Contributions to Automatic Parallelization Probably in Support of Computer Algebra

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This talk is based on research projects of my former PhD students: Alexander Brandt (University of Nova Scotia), Sardar Anisul Haque (Otterbein University) Ruijuan Jing (Jiangsu University) Xiaohui Chen (HUAWEI), Davood Mohajerani (Untether AI), Wei Pan (NVIDIA), Delaram Talaashrafi (NVIDIA), Linxiao Wang (HUAWEI), Ning Xie (HUAWEI),

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Special thanks go to Alexander Brandt who helped who is leading the development of the Basic Polynomial Algebra Subprograms (BPAS) [1].
Tentative Plan

1. Parallel programming patterns
2. Their BPAS implementation (this part is skipped)
3. Parallelization and automatic parallelization
4. Pipeline pattern detection
5. Automatic GPU offloading
6. Automatic determination of optimal launch parameters for GPU kernels
Outline

1. Parallel programming patterns
2. BPAS implementation of parallel patterns
   2.1 Multi-threading in C++
   2.2 Implementing a thread pool
   2.3 Parallel patterns with ExecutorThreadPool
3. Parallelization and automatic parallelization
4. Pipeline pattern detection
5. Automatic determination of optimal launch parameters for GPU kernels
   5.1 Overview
   5.2 GPUs and the CUDA programming model
   5.3 Klaraptor
   5.4 Experimentation
6. Automatic prefetching of data to GPU’s shared memory
7. Conclusions
Parallel map and workpile

**Map** is the possibly the best known parallel programming pattern

- It executes a function on each item in a collection concurrently.
- With multiple Maps, tasks may execute in *lockstep*.

**Workpile** generalizes Map to a *queue of a tasks*, allowing tasks to add more tasks, thus enabling *load-balancing* as tasks start asynchronously.

- one possible implementation of workpile is a *thread pool*
Parallel map and workpile in computer algebra

Opportunities exist and have been studied in
- completion algorithms (Buchberger algorithm, the characteristic set method, etc.)
- dynamic evaluation (computations with regular chains)
Divide-and-conquer and fork-join

- **Divide**: Divide a problem into sub-problems, solving each recursively.

- **Combine**: Combine sub-solutions to produce a full solution.

- **Fork**: Execute multiple recursive calls in parallel (divide).

- **Join**: Merge parallel execution back into serial execution (combine).

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Marc Moreno Maza
Contributions to Automatic Parallelization
RTCA 2023
Opportunities exist and have been intensively studied in
- plain dense linear algebra and plain dense polynomial algebra,
- FFTs,
- asymptotically fast algorithms for matrices, polynomials, series, etc.
- real root isolation, etc.
Generators and pipelines

Generators

- A generator function (i.e. iterator) yields data items one at a time, allowing the function’s control flow to resume on its next execution.

Asynchronous Generators, Producer-Consumer

- *async generators* can concurrently produce items while the generator’s caller is consuming items, creating a producer-consumer pair

Pipeline

- By connecting many producer-consumer pairs we create a *pipeline*
- Pipelines need not be linear, they can be *directed acyclic graphs*
Generators and pipelines in computer algebra

Opportunities exist and but have received too little attention:

- **linear pipeline**: factorization over a finite field with its usual 3 stages (squarefree, distinct degree, equal degree),
- **non-linear pipeline**: computations of inverses and polynomial GCDs over direct product of fields with (at most) $2n$ stages for towers of $n$ field extensions,
- **another non-linear pipeline**: iterative methods in numerical linear algebra.

In this talk, we are interested in discovering pipelines from a sequence of for-loop nests accessing/updating arrays.
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Compiler-Level automatic parallelization

- **Cilk** and **OpenMP** provide automatic parallelization though compiler extensions
- Very easy but flexibility more challenging

```c
void mergeSort(int* A, int i, int j) {
    //... base case, k
    cilk_spawn mergeSort(A, i, k);
    mergeSort(A, k, j);
    cilk_sync
    merge(A, i, k, j);
}
```
Fork-Join parallelism with BPAS

- Object-oriented
- Standard C++, no compiler extensions
- Extends the *Thread Support Library* of C++11

```cpp
void mergeSort(int* A, int i, int j) {
    //... base case, k
    threadID id;
    ExecutorThreadPool& pool =
        ExecutorThreadPool::getThreadPool();

    pool.obtainThread(id);
    pool.executeTask(id, std::bind(mergeSort, A, i, k));
    mergeSort(A, k, j);

    pool.returnThread(id);

    merge(A, i, k, j);
}
```
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Threading primitives

C++11 introduced the *Thread Support Library*

- **std::thread**
  - C++ class encapsulating a thread (often a pthread) and its low-level spawn and join

- **std::mutex**
  - shared object between threads to indicate *mutual exclusion* to a *critical region.*
  - mutex is *locked* or *owned* by at most one thread at a time.

