## Inverse Preference Elicitation

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## Outline

- Part I
  - Motivation: Symptoms and Side-Effects in Schizophrenia
  - Background: Predictive Models and Optimal Decision Rules
  - Contribution: Inverse Preference Elicitation
- Part II
  - IPE for Sequences of Actions
  - Results: Exploratory Analysis of the CATIE Antipsychotic trial
  - Discussion and Future Work:
    - Experimental evaluation using Mechanical Turk
    - Other extensions

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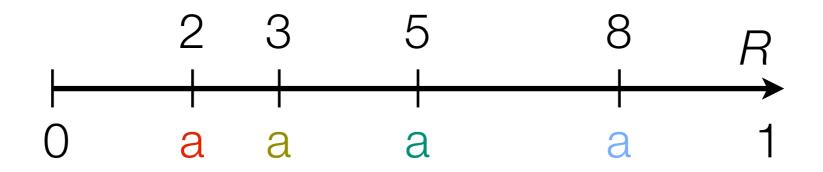
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- Treatments that provide the best symptom reduction induce the worst weight gain, and vice-versa
- Different doctors and patients have very different preferences about relative importance of outcomes
  - How can we recommend a treatment that accommodates these preferences?

### Outcome Predictions and Decision Rules - Single Outcome

- Identify an outcome of interest (reward) R, predictive patient features (state) S, and a set of treatments (actions) A
- Construct a predictive model
  - Input: (S, A) Output: Prediction of R
- Could be done by regressing *R* on (*S*, *A*) for example
- e.g., have (S, A, R) for each individual, A is randomized

#### **Optimal Decision Rule - Single Outcome**

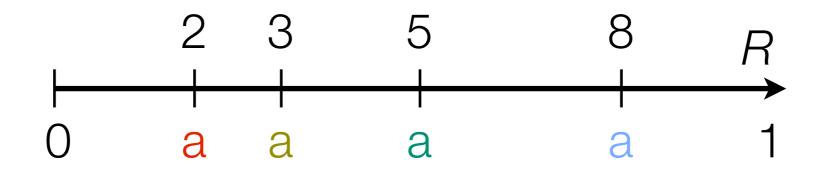
#### Predicted R for patient with S=s



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 A model makes predictions of the reward of a patient with state s under 4 different actions, a, a, a, a, and a.

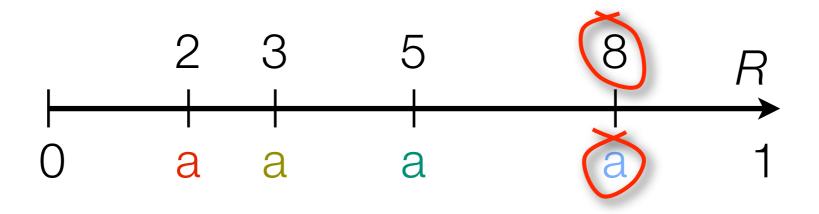
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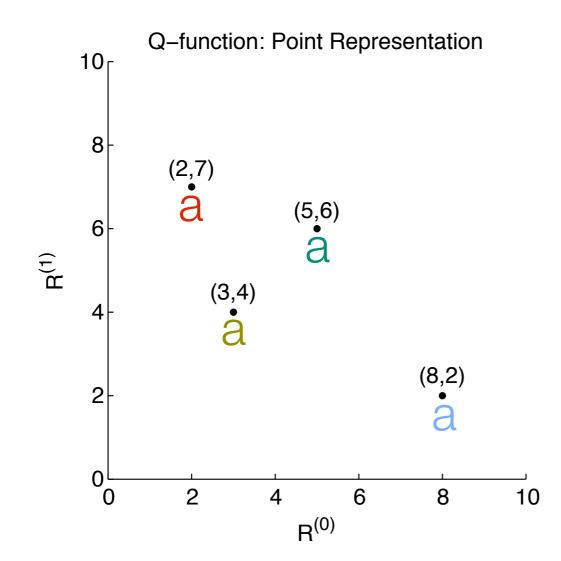
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- A model makes predictions of the reward of a patient with state s under 4 different actions, a, a, a, a, and a.
- For the patient with the predictions shown below, action a looks best, with a predicted reward of 8. (Higher rewards are better.)

