Computing with Semi-Algebraic Sets Represented by Triangular Decomposition

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joint work with
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Plan

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Related work

Triangular decomposition of an algebraic system: W.T. Wu, D.M. Wang, S.C. Chou, X.S. Gao, D. Lazard, M. Kalkbrener, L. Yang, J.Z. Zhang, D.K. Wang, M. Moreno Maza, . . .

Decomposition of a semi-algebraic system (SAS): CAD (G.E. Collins, et.al)

Our previously work:
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Motivation

- Investigate geometrically intrinsic aspects of the decomposition
- Improve the algorithm: better runing time, better output
- Realize set-theoretic operations on semi-algebraic sets

Triangular decomposition of a semi-algebraic system

Example

`RealTriangularize`$([ax^2 + x + b = 0])$ w.r.t. $b \prec a \prec x$ consist of 3 *regular semi-algebraic systems* :

$$\begin{cases} ax^2 + x + b = 0 \\ a \neq 0 \wedge 4ab < 1 \end{cases}, \quad \begin{cases} x + b = 0 \\ a = 0 \end{cases}, \quad \begin{cases} 2ax + 1 = 0 \\ 4ab - 1 = 0 \\ b \neq 0 \end{cases}$$

`RealTriangularize`

- is an **analogue** of triangular decomposition of algebraic systems
- represents real solutions of a semi-algebraic system by regular semi-algebraic systems
- solves many foundamental problems related to semi-algebraic systems/sets: emptiness test, dimension, parametrization, sample points, ...

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Regular semi-algebraic system

Notation

- T: a regular chain of $\mathbb{Q}[\mathbf{x}]$
- $\mathbf{u} = u_1, \ldots, u_d$ and $\mathbf{y} = \mathbf{x} \setminus \mathbf{u}$: the free and algebraic variables of T
- $P \subset \mathbb{Q}[\mathbf{x}]$: each polynomial in P is regular w.r.t. $\mathrm{sat}(T)$
- $\mathcal{Q}$: a quantifier-free formula (QFF) of $\mathbb{Q}[\mathbf{u}]$

Definition (regular semi-algebraic system)

We say that $\mathcal{R} := [\mathcal{Q}, T, P_>]$ is a regular semi-algebraic system (RSAS) if:

(*i*) the set $S = Z_{\mathbb{R}}(\mathcal{Q}) \subset \mathbb{R}^d$ is non-empty and open,

(*ii*) the regular system $[T, P]$ specializes well at every point u of S

(*iii*) at each point u of S, the specialized system $[T(u), P(u)_>]$ has at least one real zero.

Notions related to generating RSAS

Pre-regular semi-algebraic system

Let $B \subset \mathbb{Q}[\mathbf{u}]$. A triple $[B_{\neq}, T, P_{>}]$ is called a *pre-regular semi-algebraic system* (PRSAS) if $\forall u \in B_{\neq}$, $[T, P]$ specializes well at u.

Definition (border polynomial)

Let R be a squarefree regular system $[T, P]$. The *border polynomial set* of R, denoted by $\mathrm{bps}(R)$, is the set of *irreducible factors* of

$$\prod_{f \in P \cup \{\mathrm{diff}(t, \mathrm{mvar}(t)) \mid t \in T\}} \mathrm{res}(f, T).$$

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$[T, P_{>}, H_{\neq}]$: $[\mathrm{bps}([T, H \cup P])_{\neq}, T, P_{>}]$ is a PRSAS

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Lemma (Property of the border polynomial set)

Let $B := \mathrm{bps}([T, P])$.