- **std::lock_guard, std::unique_lock**
  - temporary object wrapping a mutex whose object lifetime automatically locks and unlocks the mutex.
  - the constructor **blocks** and only returns once the shared mutex is successfully owned by the calling thread.

- **std::condition_variable**
  - blocks the current thread and temporarily releases a lock
  - receives notification from another thread to awaken the blocked thread
std::function

Functors, function objects, callable objects

- First-class objects which are callable using normal function syntax
- Are often constructed by passing function names, function pointers
- std::bind binds arguments to a function or function object, returning a function object which requires fewer arguments

```cpp
void printInteger(int a) {
    std::cout << a << std::endl;
}

// Function object from function name
std::function<void(int)> f_printInt(printInteger);
f_printInt(12);

// Function object binding arguments to function name
std::function<void()> f_print42(std::bind(printInteger, 42));
f_print42();
```
Parallel overheads

Creating and managing multiple threads of execution can be expensive

■ Every thread spawn requires non-insignificant amount of time
■ If more threads are active than the hardware supports, over-subscription occurs and repeated context switching slows down the program
■ Thread synchronization, locking mutexes, accessing critical regions require special care

Thread pools mitigate the first two, by supplying a fixed number of long-running threads.

Parallel programming patterns are algorithmic designs for efficient thread scheduling and minimizing locking
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Long-Running threads

Threads typically terminate once their assigned function/code block finishes

In order to implement and benefit from a thread pool, we require a mechanism which allows threads to:

1. Remain active until explicitly told to exit (or the entire program exits)
2. Receive new code blocks to execute on demand

*FunctionExecutorThreads* are such long-running threads which receive functions or code blocks and executes them asynchronously.
FunctionExecutorThread usage

```cpp
int A[N];
int* ret = new int();
FunctionExecutorThread t;

t.sendRequest( [=]() void -> {
    int s = 0;
    for (int i = 0; i < N; ++i) {
        s += A[i];
    }
    *ret = s;
});

doSomethingElse();

// make sure result is available before continuing
t.waitForThread();

std::cout << "sum: " << *ret << std::endl;
```
Object streams

The key to the implementation of `FunctionExecutorThread` is the `AsyncObjectStream` class. It provides:

1. a queue for tasks (or any object) and
2. a blocking mechanism to keep the `FunctionExecutorThread` alive and idle when waiting for tasks

- Actually a class template for any kind of object being passed between two threads
- Implements a queue satisfying the **producer-consumer problem**
- A `std::queue` combined with a mutex and condition variable
Thread pools

A **thread pool** manages a collection of long-running threads and a queue of tasks

- spawn all threads once at the beginning of program
- idle threads receive and execute tasks as required
- if all threads busy, tasks are added to queue
ExecutorThreadPool

- A thread pool built using FunctionExecutorThreads
- An internal queue of tasks and queue of threads
- When threads are busy, they are temporarily removed from the pool
- When all threads busy, tasks are added to task queue

```cpp
class ExecutorThreadPool {

private:
  std::deque<FunctionExecutorThread*> threadPool;
  std::deque<std::function<void()>> taskPool;
  std::mutex  m_mutex;
  std::condition_variable m_cv; // used in waitForThreads

  void tryPullTask();
  void putBackThread(FunctionExecutorThread* t);

public:
  void addTask(std::function<void()> f);
  void waitForThreads();
};
```
ExecutorThreadPool: flexible usage