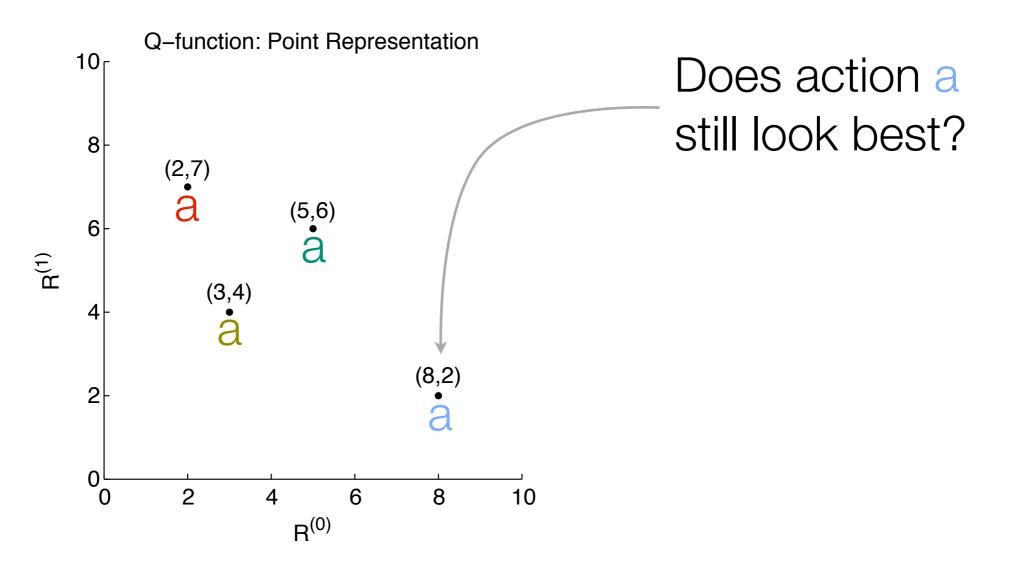




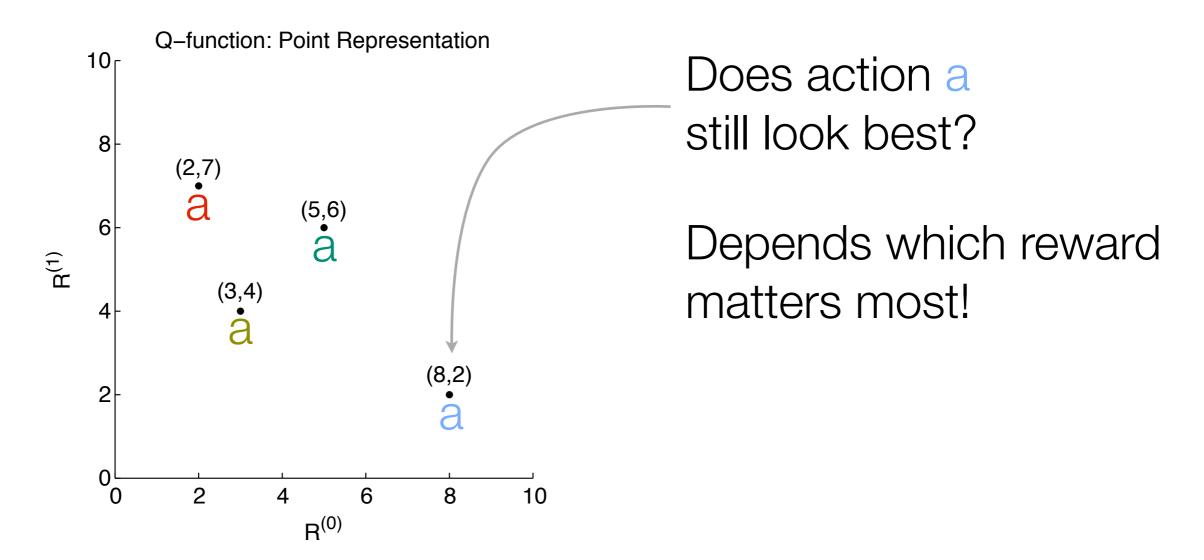
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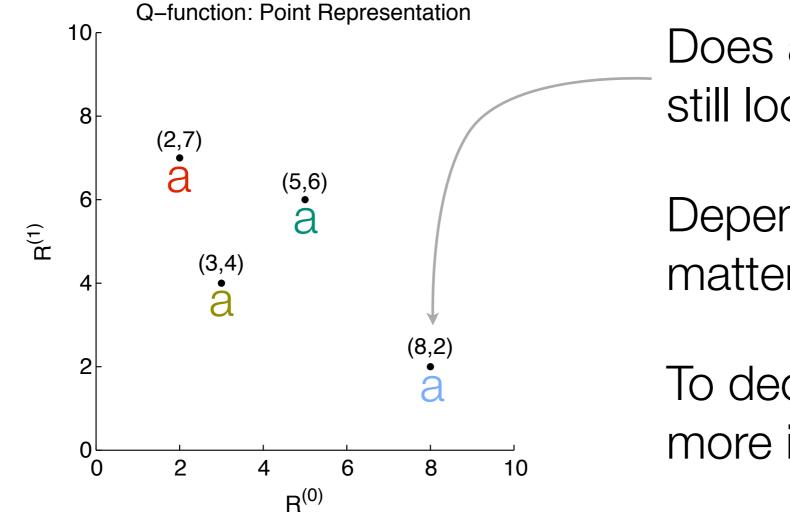
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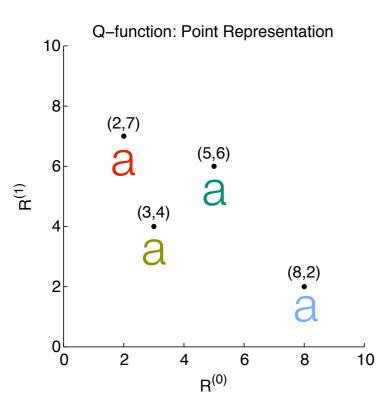


Does action a still look best?

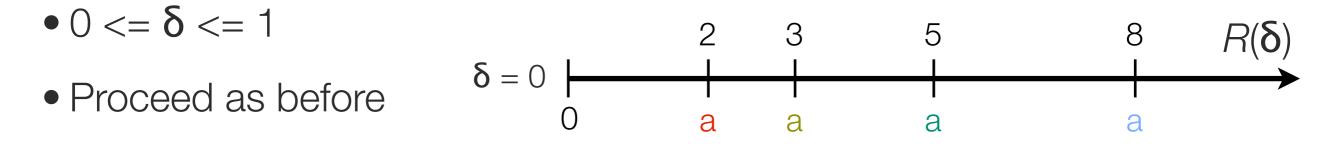
Depends which reward matters most!

