Reward Preferences
in Reinforcement Learning

Dan Lizotte
Postdoctoral Fellow, University of Michigan Statistics

Joint work with Michael Bowling†, Susan A. Murphy‡
†University of Alberta, ‡University of Michigan

Montefiore Institute
University of Liège
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About Me

Learning Treatment Policies from Data
- Informing treatment decisions in a non-myopic way
- Accommodating preferences
- Measures of confidence

Global Optimization
- Deciding which domain points to investigate next
- Fundamentals: Experimental evaluation, quantifying difficulty, invariant methods
- Application: Robot gait learning

Budgeted/Active Learning
- Deciding which data items to purchase next
- Budgeted learning in the naïve Bayes setting
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Health Informatics

- Development and application of computer science methods for problems that arise in health care.
A Healthcare “Problem”: Evidence-Based Medicine

• “It’s about integrating individual clinical expertise and the best external evidence.”\(^1\)

• “Evidence based medicine is not restricted to randomised trials and meta-analyses.”

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Motivation

A Healthcare “Problem”: Evidence-Based Medicine

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How do we provide the “best external evidence?”

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  - Input: Patient state, Output: Recommended treatment
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• Approach: Modify methods and algorithms that recommend a single treatment to produce richer information about available treatments
Example: Reinforcement Learning for Chronic Disease Management

- Chronic diseases are *managed*, not cured
  - Type 2 diabetes
  - Major depressive disorder
  - Schizophrenia
  - ...

- Treatment decisions should be:
  - Personalized
    - Current treatment is chosen based on current patient state
  - Non-myopic
    - Current treatment is chosen conditioned on future treatment strategy

- Reinforcement Learning (RL) methods can be used to learn a personalized, non-myopic treatment policy from data
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Challenges in Using Reinforcement Learning to Provide Evidence

- “Output-Related” Challenges
  - Conveying strength of evidence
  - Accommodating preferences
  - ...

- These are in addition to more usual “Input-Related” Challenges
  - Curse of dimensionality
  - Discovering good features
  - Missing data/partial observability
  - Causal inference
  - Computational tractability
  - ...

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Preferences in Schizophrenia Treatment

- Many treatments available for managing schizophrenia (dozens)

- Consider two important objectives or rewards:
  - Symptom reduction, weight control

- No treatment is best by both measures

- Different doctors and patients have very different preferences about relative importance of rewards

- Preference information is absent from large schizophrenia datasets
The Inverse Preference Elicitation Project

- How can we provide salient information about available treatments that is non-myopic and that accommodates these preferences?

1. Augment Q-Learning to allow for different reward preferences
   - Formalize preferences as a multi-objective optimization problem

2. Develop an algorithm tailored to randomized trial data that provides information for each treatment for all preferences simultaneously
Example: Decision Aid for Choosing Antipsychotics

- Possible decision aid:

<table>
<thead>
<tr>
<th>Initial Symptoms</th>
<th>Preference</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Symptom Relief</td>
<td>Weight Control</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Strong</td>
<td>Mild</td>
<td>Mild</td>
</tr>
<tr>
<td>Good</td>
<td>Olan</td>
<td>Olan or Zip</td>
<td>Zip</td>
</tr>
<tr>
<td>Moderate</td>
<td>Olan</td>
<td>Olan or Zip</td>
<td>Zip</td>
</tr>
<tr>
<td>Bad</td>
<td>Olan</td>
<td>Olan</td>
<td>Risp or Zip</td>
</tr>
</tbody>
</table>

Olan = Olanzapine, Zip = Ziprasidone, Risp = Risperidone

- This is harder than it looks
Q-Learning - Scalar Reward, Two Time Points

• Randomized trial data: \((S_1, A_1, S_2, A_2, R)\) for each individual
  • \(S_t \in S_t\) - “State” - Patient features (prior treatments, test results, …)
  • \(A_t \in A_t\) - “Action” - Treatment assigned by exploration policy
  • \(R \in \mathbb{R}\) - “Reward” - Scalar clinical outcome, depends on \((S_2, A_2)\)

• Want to find \(\pi^*\) that produces maximal expected reward

• A “policy” \(\pi = \{\pi_1, \pi_2\}\) chooses actions given state
  • \(\pi_t : S_t \rightarrow A_t\)
  • \(\pi_1\) influences distribution of \(S_2\) by choosing \(A_1\)
  • \(\pi_2\) influences distribution of \(R\) by choosing \(A_2\)
Q-Learning - Dynamic Programming

- Q-Learning is Dynamic Programming. Determine \( \pi_2^* \), then \( \pi_1^* \).