- In support of certain parallel patterns, clients can (temporarily) obtain ownership of threads from the pool, rather than using addTask
- Abstract away actual threads through thread IDs
- Once thread obtained, repeat Steps 2–3 as often as necessary

```cpp
class ExecutorThreadPool {

    // Storage for threads removed from pool by obtainThread
    std::vector<FunctionExecutorThread*> occupiedThreads;

    // Step 1: obtain a thread's ID, removing it from the pool
    void obtainThread(threadID& id);

    // Step 2: execute a task on a particular thread
    void executeTask(threadID id, std::function<void()>& f);

    // Step 3 (optional): wait for thread to become idle
    void waitForThread(threadID id);

    // Step 4: return thread to pool (waits before returning)
    void returnThread(threadID id);
}
```
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Fork-Join with ExecutorThreadPool

```cpp
void mergeSort(int* A, int i, int j) {
    if (j <= i) { return; }
    int k = i + (j-1)/2;
    mergeSort(A, i, k);
    mergeSort(A, k, j);
    merge(A, i, k, j);
}
```
Workpile with ExecutorThreadPool

```cpp
void processInt(std::queue<int> B, int a) {
    a -= 10;
    if (a > 0) {
        getThreadPool().addTask(std::bind(processInt, B, a));
    } else {
        B.push(a);
    }
}

void WorkpileExample(std::queue<int> B, std::queue<int> A) {
    ExecutorThreadPool& pool = getThreadPool();
    while (!A.empty()) {
        pool.addTask(std::bind(processInt, B, A.front()));
        A.pop();
    }
    pool.waitForAllThreads();
}
```
AsyncGenerator and AsyncObjectStream

We want an object-oriented approach to create and use generators.

AsyncObjectStream already solves the producer-consumer problem.

- It provides a queue which blocks and notifies the consumer as data is produced, implemented using a condition variable
- As a class template, can be used within AsyncGenerator to yield any type of object

```cpp
template <class Object>
class AsyncObjectStream {
    void addResult(Object&& res); // Producer

    void resultsFinished(); // Producer

    bool getNextObject(Object& res); // Consumer

    void streamEmpty(); // Consumer
};
```
**AsyncGenerator**

*AsyncGenerator* is itself a class template, templated by `Object`, the type of object to generate.

- The *AsyncGenerator* acts as interface between producer and consumer
- The consumer constructs the *AsyncGenerator*, passing the constructor the producer’s function and arguments
- The producer’s signature should be:

  ```
  void producerFunction(..., AsyncGenerator<Object>&);
  ```
- The *AsyncGenerator* being constructed inserts itself into the producer’s list of arguments so that it has reference to the generator object
void FibonacciGen(int n, AsyncGenerator<int>& gen) {
    int Fn_1 = 0;
    int Fn = 1;
    for (int i = 0; i < n; ++i) {
        gen.generateObject(Fn_1); // yield Fn_1 and continue
        Fn = Fn + Fn_1;
        Fn_1 = Fn - Fn_1;
    }
    gen.setComplete();
}

void Fib() {
    int n;
    std::cin >> n;
    AsyncGenerator<int> gen(FibonacciGen, n);

    int fib;
    // get one integer at a time until generator is finished
    while (gen.getNextObject(fib)) {
        std::cerr << fib << std::endl;
    }
}
Cooperative parallelism

With several simultaneous clients of `ExecutorThreadPool` (workpile, fork-join, generators), some tasks should be given priority.

- Some tasks are more **coarse-grained**, offer more potential speed-up
- Some tasks may expose more parallelism and should be executed first

Often, parallelism coming from Fork-Join or Map is preferred over Producer-Consumer.

- **Goal**: allow Fork-Join and Map to access thread pool threads over Producer-Consumer while still keeping the latter possible when there are idle threads
- **Solution**: priority tasks
  - `addTask()` vs `addPriorityTask()`
  - If all threads busy, `addPriorityTask()` temporarily spawns new thread to start execution immediately
The BPAS library

- A high-performance polynomial algebra library
  - Core of library written in C, wrapped in C++ interface for usability and object-oriented programming

- Optimized algorithms and data structures, data locality, and parallelism
  - Sparse multivariate polynomials [3], dense univariate and bivariate [21]
  - Triangular decomposition of polynomial systems [2, 4]

- User-friendly, object-oriented interface based on template meta-programming [5]
  - A natural encoding of the algebraic hierarchy
  - “Dynamic” creation of algebraic types through composition
  - Compile-time type safety between algebraic types

- Generic support for parallel programming and parallel patterns (this talk)

http://www.bpaslib.org/
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Parallelization and automatic parallelization

- Parallelization usually refers to the programmer applying the fork-join pattern to a divide-and-conquer algorithm, or turning a `for` into `parallel-for`.