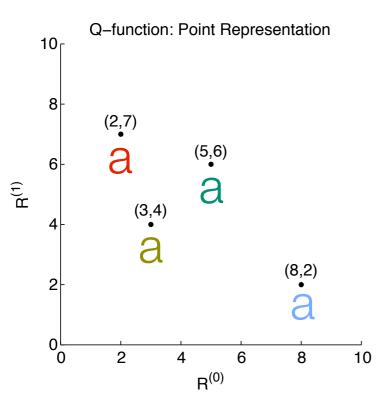
To decide, we need more information.

- Define a new reward  $R(\delta) = (1 \delta) \cdot R^{(0)} + \delta \cdot R^{(1)}$
- 0 <= δ <= 1
- Proceed as before

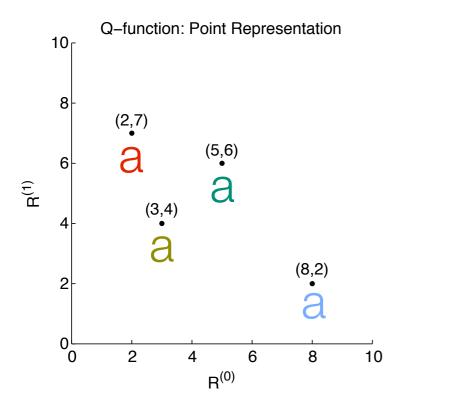


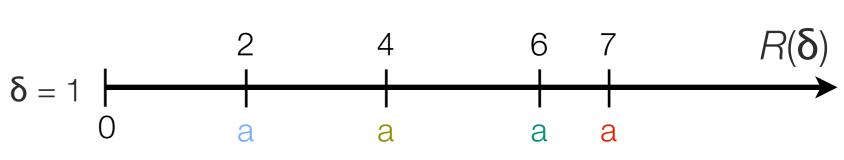
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4

R<sup>(0)</sup>

6

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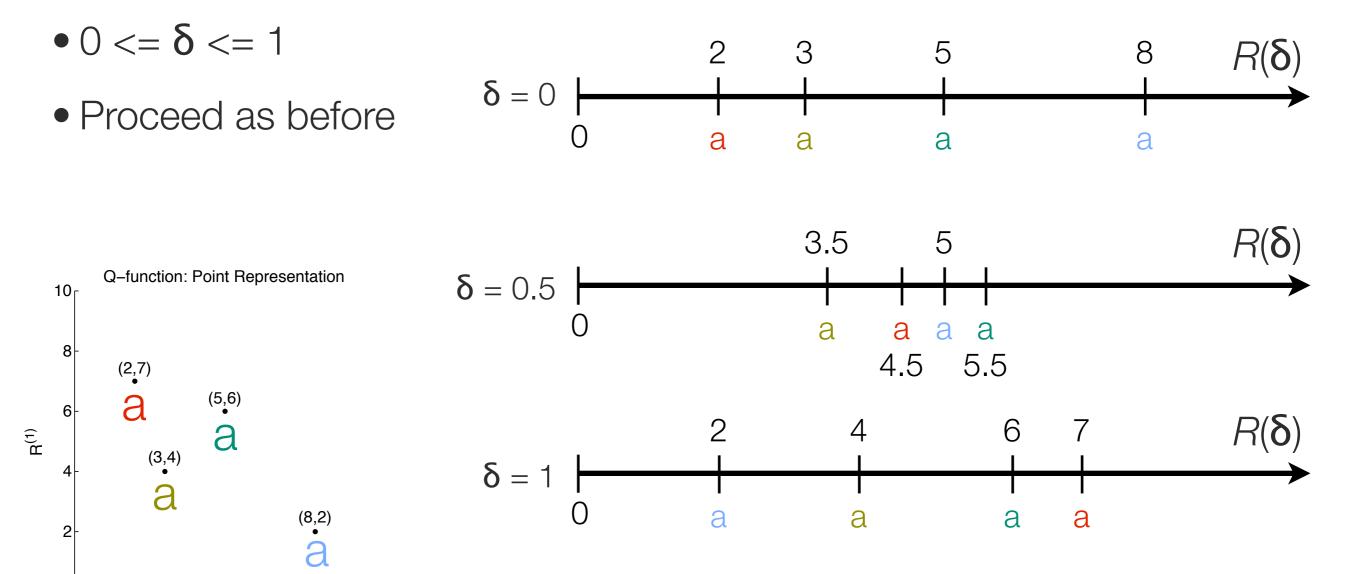
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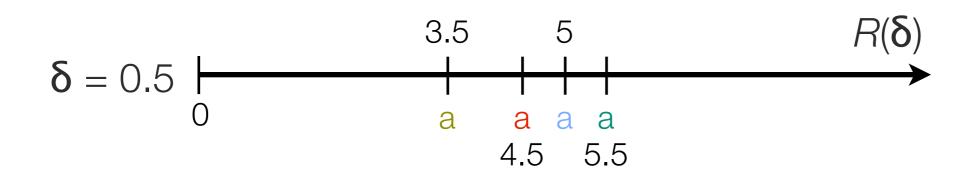
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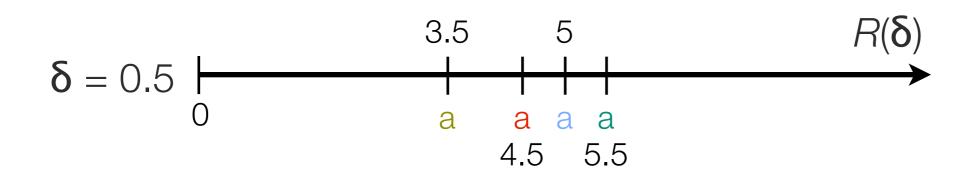
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- Note that this approach does not have anything to do with the actions that are actually available.