**Time 2**

- Define \( Q_2(s_2, a_2) = E[R|S_2 = s_2, A_2 = a_2] \)
  - For state \( s_2 \), this is quality of each \( a_2 \in \mathcal{A}_2 \).
- \( \pi_2^*(s_2) = \arg\max_{a_2} Q_2(s_2, a_2) \)
- \( V_{2}^{\pi_2^*}(s_2) = \max_{a_2} Q_2(s_2, a_2) \) the value of being in \( s_2 \)

**Time 1**

- Define \( Q_1(s_1, a_1) = E_{S_2}[V_{2}^{\pi_2^*}(S_2)|S_1 = s_1, A_1 = a_1] \)
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- Q-functions are conditional expectations
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Q-Learning from Data: Repeated Regression

- Estimate each $Q_t$ by linear regression on features $\phi_t(s_t, a_t)$

**Time 2:** $Q_2(s_2, a_2) = E_R[R|S_2 = s_2, A_2 = a_2]$

- Regress $R$ on features $\phi_2(S_2, A_2)$ [$R$ might be symptom reduction] to obtain $\hat{Q}_2(s_2, a_2; \hat{\beta}_2) = \hat{\beta}_2^T \phi_2(s_2, a_2)$
- Set $\hat{\pi}_2^* = \arg\max_{a_2} \hat{Q}_2(s_2, a_2), \quad \hat{V}_{2}^{\hat{\pi}_2^*}(s_2) = \max_{a_2} \hat{Q}_2(s_2, a_2)$

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- $\hat{\pi}_1^*(s_1) = \arg\max_{a_1} \hat{Q}_1(s_1, a_1)$
Q-Learning - Beyond Scalar Rewards

- Learned policy $\hat{\pi}_t^*(s_t) = \arg\max_{a_t} \hat{Q}_t(s_t, a_t)$ is constructed to maximize long-term expected reward, i.e. is non-myopic

- Dynamic programming “trick” is to maximize over $a_2$ first
  - Key step: Set $\hat{V}_2^\pi(s_2) = \max_{a_2} \hat{Q}_2(s_2, a_2)$
  - Only makes sense if $R$ is scalar

- What if there is more than one $R$ of interest?
Time 2 Policy - Two Rewards

- Suppose two rewards $R^{(0)}$ and $R^{(1)}$ are of interest, e.g. symptom reduction and weight control
- Below, $(\hat{Q}^{(0)}_2, \hat{Q}^{(1)}_2)$ for patient with $S_2 = s_2$, four different actions
- What should $\hat{\pi}^*_2(s_2)$ be?
Contribution

Formalizing Preference

- Define a scalar reward \( R(\delta) = (1 - \delta)R(0) + \delta R(1) \)
- \( 0 \leq \delta \leq 1 \)
- Proceed as before to get \( \hat{Q}_2(\delta) \)

\[ \delta = 0 \]
\[ \hat{Q}_2^{(0)} \]
\[ \hat{Q}_2^{(1)} \]
\[ \hat{Q}_2^{(\delta)} \]

\[ \delta = 0.5 \]
\[ \hat{Q}_2^{(0)} \]
\[ \hat{Q}_2^{(1)} \]
\[ \hat{Q}_2^{(\delta)} \]

\[ \delta = 1 \]
\[ \hat{Q}_2^{(0)} \]
\[ \hat{Q}_2^{(1)} \]
\[ \hat{Q}_2^{(\delta)} \]

- \( \delta \) represents “How much do I care about \( R(1) \)?”
Preference Elicitation

• Function $R^{(\delta)}$ is an *Aggregate Objective Function* (many names...) familiar in multi-objective optimization

• Preference Elicitation Approach:
  • Figure out the decision maker’s $\delta$
  • Define a scalar reward $R^{(\delta)} = (1 - \delta) \cdot R^{(0)} + \delta \cdot R^{(1)}$
  • Use Q-learning to estimate the optimal policy for that reward

• Resulting policy is Pareto optimal

• E.g., for $\delta = 0.5$
Preference Elicitation

- E.g., “Consider two actions. You can have (8, 5), or you can have (5, x). What value of x makes you indifferent to this choice?”

- Find $\delta$ so that $R^{(\delta)}$ is equal for the two points
  - $(1 - \delta) \cdot 8 + \delta \cdot 5 = (1 - \delta) \cdot 5 + \delta \cdot x$

- Doubt about whether or not this actually works
- Has nothing to do with the actions that are actually available, i.e. does not provide salient information about available treatments.

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2Actual question would be more subtle.
Contribution: Inverse Preference Elicitation

• Preference Elicitation
  • “Give me your $\delta$, I will tell you the right action.”
  
