UNIVERSITY OF WESTERN ONTARIO

Computer Science 209b, Winter 2001

Applied Logic for Computer Science

ASSIGNMENT 10
Solution

1. First, in order to use resolution, we have to transform all the premises and the negation of the conclusion in conjunctive normal form.

$p_1 : \neg P \vee Q$
$p_2 : P \vee R$
$p_3 : (\neg Q \vee S) \wedge (\neg R \vee S)$
$\neg C : \neg S$

Secondly, we transform the obtained formulas into disjunctive clauses:
p_1 gives $\neg P \vee Q$
p_2 gives $P \vee R$
p_3 gives $\neg Q \vee S$ and $\neg R \vee S$
$\neg C$ gives $\neg S$

Thirdly, apply resolution with the set of support strategy: The initial set of support is $S_1 = \{\{\neg S\}\}$ (the 5th clause from below).

1.$\neg P \vee Q$
2.$P \vee R$
3.$\neg Q \vee S$
4.$\neg R \vee S$
5.$\neg S$
6.$\neg R$ resolve (4,5)
7.P resolve (2,6)
8.Q resolve (1,7)
9.S resolve (3,8)
10.0 resolve (5,9)

$S_1 = \{5\}$
$S_2 = \{5, 6\}$
$S_3 = \{5, 6, 7\}$
$S_4 = \{5, 6, 7, 8\}$
$S_5 = \{5, 6, 7, 8, 9\}$

The fact that we reached a contradiction means that the set of clauses obtained from $p_1 \wedge p_2 \wedge p_3 \wedge \neg C$ is not satisfiable, which in turn means that $p_1 \wedge p_2 \wedge p_3 \models C$, and the argument is correct.

2.
$\{P,\neg R\}_1, \{Q,\neg R\}_2, \{Q,\neg S\}_3, \{\neg P, T\}_4, \{\neg Q,\neg T\}_5,$
$\{\neg Q, R, T\}_6, \{P, S, \neg T\}_7, \{\neg P, Q, R\}_8, \{Q, R, S, T\}_9$

S_1 =all clauses 1-9
$S_1' = S_1$
T_1=clauses 1,4,7,8
Eliminate P:
U_1 consists of : $\{\neg R, T\}$ resolve 1,4
$\{\neg R, Q, R\}$ resolve 1,8
$\{T, S, \neg T\}$ resolve 4,7
$\{S, \neg T, Q, R\}$ resolve 7,8

$S_2 = (S_1 - T_1) \cup U_1$ and therefore S_2' consists of:
$\{Q,\neg R\}_1, \{Q,\neg S\}_2, \{\neg Q,\neg T\}_3, \{\neg Q, R, T\}_4, \{Q, R, S, T\}_5, \{\neg R, T\}_6, \{S, \neg T, Q, R\}_7$
T_2 clauses 1,2,3,4,5,6,7,

Eliminate Q:
U_2 consists of : $\{\neg R, \neg T\}$ resolve 1,3
$\{\neg R, R, T\}$ resolve 1,4
$\{\neg S, \neg T\}$ resolve 2,3
$\{\neg S, R, T\}$ resolve 2,4
$\{\neg T, R, S, T\}$ resolve 3,5
$\{S, \neg T, R\}$ resolve 3,7
$\{R, T, S\}$ resolve 4,5
$\{R, T, S, \neg T\}$ resolve 4,7

$S_3 = (S_2 - T_2) \cup U_2$ and after eliminating clauses that contain complementary literals we obtain:
$S_3' = \{\neg R, T\}_1, \{\neg R, \neg T\}_2, \{\neg S, \neg T\}_3, \{\neg S, R, T\}_4, \{S, \neg T, R\}_5, \{R, T, S\}_6$
T_3 consists of clauses 1,2,4,5,6

Eliminate R:
U_3 consists of $\{T, \neg S\}$ (1,4)
$\{T, S, \neg T\}$ (1,5)
$\{T, S\}$ (1,6)
$\{\neg T, \neg S, T\}$ (2,4)
$\{\neg T, S\}$ (2,5)
$\{\neg T, T, S\}$ (2,6)

$S_4 = (S_3 - T_3) \cup U_3$ and after eliminating clauses containing complementary literals we obtain:
$S_4' = \{\neg S, \neg T\}_1, \{T, \neg S\}_2, \{T, S\}_3, \{\neg T, S\}_4$
T_4=clauses 1,2,3,4

Eliminate S:
U_4 consists of $\{\neg T, T\}$ (1,3)

$\{\neg T\}$ (1,4)
$\{T\}$ (2,3)
$\{T, \neg T\}$ (2,4)
$S_5 = (S_4 - T_4) \cup U_4$ and after eliminating clauses containing complementary literals we obtain:
$S'_5 = \{\neg T\}_1, \{T\}_2$
T_5 = clauses 1,2

