preferences?

1. Augment Q-Learning to allow for different reward preferences
   - Formalize preferences as a multi-objective optimization problem

2. Develop an algorithm tailored to randomized trial data that provides information for each treatment for all preferences simultaneously
Example: Decision Aid for Choosing Antipsychotics

- Possible decision aid:

<table>
<thead>
<tr>
<th>Initial Symptoms</th>
<th>Preference</th>
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<tbody>
<tr>
<td></td>
<td>Symptom Relief</td>
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<tr>
<td></td>
<td>Strong</td>
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<tr>
<td>Good</td>
<td>Olan</td>
</tr>
<tr>
<td>Moderate</td>
<td>Olan</td>
</tr>
<tr>
<td>Bad</td>
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Olan = Olanzapine, Zip = Ziprasidone, Risp = Risperidone

- This is harder than it looks
Q-Learning - Scalar Reward, Two Time Points

- Randomized trial data: \((S_1, A_1, S_2, A_2, R)\) for each individual
  - \(S_t \in S_t\) - “State” - Patient features (prior treatments, test results, ...)
  - \(A_t \in A_t\) - “Action” - Treatment assigned by exploration policy
  - \(R \in \mathbb{R}\) - “Reward” - Scalar clinical outcome, depends on \((S_2, A_2)\)

- Want to find \(\pi^*\) that produces maximal expected reward

- A “policy” \(\pi = \{\pi_1, \pi_2\}\) chooses actions given state
  - \(\pi_t : S_t \rightarrow A_t\)
  - \(\pi_1\) influences distribution of \(S_2\) by choosing \(A_1\)
  - \(\pi_2\) influences distribution of \(R\) by choosing \(A_2\)
Q-Learning - Dynamic Programming

- Q-Learning is Dynamic Programming. Determine $\pi_2^*$, then $\pi_1^*$.

**Time 2**

- Define $Q_2(s_2, a_2) = E[R|S_2 = s_2, A_2 = a_2]$
  - For state $s_2$, this is *quality* of each $a_2 \in A_2$.
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Q-Learning from Data: Repeated Regression

- Estimate each $Q_t$ by linear regression on features $\phi_t(s_t, a_t)$

**Time 2:**

\[ Q_2(s_2, a_2) = E_R[R|S_2 = s_2, A_2 = a_2] \]

- Regress $R$ on features $\phi_2(S_2, A_2)$ [$R$ might be symptom reduction] to obtain
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- $\hat{\pi}_1^*(s_1) = \text{argmax}_{a_1} \hat{Q}_1(s_1, a_1)$
Q-Learning - Beyond Scalar Rewards and Values

- Learned policy \( \hat{\pi}^*_t(s_t) = \arg\max_{a_t} \hat{Q}_t(s_t, a_t) \) is constructed to maximize long-term expected reward, i.e. is non-myopic

- Dynamic programming “trick” is to maximize over \( a_2 \) first
  - Key step: Set \( \hat{V}_2^{\hat{\pi}^*_2}(s_2) = \max_{a_2} \hat{Q}_2(s_2, a_2) \)
  - Only makes sense if \( R \) is scalar

- What if there is more than one \( R \) of interest?
Time 2 Policy - Two Rewards

- Suppose two rewards $R^{(0)}$ and $R^{(1)}$ are of interest, e.g. symptom reduction and weight control.
- Below, $(\hat{Q}^{(0)}_2, \hat{Q}^{(1)}_2)$ for patient with $S_2 = s_2$, four different actions.
- What should $\hat{\pi}^*_2(s_2)$ be?
Formalizing Preference

- Define a scalar reward $R(\delta) = (1 - \delta)R^{(0)} + \delta R^{(1)}$
- $0 \leq \delta \leq 1$
- Proceed as before to get $\hat{Q}_2(\delta)$

- $\delta$ represents “How much do I care about $R^{(1)}$?”
Preference Elicitation

- Function $R^{(\delta)}$ is an *Aggregate Objective Function* (many names…) familiar in multi-objective optimization

- Preference Elicitation Approach:
  - Figure out the decision maker’s $\delta$
  - Define a scalar reward $R^{(\delta)} = (1 - \delta) \cdot R^{(0)} + \delta \cdot R^{(1)}$
  - Use Q-learning to estimate the optimal policy for that reward

- Resulting policy is Pareto optimal

- E.g., for $\delta = 0.5$

![Diagram showing preference elicitation for $\delta = 0.5$.]
Preference Elicitation

• E.g., “Consider two actions. You can have (8, 5), or you can have (5, x). What value of x makes you indifferent to this choice?”