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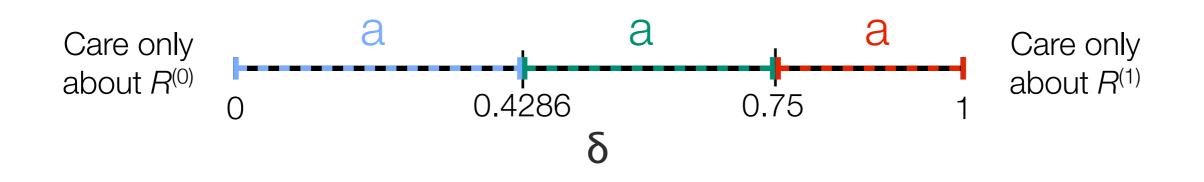
#### • Inverse Preference Elicitation

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  - $\bullet$  In fact, each action is optimal over a range of  $\delta$

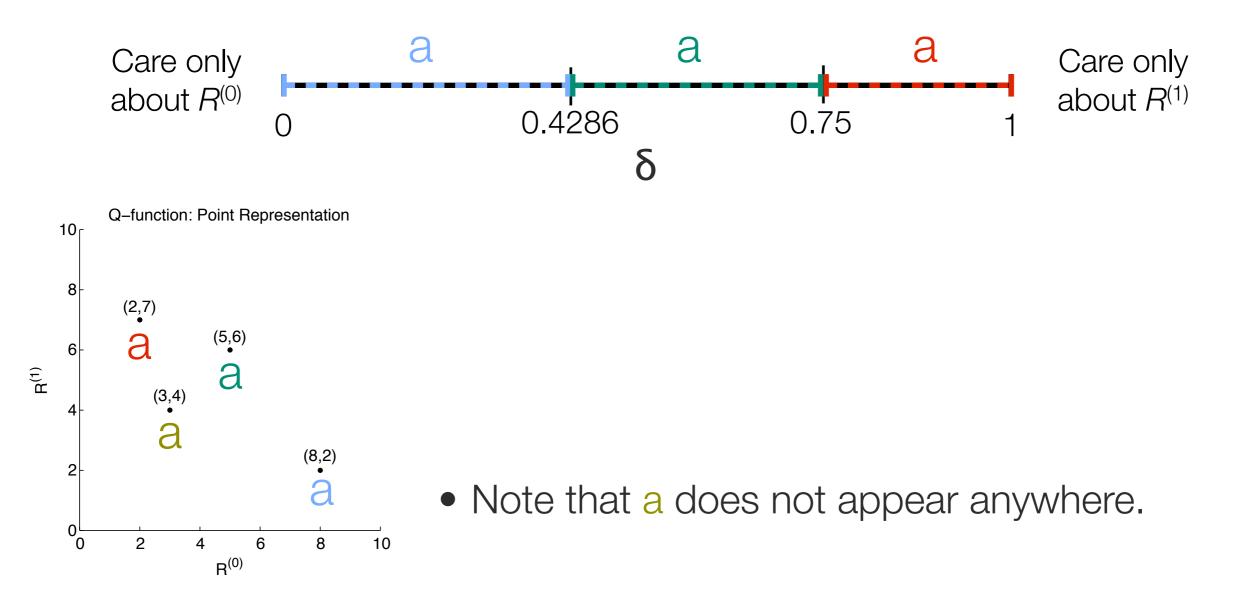
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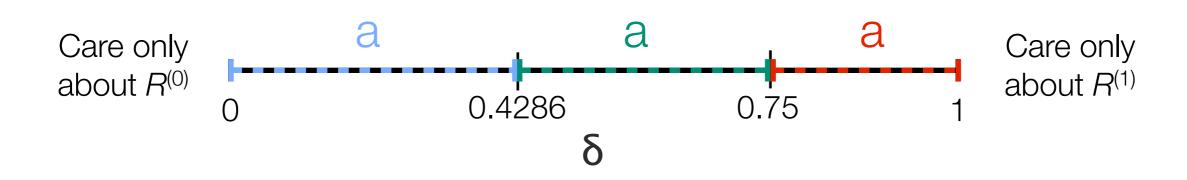


#### Possible Decision Aid

Care only about  $R^{(0)}$ 

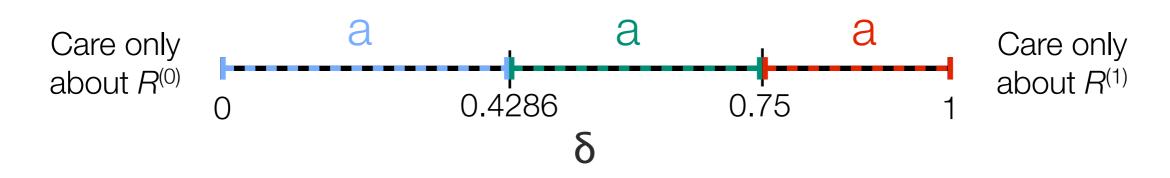
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"I am concerned..."