• Infinite number of $\delta$, but only 4 actions
  
• Inverse Preference Elicitation
  • “Given each available action, I will tell you the $\delta$ for which that action is optimal.”
  • This is our salient information
Contribution: Inverse Preference Elicitation

- “Given each available action, I will tell you the $\delta$ for which that action is optimal.”
- Each action is optimal over a range of $\delta$
Contribution: Inverse Preference Elicitation

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Note action $a$ does not appear
Contribution: Inverse Preference Elicitation

- “Given each available action, I will tell you the $\delta$ for which that action is optimal.”
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Note action a does not appear
Can see sensitivity of action choice w.r.t. preference
Contribution: Inverse Preference Elicitation

- “Given each available action, I will tell you the $\delta$ for which that action is optimal.”
- Each action is optimal over a range of $\delta$

- Note action $a$ does not appear
- Can see sensitivity of action choice w.r.t. preference
- Decision aid from the beginning is a coarsened version of a picture like this
Non-Myopic Inverse Preference Elicitation

- At each timepoint $t$, define $Q_t(s_t, a_t; \hat{\beta}_t(\delta)) = \hat{\beta}_t^T(\delta)\phi_t(s_t, a_t)$

- Perform Q-learning for all $\delta$ simultaneously

- At Time 2, finding ranges of $\delta$ is straightforward
  - Use convex hull to identify regions of $\delta$

- At Time 1, things get interesting

- **Challenge**: represent $Q_1(s_1, a_1; \hat{\beta}_1(\delta))$
  - Exactly
  - Economically
Q-Learning for all $\delta$: Time 2

Time 2: $Q_2(s_2, a_2; \delta) = E_R[R^{(\delta)}|S_2 = s_2, A_2 = a_2]$

- For all $\delta \in [0, 1]$, regress $R^{(\delta)}$ on features $\phi_2(S_2, A_2)$ giving
  
  \[ \hat{Q}_2(s_2, a_2; \hat{\beta}_2(\delta)) = \hat{\beta}_2^T(\delta)\phi_2(s_2, a_2) \]

- $\hat{\beta}_2(\delta) = (\Phi_2^T \Phi_2)^{-1} \Phi_2^T((1 - \delta)\bar{R}(0) + \delta\bar{R}(1))$

- Notice that $\hat{Q}_2(s_2, a_2; \hat{\beta}_2(\delta)) = \phi_2(s_2, a_2)^T\hat{\beta}_2(\delta)$ is linear in $\delta$, so only compute $\hat{\beta}_2(0)$ and $\hat{\beta}_2(1)$. 
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- Notice that $\hat{Q}_2(s_2, a_2; \hat{\beta}_2(\delta)) = \phi_2(s_2, a_2)^T \hat{\beta}_2(\delta)$ is linear in $\delta$, so only compute $\hat{\beta}_2(0)$ and $\hat{\beta}_2(1)$.

\[ S_2 = s_2 \]

![Graph showing Q-values](image-url)
**Q-Learning for all $\delta$: Time 1**

**Time 1: $Q_1(s_1, a_1; \delta) = E_{S_2}[V_2(S_2, \delta)|S_1 = s_1, A_1 = a_1]$**

- For all $\delta \in [0, 1]$, set $\hat{V}_2(s_2; \delta) \triangleq \max_{a_2} \hat{Q}_2(s_2, a_2; \delta)$. Note $\hat{V}_2(s_2; \delta)$ is piecewise linear in $\delta$.

- For all $\delta \in [0, 1]$, regress $\hat{V}_2(S_2; \delta)$ on features $\phi_1(S_1, A_1)$ giving
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  \]

- $\hat{\beta}_1(\delta) = (\Phi_1^T \Phi_1)^{-1} \Phi_1^T \hat{V}_2(S_2; \delta)$

- Notice that $\hat{\beta}_1(\delta)$ and $\hat{Q}_1(s_1, a_1; \hat{\beta}_1(\delta))$ are linear over regions of $\delta$ where elements of $\hat{V}_2(s_2; \delta)$ are all simultaneously linear.
Q-Learning for all $\delta$: Time 1

- Notice $\hat{V}_2(s_2; \delta)$ is piecewise linear in $\delta$. 

![Graph showing $S_2 = s_2$ and $\hat{V}_2(s_2, \delta)$]
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**Q-Learning for all $\delta$: Time 1**

- Notice that $\hat{\beta}_1(\delta)$ and $\hat{Q}_1(s_1, a_1; \hat{\beta}_1(\delta))$ are linear over regions of $\delta$ where $\hat{V}_2(s_2; \delta)$ is simultaneously linear for all $s_2$.

- Only need to evaluate $\hat{\beta}_1(\delta)$ at union of knots in $\hat{V}_2(S_2; \delta)$
Convexity of $\hat{Q}_t, \hat{V}_t$

- $\hat{Q}_1(s_1, a_1; \hat{\beta}_1(\delta))$ is a weighted combination of the $\hat{V}_2(S_2; \delta)$, with weights $\phi_1(s_1, a_1)^T(\Phi_1^T\Phi_1)^{-1}\Phi_1^T$

- Note that $\hat{Q}_1(s_1, a_1; \hat{\beta}_1(\delta))$ may not be convex in $\delta$!