• Find $\delta$ so that $R^{(\delta)}$ is equal for the two points
  - $(1 - \delta) \cdot 8 + \delta \cdot 5 = (1 - \delta) \cdot 5 + \delta \cdot x$

• Doubt about whether or not this actually works
• Has nothing to do with the actions that are actually available, i.e. does not provide salient information about available treatments.

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\(^2\)Actual question would be more subtle.
Contribution: Inverse Preference Elicitation

- Preference Elicitation
  - “Give me your $\delta$, I will tell you the right action.”

- Infinite number of $\delta$, but only 4 actions

- Inverse Preference Elicitation
  - “Given each available action, I will tell you the $\delta$ for which that action is optimal.”
  - This is our salient information
Contribution: Inverse Preference Elicitation

- “Given each available action, I will tell you the $\delta$ for which that action is optimal.”
- Each action is optimal over a range of $\delta$
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- “Given each available action, I will tell you the $\delta$ for which that action is optimal.”
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![Diagram showing care for $R^{(0)}$ and $R^{(1)}$, with actions a at different $\delta$ values and corresponding $\hat{Q}$ values.]

- Note action $a$ does not appear
Contribution: Inverse Preference Elicitation

- “Given each available action, I will tell you the $\delta$ for which that action is optimal.”
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- Care only about $R^{(0)}$
- Care only about $R^{(1)}$

- Note action $a$ does not appear
- Can see sensitivity of action choice w.r.t. preference
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- “Given each available action, I will tell you the $\delta$ for which that action is optimal.”
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- Note action $a$ does not appear
- Can see sensitivity of action choice w.r.t. preference
- Decision aid from the beginning is a coarsened version of a picture like this
Non-Myopic Inverse Preference Elicitation

- At each timepoint $t$, define $\hat{Q}_t(s_t, a_t; \hat{\beta}_t(\delta)) = \hat{\beta}_t^T(\delta) \phi_t(s_t, a_t)$
  - Before, each $\hat{Q}_t(s_t, a_t)$ was a scalar. Now they are functions of $\delta$.

- Perform Q-learning for all $\delta$ simultaneously

- At Time 2, finding ranges of $\delta$ is straightforward
  - Use convex hull to identify regions of $\delta$

- At Time 1, things get interesting

- **Challenge:** represent $\hat{Q}_1(s_1, a_1; \hat{\beta}_1(\delta))$
  - Exactly
  - Economically
Q-Learning for all $\delta$: Time 2

- Notice that $\hat{Q}_2(s_2, a_2; \hat{\beta}_2(\delta)) = \phi_2(s_2, a_2)^T\hat{\beta}_2(\delta)$ is linear in $\delta$, so only compute $\hat{\beta}_2(0)$ and $\hat{\beta}_2(1)$.
Q-Learning for all $\delta$: Time 1

- Notice $\hat{V}_2(s_2; \delta)$ is piecewise linear in $\delta$. 

![Graph showing $\hat{V}_2(s_2, \delta)$ with annotations and labels for expected values.](image)
Q-Learning for all $\delta$: Time 1

- Notice that $\hat{\beta}_1(\delta)$ and $\hat{Q}_1(s_1, a_1; \hat{\beta}_1(\delta))$ are linear over regions of $\delta$ where $\hat{V}_2(s_2; \delta)$ is simultaneously linear for all $s_2$.