- *For any* $u \in Z_{\mathbb{C}}(B_{\neq})$*:* R *specializes well at* u.
- *Let* $S := [T, P_{>}]$, C *be a connected component of* $Z_{\mathbb{R}}(B_{\neq})$ *in* $\mathbb{R}^d$. *Then for any two points* $\alpha_1, \alpha_2 \in C$:

$$\#Z_{\mathbb{R}}(S(\alpha_1)) = \#Z_{\mathbb{R}}(S(\alpha_2)).$$

The notion of a fingerprint polynomial set

$$\mathcal{M} = [B_{\neq}, T, P_{>}] \quad \xrightarrow{FPS} \quad D, \mathcal{R}$$

Definition (fingerprint polynomial set)

A polynomial set $D \subset \mathbb{Q}[\mathbf{u}]$ is a fingerprint polynomial set (FPS) of $\mathcal{M}$ if:

(*i*) for all $\alpha \in \mathbb{R}^d$, $b \in B$: $\alpha \in Z_{\mathbb{R}}(D_{\neq}) \Rightarrow b(\alpha) \neq 0$

(*ii*) for all $\alpha, \beta \in Z_{\mathbb{R}}(D_{\neq})$, if for all $p \in D$, $\mathrm{sign}(p(\alpha)) = \mathrm{sign}(p(\beta))$:

$$\#Z_{\mathbb{R}}(\mathcal{M}(\alpha)) > 0 \quad \Leftrightarrow \quad \#Z_{\mathbb{R}}(\mathcal{M}(\beta)) > 0.$$

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The polynomial set $\{a, 1 - 4ab\}$ is an FPS of

$$\mathcal{M} = [\{a \neq 0, 1 - 4ab \neq 0\}, \{ax^2 + x + b = 0\}, \{\ \}].$$

Generate RSAS from $\mathcal{M}$: $\{\}$, $[\{a \neq 0 \wedge 1 - 4ab > 0\}, \{ax^2 + x + b = 0\}, \{\ \}]$

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Lemma (A theoretical FPS, [CDMMXX10])

The polynomial set oaf(B) *is an FPS of the PRSAS* $\mathcal{M}$.

(oaf is the *open and augmented projection*, defined in [CDMMXX10])

Algorithm: GenerateRSAS

Input: A PRSAS $\mathcal{M} = [B_{\neq}, T, P_{>}]$
Output: An FPS D of $\mathfrak{S}$ and RSAS $\mathcal{R}$

$$Z_{\mathbb{R}}(\mathcal{M}) \setminus Z_{\mathbb{R}}(D_{\neq}) = Z_{\mathbb{R}}(\mathcal{R})$$

initialize $D := B$
loop
  $S :=$ SamplePoints($Z_{\mathbb{R}}(D_{\neq})$), $C_1 := \{\ \}$, $C_0 := \{\ \}$
  for $s \in S$ do
    if $\#Z_{\mathbb{R}}(\mathcal{M}(s)) > 0$ then
      $C_1 := C_1 \cup \{\text{sign}(D(s))\}$
    else
      $C_0 := C_0 \cup \{\text{sign}(D(s))\}$
    end if
  end for
  if $C_1 \cap C_0 = \emptyset$ then
    return D, $[\mathtt{qff}(C_1), T, P_{>}]$
  else
    add more polynomials from $\text{oaf}(B)$ to D
  end if
end loop

Main contributions

- The minimality of border polynomial sets for certain type of regular chains/systems
- The notion of an effective boundary: invariant of a parametric system; improve the FPS construction process
- Relaxation technique in the RSAS generating process: to reduce recursive calls
- Improve decomposition algorithm based on an incremental process
- Difference and Intersection set-theoretic operations for SASes

Plan

Border polynomial: entrance to the "real" world

Border polynomials are at the core of our decomposition algorithm: generating PRSAS, constructing FPS

Border polynomial sets have an "algorithmic" nature: triangular decomposition are not canonical

Two natural questions:

- Can we compute regular systems having smaller border polynomial sets?
- Can we make better use of the computed border polynomial set in the FPS construction?

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Canonical regular chains

Consider two regular chains T, T^* with $\mathrm{sat}(T) = \mathrm{sat}(T^*)$:

$$T = \left\{ \begin{array}{c} x^2 - 2 \\ (a^2 - xa)y - xa + 2 \end{array} \right. \qquad T^* = \left\{ \begin{array}{l} x^2 - 2 \\ ay - x \end{array} \right.$$

bps	$\{a, a^2 - 2\}$	$\{a\}$

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Definition

Let T be a regular chain of $\mathbb{Q}[\mathbf{x}]$. We say that T is *canonical* ff

- (i) T is strongly normalized,
- (ii) T is reduced,
- (iii) the polynomials in T are primitive and monic.