Mostly about R <sup>(0)</sup>		Somewhat more about $R^{(1)}$ than $R^{(0)}$	Mostly about <i>R</i> <sup>(1)</sup>
a	a or a	a	a

## Preference Elicitation vs. Inverse Preference Elicitation

- Inverse Preference Elicitation
  - Method for choosing an action when faced with multiple rewards
  - Provides information about available actions
  - Choice among a small
     **number** of alternatives

- Preference Elicitation
  - Method for choosing an action when faced with multiple rewards
  - Provides **no** information about available actions
  - Choice among an **infinite** number of alternatives

#### "I am concerned..."

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## End Part I

- We've covered:
  - Optimal Decision Rules
  - Mathematizing Preference
  - Preference Elicitation
  - Inverse Preference Elicitation
- Pause for questions and discussion?

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  - How can we recommend a *sequence of actions* that accommodates these preferences?

### Learning a Sequence of Actions From Data

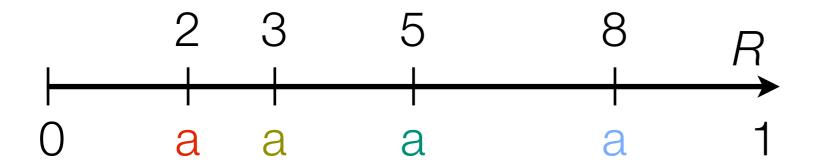
## Learning a Sequence of Actions From Data

- $(S_1, A_1, S_2, A_2, R)$  for each individual
  - S<sub>j</sub> "State" Patient covariates (previous txts, response,...)
  - $A_j$  "Action" Treatment offered to the patient
  - R "Reward" Clinical outcome
- Actions  $A_j$  have known randomization probability

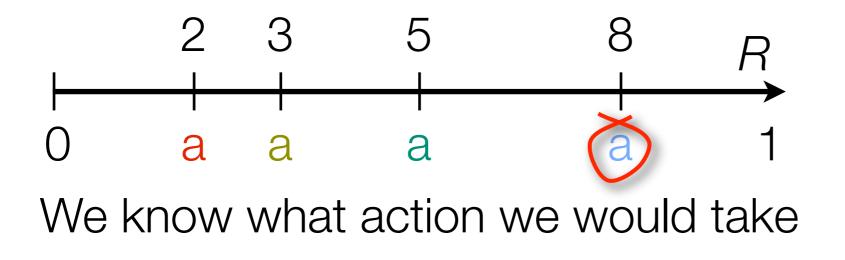
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- Let's start by looking at Stage 2:  $(S_2, A_2, R)$

- A model makes predictions of the reward of a patient with state  $S_2=s_2$  under 4 different actions, **a**, **a**, **a**, **a**, and **a**.
- For the patient with the predictions shown below, action a looks best. (Higher rewards are better.)

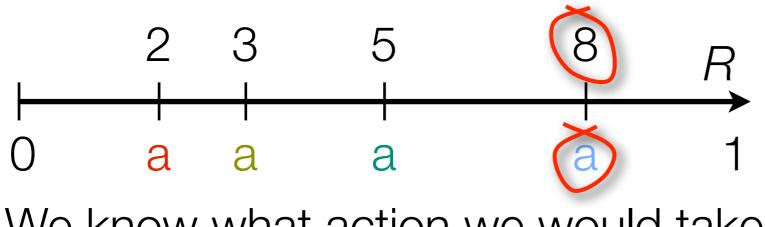


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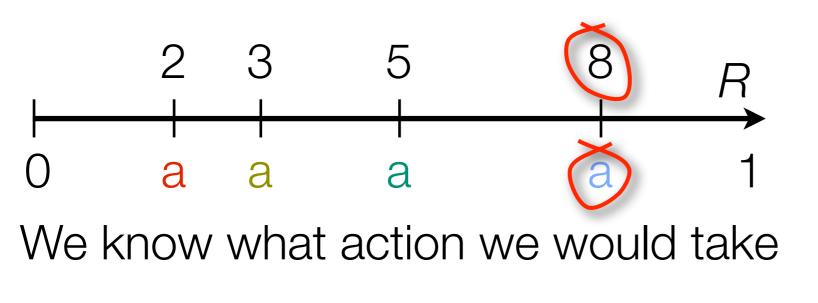
We can predict the reward *R* we will get



We know what action we would take

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We now have our decision rule for Stage 2 (we have to do the above for all s<sub>2</sub>)

• Recall: We have  $(S_1, A_1, S_2, A_2, R)$ 

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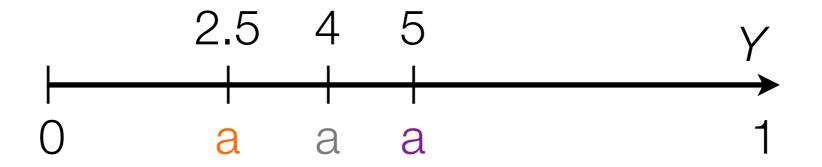
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  - Regress Y on  $S_1, A_1$