- Automatic parallelization usually refers to the compiler exposing parallelism (dependence analysis, tiling, loop transformations, etc.) scheduling tasks (load balancing, reducing parallelism overheads, etc.), generating parallel code (allocating resources, in particular memory).
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Dependence analysis in the polyhedral model

Cholesky’s LU decomposition:

```plaintext
for (i = 1; i <= n; i++) {
    x = a[i][i];
    for (k = 1; k < i; k++)
        x = x - a[i][k] * a[i][k];
    p[i] = 1.0/sqrt(x);
    for (j = i + 1; j <= n; j++) {
        x = a[i][j];
        for (k = 1; k < i; k++)
            x = x - a[j][k] * a[i][k];
        a[j][i] = x * p[i];
    }
}
```
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        for (k = 1; k < i; k++)
            x = x - a[j][k] * a[i][k];
    a[j][i] = x * p[i];
}
```

**system 1:**

\[
\begin{align*}
1 & \leq i \leq n \\
i + 1 & \leq j \leq n \\
1 & \leq k \leq i - 1 \\
i^\prime & \leq i \leq n \\
i^\prime + 1 & \leq j^\prime \leq n \\
j = j^\prime, k = i^\prime \\
i & < i^\prime
\end{align*}
\]

**system 2:**

\[
\begin{align*}
1 & \leq i \leq n \\
i + 1 & \leq j \leq n \\
1 & \leq k \leq i - 1 \\
i^\prime & \leq i \leq n \\
i^\prime + 1 & \leq j^\prime \leq n \\
j = j^\prime, k = i^\prime \\
i & = i^\prime, j < j^\prime
\end{align*}
\]

**system 3:**

\[
\begin{align*}
1 & \leq i \leq n \\
i + 1 & \leq j \leq n \\
1 & \leq k \leq i - 1 \\
i^\prime & \leq i \leq n \\
i^\prime + 1 & \leq j^\prime \leq n \\
j = j^\prime, k = i^\prime \\
i & = i^\prime, j = j^\prime
\end{align*}
\]
Automatic parallelization: plain multiplication

Serial dense univariate polynomial multiplication

```
for(i=0; i<=n; i++){
    c[i] = 0; c[i+n] = 0;
    for(j=0; j<=n; j++)
        c[i+j] += a[i] * b[j];
}
```

Dependence analysis suggests to set \( t(i, j) = n - j \) and \( p(i, j) = i + j \).
Automatic parallelization: plain multiplication

Serial dense univariate polynomial multiplication

```c
for(i=0; i<=n; i++){
    c[i] = 0; c[i+n] = 0;
    for(j=0; j<=n; j++)
        c[i+j] += a[i] * b[j];
}
```

Dependence analysis suggests to set $t(i, j) = n - j$ and $p(i, j) = i + j$.

Asynchronous parallel dense univariate polynomial multiplication

```c
parallel_for (p=0; p<=2*n; p++){
    c[p] = 0;
    for (t=max(0,n-p); t<= min(n,2*n-p); t++)
}
```
Generating parametric code & use of tiling techniques

```c
parallel_for (p=0; p<=2*n; p++){
    c [ p ] =0;
    for (t=max(0,n-p); t<= min(n,2*n-p);t++)
        C [ p ] = C [ p ]
        + A [ t+p-n ] * B [ n-t ] ;
}
```

Improving the parallelization

- The above generated code is not practical for multicore implementation: the number of processors is in $\Theta(n)$. (Not to mention poor locality!) and the work is unevenly distributed among the workers.
- We group the virtual processors (or threads) into 1D blocks, each of size $B$. Each thread is known by its block number $b$ and a local coordinate $u$ in its block.
- Blocks represent good units of work which have good locality property.
- This yields the following constraints: $0 \leq u < B$, $p = bB + u$. 
Generating parametric code: using tiles

We apply Regular-Chains:-QuantifierElimination on the left system (in order to get rid off $i,j$) leading to the relations on the right:

\[
\begin{align*}
& o < n \\
& 0 \leq i \leq n \\
& 0 \leq j \leq n \\
& t = n - j \\
& p = i + j \\
& 0 \leq b \\
& o \leq u < B \\
& p = bB + u,
\end{align*}
\]

From where we derive the following program:

```c
for (p=0; p<=2*n; p++) c [ p ] = 0;
parallel_for (b=0; b<= 2 n / B; b++) {
  parallel_for (u=0; u<=min(B-1, 2*n - B * b); u++) {
    p = b * B + u;
    for (t=max(0,n-p); t<=min(n,2*n-p) ;t++)
      c [ p ] = c [ p ] + a [ t+p-n ] * b [ n-t ];
  }
}
```

B > 0
\[
\begin{align*}
& n > 0 \\
& 0 \leq b \leq 2n/B \\
& 0 \leq u < B \\
& 0 \leq u \leq 2n - Bb \\
& p = bB + u,
\end{align*}
\]

\(1\)
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This is a joint work with:

- Delaram Talaashrafi (NVIDIA)
- Johannes Doerfert (Lawrence Livermore National Laboratory)

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The problem

The polyhedral model

- The polyhedral model is effective for optimizing loop nests using different methods: dependence analysis, loop tiling (blocking), …
- They all optimize for-loop nests on a per-loop basis.

Our work

- is about exploiting cross-loop parallelization, through tasking.
- is done by detecting pipeline pattern between iteration blocks of different loop nests.

Polly

**Polly** is an LLVM-based framework, which applies polyhedral transformations: analysis, transformation, scheduling, code generation.

We use **OpenMP** task construct and the depend clause to exploit the detected parallelism.
Motivating example

- The Statement $S$ of the first loop nest updates the array $A$
- The Statement $T$ of the second loop nest updates the array $B$ and uses the array $A$
- Before the first loop completes one can start executing the second.

```c
for(i=0; i<N-1; i++)
  for(j=0; j<N-1; j++)
    S: A[i][j]=f(A[i][j], A[i][j+1], A[i+1][j+1]);