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Theorem (Properties of a canonical regular chain)

Given T a regular chain, then there exists a unique canonical regular chain T^* s.t. $\mathrm{sat}(T^*) = \mathrm{sat}(T)$. Moreover, $\mathrm{bps}(T^*) \subseteq \mathrm{bp}(T)$ holds.

Canonical vs practical

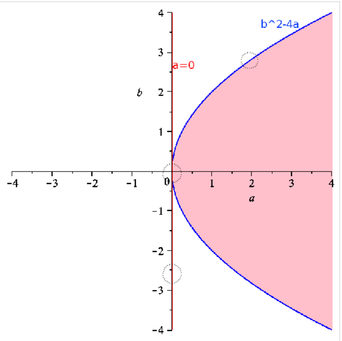
Canonical regular chains are good for theoretical analysis, but more **expensive** to compute in practice.

Where the number of real solutions does change?

Border polynomial set: more about the number does not change

Example

Consider the PRSAS $\mathcal{M} = [\{a \neq 0\}, \{ax^2 + bx + 1 = 0\}, \{\}]$:
$\text{bps}(\mathcal{M}) = \{a, b^2 - 4a\}$.



not every border polynomial factor true boundary" of change

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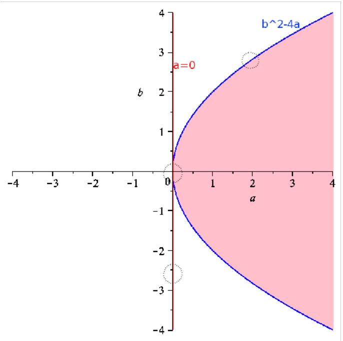

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Effective boundary

Consider $\mathfrak{S} = [T, P_>]$ where $\mathbf{u} = u_1, \ldots, u_d$ are the free variables of T.

Definition (irreducible effective boundary)

Let $\mathbf{h}$ be a hypersurface defined by an irreducible polynomial in $\mathbf{u}$. We call $\mathbf{h}$ an *irreducible effective boundary* if there exists an open ball $O \subset R^d$ satisfying

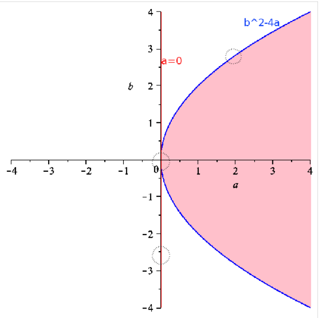
(i) $O \setminus \mathbf{h}$ consists of two connected components O_1, O_2;

(ii) for $i = 1, 2$ and any two points $\alpha_1, \alpha_2 \in O_i$: $\#Z_{\mathbb{R}}(\mathfrak{S}(\alpha_1)) = \#Z_{\mathbb{R}}(\mathfrak{S}(\alpha_2))$;

(iii) for any $\beta_1 \in O_1, \beta_2 \in O_2$: $\#Z_{\mathbb{R}}(\mathfrak{S}(\beta_1)) \neq \#Z_{\mathbb{R}}(\mathfrak{S}(\beta_2))$.

Denote by $\mathcal{E}(\mathfrak{S})$ the union of all irreducible effective boundaries of $\mathfrak{S}$.

Properties of effective boundaries

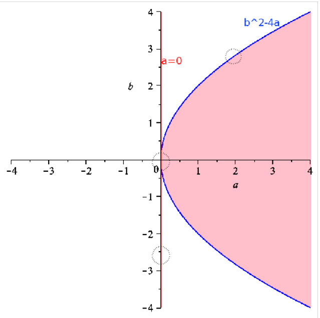


Proposition

We have $\mathcal{E}(\mathfrak{S}) \subseteq Z_{\mathbb{R}}(\prod_{f \in \text{bps}(\mathfrak{S})} f = 0)$.