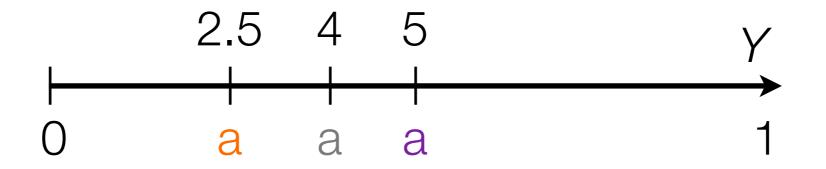
## Stage 1

- A model makes predictions of the "pseudo-outcome" Y of a patient with state  $S_1=s_1$  under 3 different actions, a, a, and a.
  - Predicts reward of patient if we choose a and then act optimally
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## Stage 1

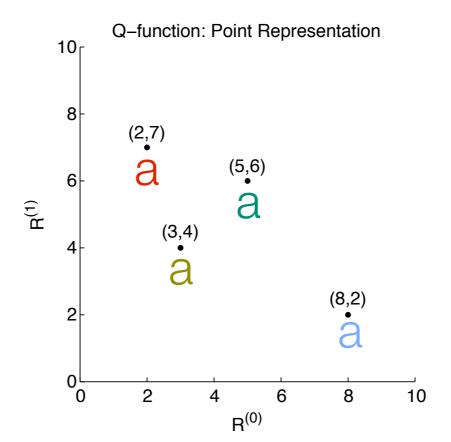
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We now have our decision rule for Stage 1

# Dynamic Programming: Multiple Rewards

- Big "trick" was constructing Y
  - Requires knowing the decision rule at stage 2
  - But what if we don't know?



• We can still use the  $\delta$  approach to make a single reward  $R(\delta)$  and proceed as before

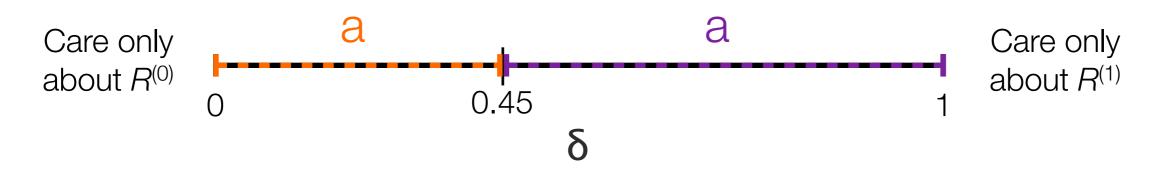
# Dynamic Programming: Inverse Preference Elicitation

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- And we can do Inverse Preference Elicitation! Algorithm is complex. [Lizotte, Bowling, Murphy 2010]