for(i=0; i<N/2-1; i++)
  for(j=0; j<N/2-1; j++)
    R: B[i][j]=g(A[i][2*j], B[i][j+1], B[i+1][j+1],
               B[i][j]);
```
Motivating example

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```
Motivating example
Motivating example

for(i=0; i<N-1; i++)
    for(j=0; j<N-1; j++)
        S: A[i][j] = f(A[i][j], A[i][j+1], A[i+1][j+1]);

for(i=0; i<N/2-1; i++)
    for(j=0; j<N/2-1; j++)
        R: B[i][j] = g(A[i][2*j], B[i][j+1], B[i+1][j+1], B[i][j]);

\[
\begin{align*}
\langle S, [0, 0] \rangle & \langle S, [0, 1] \rangle \langle S, [0, 2] \rangle \\
\langle S, \text{last} \rangle \langle R, [0, 0] \rangle & \langle R, [0, 1] \rangle \\
\end{align*}
\]
Consider two statements in a program:

- S: with iteration domain \( \mathcal{I} \), which writes a memory location in \( \mathcal{M} \), thus defining a binary relation \( Wr(\mathcal{I} \rightarrow \mathcal{M}) \)

- T: with iteration domain \( \mathcal{J} \), which reads a memory location from \( \mathcal{M} \), thus defining a binary relation \( Rd(\mathcal{J} \rightarrow \mathcal{M}) \)

The pipeline map between S and T is \( T_{S,T}(\mathcal{I} \rightarrow \mathcal{J}) \), where \( (\vec{i}, \vec{j}) \in T_{S,T} \) if and only if:

1. after running all iterations of S up to \( \vec{i} \), we can safely run all iterations of T up to \( \vec{j} \),
2. \( \vec{i} \) is the smallest vector and \( \vec{j} \) is the largest vector with Property (1).
We first find the pipeline map between all pairs of dependent statements,
then, use them to block the iteration domains and find pipeline blocking maps.

The final blocks are such that:
- each block is an atomic task,
- we can establish a pipeline relation between all blocks of all statements,
- maximize the number of blocks of different loops that can execute in parallel.

In the last step, we find dependency relations between the tasks.
Transformation Algorithm (3/3)

Optimal block of $S_3$:

Pipeline dependencies:

$\sum \{ \left[ \begin{array}{c} \mathcal{T}_1 \\ \mathcal{T}_2 \end{array} \right] \}$
Transformation Algorithm (3/3)

Transformation Algorithm (3/3)

Optimal block of $S_3$:

Pipeline dependencies:

$S_1$, $S_2$, $S_3$, $S_4$

$S_1$ iterations

$S_3$ iterations
Transformation Algorithm (3/3)

- \( S_1 \)
  - \( \tau_1 \)
  - \( S_3 \)
  - \( \tau_2 \)
- \( S_2 \)
- \( S_3 \)
  - \( S_4 \)

Optimal block of \( S_3 \):
\[
\big\{ \vdash S_3, \sum_\{ \vdash \} j_3 \}\]

Pipeline dependencies:
\[
\big\{ \vdash S_1, \sum_\{ \vdash \} i_1 \big\}, \big\{ \vdash S_2, \sum_\{ \vdash \} i_2 \big\}
\]

\( S_4 \) iterations:
\[
\big\{ \vdash j_0 \big\}, \big\{ \vdash j_2 \big\}, \big\{ \vdash j_1 \big\}
\]
Transformation Algorithm (3/3)

![Diagram showing the transformation algorithm with nodes S1, S2, S3, and S4 connected by edges τ1, τ2, and τ3.]

- **Optimal block of S3:** \[ \bigcup S_3 \]
- **Pipeline dependencies:**
  - S1
  - \( \bigcup S_1 \)
  - S2
  - \( \bigcup S_2 \)
  - S4
  - \( \bigcup S_4 \)

**Iterations:**
- S1 iterations
- S2 iterations
- S3 iterations
- S4 iterations

---

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Transformation Algorithm (3/3)

Optimal block of \( S_3 \): \( \langle S_3, \tilde{j}_3 \rangle \)
Pipeline dependencies: \( \langle S_1, \tilde{i}_1 \rangle, \langle S_2, \tilde{i}_2 \rangle \)

\( \rightarrow \text{skip slide} \)
Implementation (1/2)

Analysis passes of Polly

We **extend** the analysis passes of Polly to compute pipeline information for the iteration domains.

Scheduling

1. Create a schedule tree to iterate *over* blocks,
2. Create a schedule tree to iterate *inside* each block,
3. **Expand** the first tree with the second tree.
4. Create `pw_multi_aff_list` objects from pipeline dependency relations,
5. Add the `pw_multi_aff_list` objects as mark nodes to the schedule tree.
Implementation (2/2)

1. We generate AST from the new schedule tree.
2. The mark nodes in the schedule tree **annotates** the AST.

### Code generation

1. Outline tasks to function calls,
2. Compute unique integer numbers from `pw_multi_aff_list` objects
   - this can be used in OpenMP depend clauses.
3. Replace the tasks part in the code with call to the `CreateTask` function that:
   - gets tasks and dependencies, creates OpenMP tasks with proper depend clauses,
   - handles the order between tasks created from the same loop nest.
Evaluation

Pipelining speed-up of the tests with different access functions, and different sizes on 4 cores.
Related Works

**Pipelined multithreading generation in a polyhedral compiler [22]:**
- a limited, polyhedral model based, source-to-source method to solve a similar problem
- uses nowait and order clauses of OpenMP to exploit pipeline parallelism

**Compiling neural networks for a computational memory accelerator [15]**
- an algorithm to detect pipeline parallelism between two loop nests
- applicable on the computational memory accelerators considered in that paper
Future works

Generalize **transformation** algorithm:
- works with non-injective write functions,

Generalize our **code generation** phase:
- generate code for loops with arbitrary depth and arbitrary number of tasks per loop.

Develop an algorithm to **choose a good task granularity** when there are multiple choices.

Change the tasking layer from the OpenMP task to **other platforms**.

Take advantage of **other parallelization opportunities**, when using the cross-loop tasking.
1. Parallel programming patterns
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   5.3 Klaraptor
   5.4 Experimentation
6. Automatic prefetching of data to GPU’s shared memory
7. Conclusions

This is a joint work with Alexander Brandt, Taabish Jeshani, Davood Mohajerani, Jeeva Paudel (IBM) and Linxiao Wang.

This is a US Patent (US 10,901,713 B2)

https://github.com/orcca-uwo/KLARAPTOR
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Vector addition in CUDA