Effective border polynomial factors (ebf($\mathfrak{S}$)): $p \in \text{bps}(\mathfrak{S})$ and $Z_{\mathbb{R}}(p = 0) \subseteq \mathcal{E}(\mathfrak{S})$

Properties of effective boundaries



Theorem

For all $R_1 = [T_1, P_>]$ *and* $R_2 = [T_2, P_>]$*:*

$$\mathrm{sat}(T_1) = \mathrm{sat}(T_2) \implies \mathit{ebf}(R_1) = \mathit{ebf}(R_2).$$

Algorithmic benefits

Theorem

Given a PRSAS $\mathcal{M} = [B_{\neq}, T, P_{>}]$, *let* $D = \mathrm{oaf}(ebf([T, P_{>}]))$. *Then* $D \cup B$ *is an FPS of* $\mathcal{M}$.

Form new candidate FPS by picking polynomials from D (instead of $\mathrm{oaf}(B)$)

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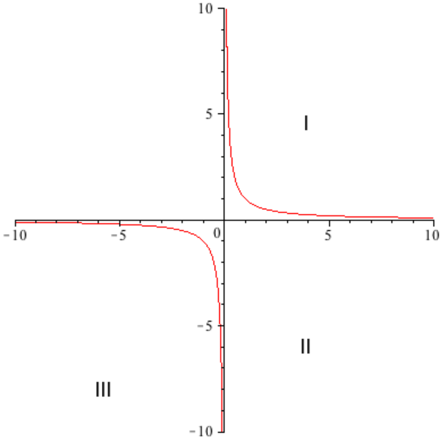
Relaxation: why?

$\mathcal{M} = [\{b\}, T, P_>]$

$b > 0$	$b < 0$
I, III	II

$\mathcal{M}$ has solutions over *I* and *II*



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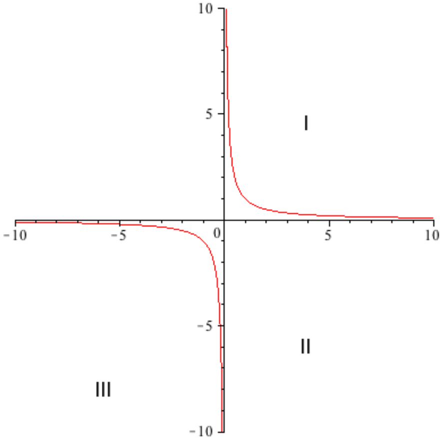
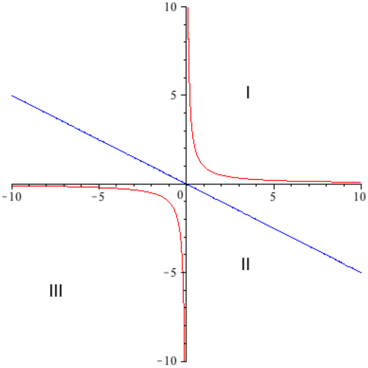


Relaxation: why?

An FPS $F = \{b, f\}$ of $\mathcal{M} = [\{b\}, T, P_>]$



Signs conditions on F:

$C_1 = b > 0 \wedge f > 0$

$C_2 = b < 0 \wedge f > 0$

$C_3 = b < 0 \wedge f < 0$

$$C_1 \vee C_2 \vee C_3 \iff I \cup II \setminus Z_{\mathbb{R}}(f = 0)$$

$$\widetilde{C_1}^f = b > 0 \wedge f \geq 0, \widetilde{C_2}^f = b < 0 \wedge f \geq 0, \widetilde{C_3}^f = b > 0 \wedge f \leq 0$$

$$\widetilde{C_1}^f \vee \widetilde{C_2}^f \vee \widetilde{C_3}^f \iff I \cup II$$

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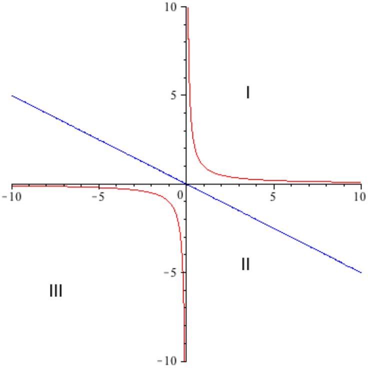

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Criterion for relaxation

Let $\mathcal{S} := [T, P_>]$, $B := \text{bps}([T, P])$, $D \subset \mathbb{Q}[\mathbf{u}]$. Let Q_0, Q_1 be QFFs of $\mathbf{u}$. Suppose