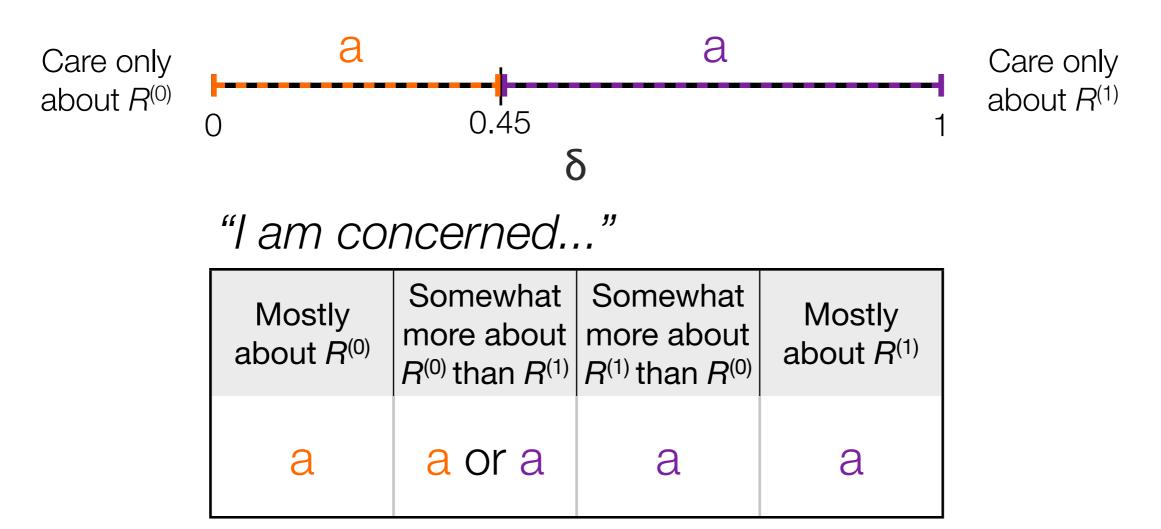
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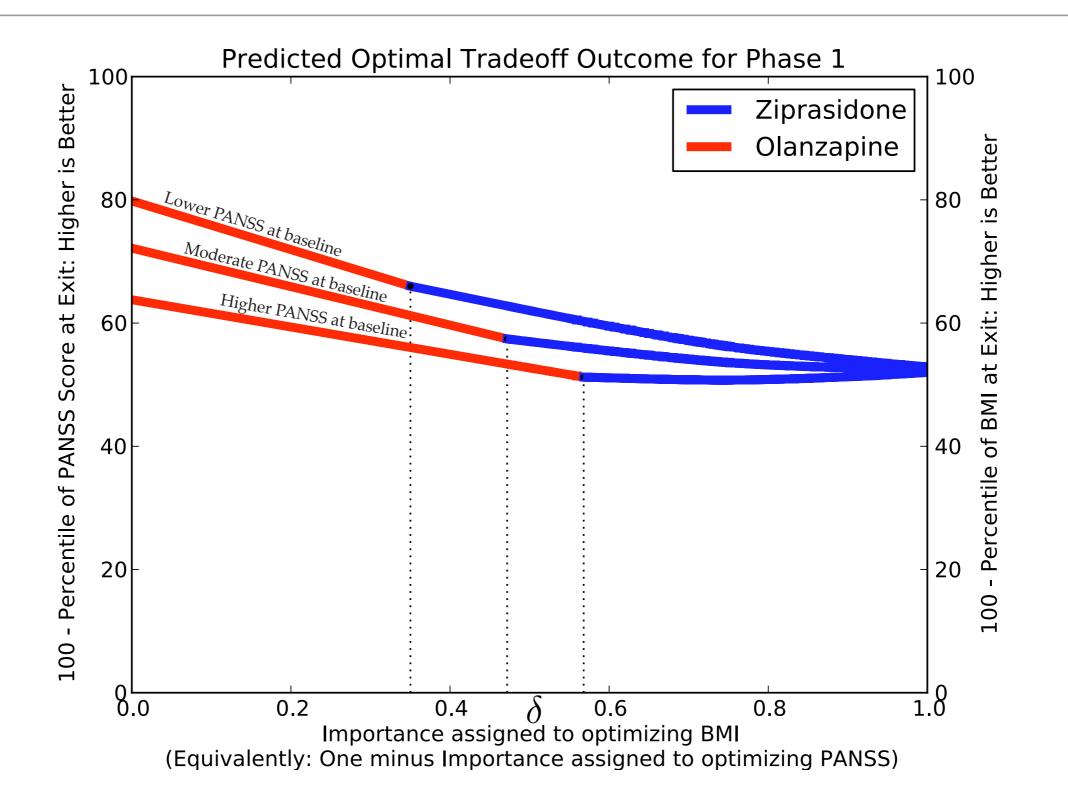
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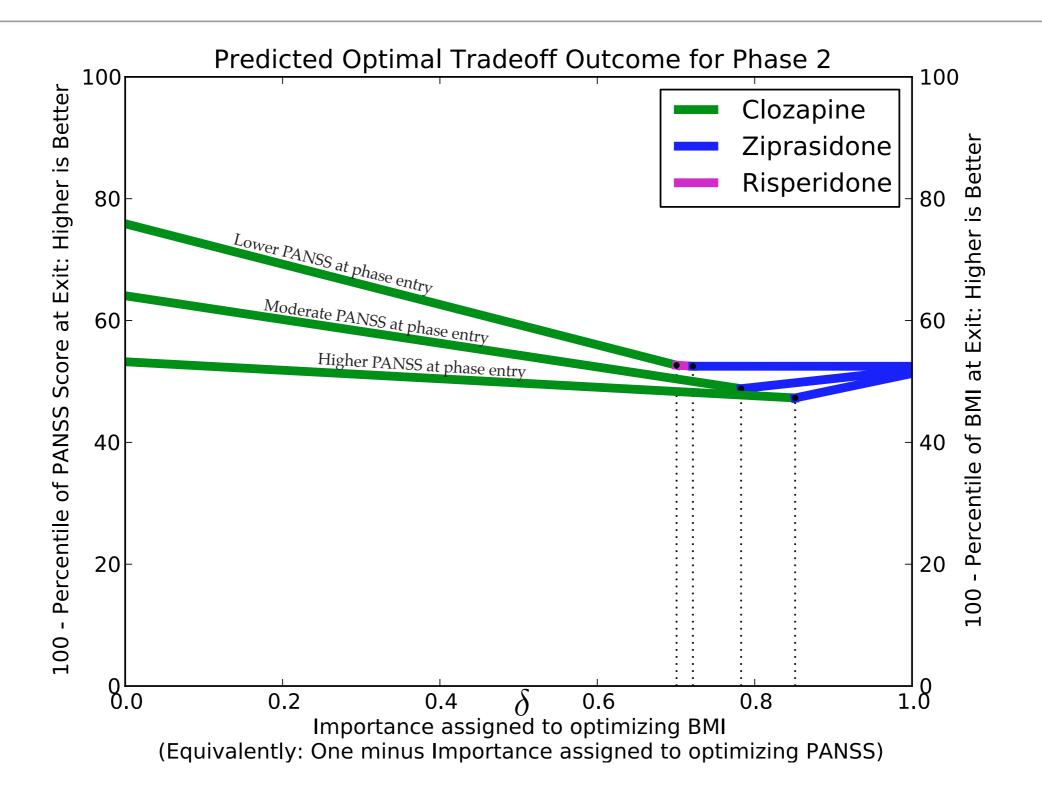
# Example: CATIE

- Large (n = 1460) comparative effectiveness trial funded by NIMH
- Compares medications for treatment of schizophrenia
- Most patients randomized two times:
  - First to one of 5 actions
  - Then, if desired, to one of 5 different actions
- Details are quite complicated
- Following is a *highly* simplified analysis
- Overall, the results are consistent with what is known in the literature
- Rewards: PANSS (symptoms) versus BMI (weight gain side-effect)

#### Example: CATIE Exploratory Analysis



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### Example: CATIE-based Decision Aid

• One possibility for a decision aid is a very coarse version of the plots:

Recommendation given State and Preference	Strong Preference for Symptom Relief over Weight Control	Mild Preference for Symptom Relief over Weight Control	Mild Preference for Weight Control over Symptom Relief	Strong Preference for Weight Control over Symptom Relief
Lower PANSS	Olanzapine	Olanzapine	Ziprasidone	Ziprasidone
at Entry to Phase 1	Olalizaplile	or Ziprasidone	Zipiasidone	
Moderate PANSS	Olanzapine	Olanzapine	Ziprasidone	Ziprasidone
at Entry to Phase 1	L	or Ziprasidone	1	1
Higher PANSS	Olanzapine	Olanzapine	Olanzapine	Ziprasidone
at Entry to Phase 1	Ĩ	1	or Ziprasidone	
Lower PANSS	Clozapine	Clozapine	Clozapine, Risperidone, or	Ziprasidone
at Entry to Phase 2			Ziprasidone	
Moderate PANSS	Clozapine	Clozapine	Clozapine	Clozapine
at Entry to Phase 2	-	-		or Ziprasidone
Higher PANSS	Clozapine	Clozapine	Clozapine	Clozapine
at Entry to Phase 2		-		or Ziprasidone

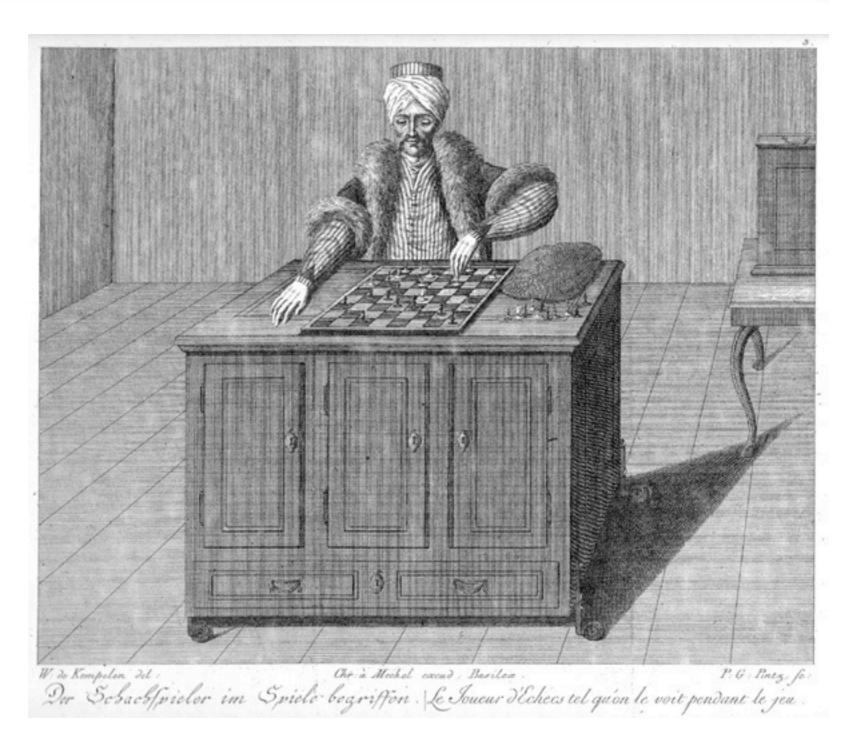
• Thanks to: Holly Wittemann, Brian Zikmund-Fisher for this idea

# Future Work

- Evaluating the "Inverse Preference Elicitation" Idea
  - MTurk Evaluation
- The Algorithms and Methods
  - Measures of Uncertainty
  - More flexible models / Approximation algorithms
  - More reward definitions
- Clinical Science Applications

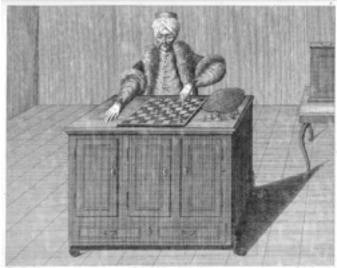
### Amazon Mechanical Turk

- Mechanism for recruiting and paying users to do "Human Intelligence Tasks" - HITs
- Popular for running survey experiments (demographics at least as good as undergrads
   [Paolacci, Chandler, lpeirotis 2010])



### Amazon Mechanical Turk

- Our experiment will compare eliciting  $\delta$  using a slider with directly eliciting an action using a decision aid.
- User will perform one of four different (similar and boring) sub-tasks, each one with different rate of pay and time duration
- The choice of action determines the sub-task, and also affects the workload of all the subsequent subtasks myopic decision making is sub-optimal.
- Competing preferences:
  - Save time vs. Make money
- We will compare the appeal of the two methods
- Plan to go live January 2011



Der Scharthorieler im Spiele begroßen Lower Hickers eit gein le mit partant to jeu

# Future Work - Clinical Science

- 1.Schizophrenia
  - Symptom reduction versus functionality, or weight gain
- 2.Major Depressive Disorder
  - Symptom reduction versus weight gain, other side-effects
- 3.Type 2 Diabetes
  - Future disease complications versus drug side-effects

# Questions

• Supported by National Institute of Health grants R01 MH080015 and P50 DA10075



- Daniel J. Lizotte, Michael Bowling, and Susan A. Murphy. *Efficient Reinforcement Learning with Multiple Reward Functions for Randomized Clinical Trial Analysis*. Proceedings of the Twenty-Seventh International Conference on Machine Learning (ICML), 2010.
- Related work:

Barrett, L. and Narayanan, S. *Learning all optimal policies with multiple criteria.* In Proceedings of the 25th International Conference on Machine Learning 2008.