Eliminate T
S_6=the empty clause
As we obtained the empty clause (i.e a contradiction from resolving $\neg T$ with T) the original set of clauses is not satisfiable (falsehood is never satisfiable)
3.
$p_1 : \forall x(D(x) \to \neg W(x))$
$p_2 : \forall x(O(x) \to W(x))$
$p_3 : \forall x(P(x) \to D(x))$
$C : \forall x(P(x) \to \neg O(x))$
First we transform the premises and negation of conclusion in conjunctive normal form:
$p_1 : \forall x(\neg D(x) \vee \neg W(x))$
$p_2 : \forall x(\neg O(x) \vee W(x))$
$p_3 : \forall x(\neg P(x) \vee D(x))$
$\neg C : \exists x(P(x) \wedge Q(x))$
Secondly, transform $p_1, p_2, p_3, \neg C$ into $\exists-$free prenex normal form:
p_1, p_2, p_3 are already in $\exists$-free prenex normal form

$\neg C : P(a) \wedge Q(a)$
Third, transform $p_1, p_2, p_3, \neg C$ into clauses:
1.$\neg D(x) \vee \neg W(x)$
2.$\neg O(x) \vee W(x)$
3.$\neg P(x) \vee D(x)$
4.$P(a)$
5.$Q(a)$
Fourth, use the resolution procedure:
1.$\neg D(x) \vee \neg W(x)$
2.$\neg O(x) \vee W(x)$
3.$\neg P(x) \vee D(x)$
4.$P(a)$
5.$Q(a)$
6.$\neg P(a) \vee D(a), (3, \forall-, x = a)$
7.$D(a)$ resolve (4,6)
8.$\neg D(a) \vee \neg W(a)$ $(1, \forall-, x = a)$
9.$\neg W(a)$ resolve (7,8)
10.$\neg O(a) \vee W(a)$ $(2, \forall-, x = a)$
11.$\neg O(a)$ resolve (9,10)

12.0 (false) resolve (5,11)
As we reached a contradiction (0= false) from $p_1 \wedge p_2 \wedge p_3 \wedge \neg C$ this means that the argument $p_1, p_2, p_3 \models C$ is valid.

4. First, transform p_1, p_2 and$\neg C$ into $\exists-$free prenex normal form:
$p_1 : \exists x \forall y[P(x) \wedge (\neg Q(y) \vee \neg R(x, y))]$ =prenex normal form
$P(b) \wedge (\neg Q(y) \vee \neg R(b, y))$ = $\exists$-free and quantifier-free PNF
$p_2 : \forall x \forall y[\neg P(x) \vee \neg S(y) \vee R(x, y)]$ -PNF
$\neg P(x) \vee \neg S(y) \vee R(x, y)$ - $\exists$-free and quantifier-free PNF
$\neg C : \exists x(Q(x) \wedge S(x))$ -PNF
$Q(a) \wedge S(a)$: $\exists-$free and quantifier-free PNF
Transform into clauses and use resolution:
1.$P(b)$
2.$\neg Q(y) \vee \neg R(b, y)$
3.$\neg P(x) \vee \neg S(y) \vee R(x, y)$
4.$Q(a)$
5.$S(a)$
7.$\neg P(x) \vee \neg S(a) \vee R(x, a) \quad (3, \forall-, y = a)$
8.$\neg P(x) \vee R(x, a)$ resolve (5,7)
9.$\neg P(b) \vee R(b, a) \quad (8, \forall-, x = b)$
10.$R(b, a)$ resolve (1,9)
11.$\neg Q(a) \vee \neg R(b, a) \quad (2, \forall-, y = a)$
12.$\neg Q(a)$ resolve (10, 11)
13.0$(false)$ resolve (4,12)
We obtained false as a result of resolution, the argument is correct.

5.
Premise 1: $\forall x((R(x) \vee B(x)) \wedge \neg(R(x) \wedge B(x)))$
Premise 2: $R(a)$
Conclusion: $\neg B(a)$
clauses from premises 1: $R(x) \vee B(x)$ and $\neg R(x) \vee \neg B(x)$
Resolution:
1.$R(x) \vee B(x)$ from premises
2.$\neg(R(x) \vee \neg B(x)$ from premises
3.$R(a)$ from premises
4.$B(a)$ from negation of conclusion
5.$\neg R(a) \vee \neg B(a) \;\; (2, \forall-, x = a)$
6.$\neg R(a)$ resolve (4,5)
7.0 resolve (3,6)
We reached 0, so the argument is valid.