- Earlier work (Barrett and Narayanan 2008) relied on convexity, which restricts possible definitions of $\phi$
Complexity of $\hat{Q}_t$, $\hat{V}_t$

- In general, $\hat{Q}_t(s_t, a_t; \hat{\beta}_t(\delta))$ is piecewise linear in $\delta$.

- For a $T$-timepoint analysis
  - $\hat{Q}_t(s_t, a_t; \hat{\beta}_t(\delta))$ has $O(n^{T-t}|A|^{T-t})$ knots
  - New knots introduced by setting $\hat{V}_t \triangleq \max_{a_t} \hat{Q}_t(s_t, a_t; \hat{\beta}_t(\delta))$
  - $\hat{V}_t(s_t; \delta)$ has $O(n^{T-t}|A|^{(T-t)+1})$ knots
  - New knots introduced by taking union over state at the $n$ data points
  - $\hat{Q}_{t-1}(s_{t-1}, a_{t-1}; \hat{\beta}(\delta))$ has $O(n^{(T-t)+1}|A|^{(T-t)+1})$ knots

- Bookkeeping allows construction of $\hat{Q}_t$ and $\hat{V}_t$ in time linear in number of knots [Lizotte, Bowling, Murphy 2010]

- Earlier work (e.g. Barrett and Narayanan 2008) did not take advantage of piecewise linear structure, which increases computation time to quadratic in $\#$ of knots.
Example: CATIE

- Large (N = 1460) comparative effectiveness trial

- Most patients randomized two times:
  - First to one of 5 actions
  - Then, if desired, to one of 5 different actions

- Following is a *highly* simplified analysis
- Overall, the results are consistent with the literature
- Rewards: symptoms relief, weight control
Example: CATIE Inverse Preference Elicitation
Example: CATIE-based Decision Aid

- Possible decision aid: Coarse version of the plots

<table>
<thead>
<tr>
<th>Initial Symptoms</th>
<th>Preference</th>
<th>Symptom Relief</th>
<th>Weight Control</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Strong</td>
<td>Mild</td>
<td>Mild</td>
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<tr>
<td>Good</td>
<td>Olan</td>
<td>Olan or Zip</td>
<td>Zip</td>
</tr>
<tr>
<td>Moderate</td>
<td>Olan</td>
<td>Olan or Zip</td>
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</tr>
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<td>Bad</td>
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<td>Olan</td>
<td>Risp or Zip</td>
</tr>
</tbody>
</table>

Olan = Olanzapine, Zip = Ziprasidone, Risp = Risperidone

- Thanks to Holly Wittemann, Brian Zikmund-Fisher, UMich SPH
Future Work

• **Algorithms and Methods for Generating Evidence**
  • More flexible models / approximation algorithms for preferences
  • Measures of uncertainty - requires interesting optimization
    • Ask me about this!
  • “Classical” ML problems (feature selection/dimensionality reduction/model selection, feature extraction via NLP, accommodating missing data...)
  • Must still provide salient information!

• **Clinical Science Applications**
  • Schizophrenia - CATIE
  • Major Depressive Disorder - STAR*D
  • ICU data (non-randomized) - MIMIC, MIMIC II
  • EHR?
Thank You

- Supported by National Institute of Health grants R01 MH080015 and P50 DA10075
- Related work:
Confidence Intervals for Q-Learning

- Question: In state $s_t$, is there evidence that $a$ is really better than $a$?
- Classical approach: get confidence interval for
  \[
  \hat{\beta}_t^T \cdot (\phi(s_t, a) - \phi(s_t, a))
  \]
  
- For $t = T$, under mild assumptions on $R$, can use normal approximation or bootstrap
- For $t < T$, standard methods can fail even as $n \to \infty$

- Trouble arises when statistics (e.g. $\hat{\beta}_t$) are non-differentiable functions of the dataset
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Adaptive Confidence Intervals for Q-Learning

- A method that produces correct coverage:
  - Re-sample a dataset $D'$ with replacement
  - Compute $\tilde{\beta}_t = \arg \max_{\beta \text{ near } \hat{\beta}_t} f(\beta, D')$
  - Repeat
- Use distribution of $\tilde{\beta}_t$ to make C.I.

- The $\arg \max_{\beta \text{ near } \hat{\beta}_t} f(\beta, D')$ problem is interesting
  - Non-convex
  - Piecewise linear but possibly not continuous
  - Can formulate as MIP, but maybe we can do better...