```c
__global__ void vecAdd(float* A, float* B, float* C)
{
    int i = threadIdx.x + blockDim.x * blockIdx.x;
    C[i] = A[i] + B[i];
}

int main()
{
    // Run grid of N/256 blocks of 256 threads each
    vecAdd<<<N/256, 256>>>(d_A, d_B, d_C);
}
```
Launch parameters greatly impact the performance of a given kernel.

Figure: Example polybench _2DCONV_ Kernel Convolution2D_kernel for input size 8192.
Launch parameters greatly impact the performance of a given kernel. They depend on data size and cannot be determined at compile time.

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**klaraptor’s goal**: automatically and dynamically find the values of the launch parameters which optimize the performance of a CUDA kernel.

**Figure**: Example polybench

_2DCONV_ Kernel Convolution2D_kernel for input size 8192
Launch parameters greatly impact the performance of a given kernel.

They depend on data size and cannot be determined at compile time.

klaraptor’s goal: automatically and dynamically find the values of the launch parameters which optimize the performance of a CUDA kernel.

We achieve this goal for CUDA kernels implementing data-oblivious algorithms.
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GPUs are designed for massive parallelism, while CPUs focus on sequential processing.

GPUs handle massive amounts of data and perform the same operation on them simultaneously (SIMD).
Heterogeneous programming

- A CUDA program is a serial program with parallel kernels, all in C.
- The serial C code executes in a **host** (= CPU) thread
- The parallel kernel C code executes in many **device** threads across multiple GPU processing elements, called **streaming processors** (SP).

<table>
<thead>
<tr>
<th>Serial Code</th>
<th>Host</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel Kernel</td>
<td></td>
</tr>
<tr>
<td>KernelA (args);</td>
<td>Device</td>
</tr>
<tr>
<td>Serial Code</td>
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<td></td>
</tr>
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<td>KernelB (args);</td>
<td>Device</td>
</tr>
</tbody>
</table>
Blocks Run on Multiprocessors

Kernel launched by host

Device processor array

Device Memory
Streaming processors and multiprocessors

Streaming Processor

Streaming Multiprocessor

Thread

Registers

Memory

Threadblock

Per-block Shared Memory

Memory
Example: increment array elements

CPU program

```c
void increment_cpu(float *a, float b, int N) {
    for (int idx = 0; idx<N; idx++)
        a[idx] = a[idx] + b;
}
```

CUDA program

```c
__global__ void increment_gpu(float *a, float b, int N) {
    int idx = blockIdx.x * blockDim.x + threadIdx.x;
    if (idx < N)
        a[idx] = a[idx] + b;
}
```