- $B \subsetneq D$
- $Z_\mathbb{R}(Q_1) \cup Z_\mathbb{R}(Q_0) = Z_\mathbb{R}(D_{\neq})$
- $Z_\mathbb{R}(Q_1) \cap Z_\mathbb{R}(Q_0) = \emptyset$
- For all $u \in Z_\mathbb{R}(D_{\neq})$: $\mathcal{S}(u)$ has real solutions $\Leftrightarrow$ $Q_1(u)$

(The assumptions imply $Z_\mathbb{R}(Q_1)$, $Z_\mathbb{R}(Q_0)$ are both **open**)

Theorem (Criterion for relaxation)

Let $h \in D \setminus B$. The following two facts are equivalent:

(i) $Z_\mathbb{R}(\widetilde{Q_1}^h) \cap Z_\mathbb{R}(\widetilde{Q_0}^h) = \emptyset$

(ii) *For all $u \in Z_\mathbb{R}((D \setminus \{h\})_{\neq})$: $\mathcal{S}(u)$ has real solutions $\Leftrightarrow$ $\widetilde{Q_1}^h(u)$.*

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Applying the relaxation criterion

Let F be an FPS of $\mathcal{M} = [B_{\neq}, T, P_{>}]$; let C_1 (resp. C_0) be the sign conditions on F for $\mathcal{M}$ to have (resp. have no) real solutions.

Input: F, C_1, C_0
Output: D, Q_1 s.t. $Z_{\mathbb{R}}([B \cup D_{\neq}, T, P_{>}]) = Z_{\mathbb{R}}([Q_1, T, P_{>}])$

$D := F$, $Q_1 := C_1$, $Q_0 := C_0$
for $h \in F \setminus B$ **do**
  if $Z_{\mathbb{R}}(\widetilde{Q_1}^h) \cap Z_{\mathbb{R}}(\widetilde{Q_0}^h) = \emptyset$ **then**
    $D := D \setminus \{h\}$
    $Q_1 := \widetilde{Q_1}^h$, $Q_0 := \widetilde{Q_0}^h$
  end if
end for
return D, Q_1

Relaxation

Gain: running time (hard problems), less redundancy

Pay: testing $Z_{\mathbb{R}}(\widetilde{Q_1}^h) \cap Z_{\mathbb{R}}(\widetilde{Q_0}^h) = \emptyset$

A short Maple worksheet demo

An expirical fact: all polynomials in $F \setminus B$ can be relaxed

Conclusion and future work

- The minimality of border polynomial of an cananical regular chain
- A more intrinsic notion, effective boundary, for real solution classification
- Relaxation technique in our FPS based QFF constrution: less redundancy in output, solve some hard problems

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Thank you!

The decomposition algorithm

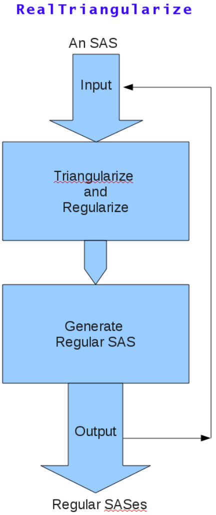
Step 1 "Algebraic" decomposition: pre-regular semi-algebraic systems

Step 2 "Real" decomposition: generate RSAS from each pre-regular semi-algebraic system $\mathcal{M}$

$$Z_{\mathbb{R}}(\mathcal{M}) \setminus Z_{\mathbb{R}}(D_{\neq}) = Z_{\mathbb{R}}(\mathcal{R})$$

Step 3 Making recursive calls: for each $f \in D$, compute and output

$\texttt{RealTriangularize}([T \cup \{f\}, P_{>}, (B)_{\neq}])$



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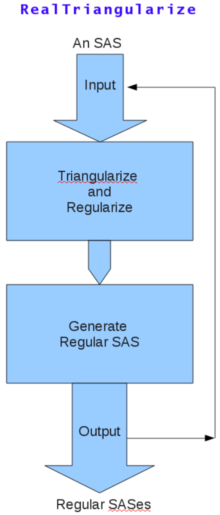

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