- Only need to evaluate $\hat{\beta}_1(\delta)$ at union of knots in $\hat{V}_2(S_2; \delta)$
Example: CATIE

- Large (N = 1460) comparative effectiveness trial

- Most patients randomized two times:
  - First to one of 5 actions
  - Then, if desired, to one of 5 different actions

- Following is a *highly* simplified analysis
- Overall, the results are consistent with the literature
- Rewards: symptoms relief, weight control
Example: CATIE Inverse Preference Elicitation

The diagram illustrates the estimated value of different symptom reductions as a function of preference for weight control. The x-axis represents the preference for weight control, ranging from 0.0 to 1.0. The y-axis shows the estimated value, ranging from 0.0 to 100.0. The graph compares the estimated value for different types of symptoms: Good Symptoms (purple line), Moderate Symptoms (red line), and Bad Symptoms (blue line). Each line represents a different medication: Risperidone, Olanzapine, and Ziprasidone.
Example: CATIE-based Decision Aid

- Possible decision aid: Coarse version of the plots

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<td>Good</td>
<td>Olan</td>
<td>Olan or Zip</td>
</tr>
<tr>
<td>Moderate</td>
<td>Olan</td>
<td>Olan or Zip</td>
</tr>
<tr>
<td>Bad</td>
<td>Olan</td>
<td>Olan</td>
</tr>
</tbody>
</table>

Olan = Olanzapine, Zip = Ziprasidone, Risp = Risperidone

- Thanks to Holly Wittemann, Brian Zikmund-Fisher, UMich SPH
Future Work

• Algorithms and Methods for Generating Evidence
  • More flexible models / approximation algorithms for preferences
  • Measures of uncertainty - requires interesting optimization
    • Ask me about this!
  • “Classical” ML problems (feature selection/dimensionality reduction/model selection, feature extraction via NLP, accommodating missing data…)
  • Must still provide salient information!

• Clinical Science Applications
  • Schizophrenia - CATIE
  • Major Depressive Disorder - STAR*D
  • ICU data (non-randomized) - MIMIC, MIMIC II
  • EHR?
Thank You

- Supported by National Institute of Health grants R01 MH080015 and P50 DA10075
- Related work:
Confidence Intervals for Q-Learning

- Question: In state $s_t$, is there evidence that $a$ is really better than $a$?
- Classical approach: get confidence interval for $\hat{\beta}_t^T \cdot (\phi(s_t, a) - \phi(s_t, a))$

- For $t = T$, under mild assumptions on $R$, can use normal approximation or bootstrap
- For $t < T$, standard methods can fail even as $n \to \infty$

- Trouble arises when statistics (e.g. $\hat{\beta}_t$) are non-differentiable functions of the dataset
- $\hat{\beta}_1$ based on $\hat{V}_2(s_2) = \max_a \hat{Q}_2(s_2, a)$
Confidence Intervals for Q-Learning

• Question: In state $s_t$, is there evidence that $a$ is really better than $a$?
• Classical approach: get confidence interval for
  \[ \hat{\beta}_t^T \cdot (\phi(s_t, a) - \phi(s_t, a)) \]
• For $t = T$, under mild assumptions on $R$, can use normal approximation or bootstrap
• For $t < T$, standard methods can fail even as $n \to \infty$

• Trouble arises when statistics (e.g. $\hat{\beta}_t$) are non-differentiable functions of the dataset
• $\hat{\beta}_1$ based on $\hat{V}_2(s_2) = \max_a \hat{Q}_2(s_2, a)$
Confidence Intervals for Q-Learning

- Question: In state $s_t$, is there evidence that $a$ is really better than $a'$?
- Classical approach: get confidence interval for
  $$\hat{\beta}_t^T \cdot (\phi(s_t, a) - \phi(s_t, a'))$$
- For $t = T$, under mild assumptions on $R$, can use normal approximation or bootstrap
- For $t < T$, standard methods can fail even as $n \to \infty$
- Trouble arises when statistics (e.g. $\hat{\beta}_t$) are non-differentiable functions of the dataset
- $\hat{\beta}_1$ based on $V_2(s_2) = \max_a Q_2(s_2, a)$
Adaptive Confidence Intervals for Q-Learning

• A method that produces correct coverage:
  • Re-sample a dataset $\mathcal{D}'$ with replacement
  • Compute $\tilde{\beta}_t = \arg \max_{\beta \text{ near } \hat{\beta}_t} f(\beta, \mathcal{D}')$
  • Repeat

• Use distribution of $\tilde{\beta}_t$ to make C.I.

• The $\arg \max_{\beta \text{ near } \hat{\beta}_t} f(\beta, \mathcal{D}')$ problem is interesting
  • Non-convex
  • Piecewise linear but possibly not continuous
  • Can formulate as MIP, but maybe we can do better...