```c
void main() {
    ..... 
    increment_cpu(a, b, N);
}
```

```c
void main() {
    ..... 
    ..... 
    dim3 dimBlock (blocksize);
    dim3 dimGrid( ceil( N / (float)blocksize) );
    increment_gpu<<<dimGrid, dimBlock>>>(a, b, N);
}
```
Data transfer between one streaming multiprocessors (SMs) and the global memory can take 100’s of clock cycles.
Estimating running time with the MWP-CWP model (1/2)

- Data transfer between one streaming multiprocessors (SMs) and the global memory can take 100’s of clock cycles
- SMs typically run several groups of threads (warps) concurrently so as to hide those delays with computations

Two quantities measure the effectiveness of this mechanism: the Memory Warp Parallelism (MWP) and the Computation Warp Parallelism (CWP), see [14].

MWP and CWP can be determined by code profiling and cannot determined by static analysis.

MWP and CWP (combined with hardware characteristics) produces a sharp estimate of the running time of a (simple) kernel.

The formula is given by a piece-wise multivariate rational function, see [14]
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Estimating running time with the MWP-CWP model (2/2)

\[
\text{Mem\_L\_Uncoal} = \text{Mem\_LD} + (\#\text{Uncoal\_per\_mw} - 1) \times \text{Departure\_del\_uncoal}
\]

\[
\text{Mem\_L\_Coal} = \text{Mem\_LD}
\]

\[
\text{Mem\_L} = \text{Mem\_L\_Uncoal} \times \text{Weight\_uncoal} + \text{Mem\_L\_Coal} \times \text{Weight\_coal}
\]

\[
\text{Weight\_uncoal} = \frac{\#\text{Uncoal\_Mem\_insts}}{\#\text{Uncoal\_Mem\_insts} + \#\text{Coal\_Mem\_insts}}
\]

\[
\text{Weight\_coal} = \frac{\#\text{Coal\_Mem\_insts}}{\#\text{Coal\_Mem\_insts} + \#\text{Uncoal\_Mem\_insts}}
\]

\[
\text{Departure\_delay} = (\text{Departure\_del\_uncoal} \times \#\text{Uncoal\_per\_mw}) \times \text{Weight\_uncoal} + \text{Departure\_del\_coal} \times \text{Weight\_coal}
\]

\[
\text{MWP\_Without\_BW\_full} = \text{Mem\_L} / \text{Departure\_delay}
\]

\[
\text{MWP\_Without\_BW} = \text{MIN}(\text{MWP\_Without\_BW\_full}, \#\text{Active\_warps\_per\_SM})
\]

\[
\text{Mem\_cycles} = \text{Mem\_L\_Uncoal} \times \#\text{Uncoal\_Mem\_insts} + \text{Mem\_L\_Coal} \times \#\text{Coal\_Mem\_insts}
\]

\[
\text{Comp\_cycles} = \#\text{Issue\_cycles} \times (\#\text{total\_insts})
\]

\[
N = \#\text{Active\_warps\_per\_SM}
\]

\[
\#\text{Rep} = \frac{\#\text{Blocks}}{\#\text{Active\_blocks\_per\_SM} \times \#\text{Active\_SMs}}
\]

If (MWP is N warps per SM) and (CWP is N warps per SM)

\[
\text{Exec\_cycles\_app} = (\text{Mem\_cycles} + \text{Comp\_cycles} + \frac{\text{Comp\_cycles}}{\#\text{Mem\_insts}} \times (\text{MWP} - 1)) \times \#\text{Rep}
\]

If (CWP >= MWP) or (Comp\_cycles > Mem\_cycles)

\[
\text{Exec\_cycles\_app} = (\text{Mem\_cycles} \times \frac{N}{\text{MWP}} + \frac{\text{Comp\_cycles}}{\#\text{Mem\_insts}} \times (\text{MWP} - 1)) \times \#\text{Rep}
\]

If (MWP > CWP)

\[
\text{Exec\_cycles\_app} = (\text{Mem\_L} + \text{Comp\_cycles} \times N) \times \#\text{Rep}
\]
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Klaraptor’s underlying theory (1/2)

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- Let $\mathcal{E}$ be a high-level performance metric (e.g. running time, memory consumption) for $\mathcal{P}$ that we want to optimize.
- Given values of $D$, our goal is to find values of the $P$ such that the execution of $\mathcal{P}$ optimizes $\mathcal{E}$. 
Klaraport’s underlying theory (2/2)

- Performance prediction models (like the MWP-CWP model) attempt to estimate $\mathcal{E}$ from a combination of $P$, $D$, and platform-specific low-level metrics $L = (L_1, \ldots, L_\ell)$ (memory throughput, cache miss rate, etc.).
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- This mathematical formula takes the form of a piece-wise rational function.
- At the runtime of $\mathcal{P}$, given specific values of $D$ and a choice of $P$, we can evaluate those functions for each metric and thus compute $\mathcal{E}$.
- Repeating this for all possible choices of $P$ (assumed to be finite in number) yields values of $P$ optimizing $\mathcal{E}$. 
Main steps of Klaraptor’s algorithm

Data collection: Run the CUDA kernel with small input sizes and various program parameters to collect low-level metrics

Rational function estimation: Determine rational functions estimating low-level metrics by solving curve fitting problems

Code generation: Link the rational program to the original CUDA program

Helper program evaluation: Evaluate the rational program with the known runtime data parameters and all meaningful program parameters to estimate program performance

Program execution: Launch the program with the selected kernel launch parameters
High-level view of Klaraptor’s implementation

We have used:

- **LLVM Pass Framework** for modifying the code at the IR level
- **NVIDIA Nsight Compute CLI** to do the data collection
- **CLAPACK and ATLAS** for the numerical computations done in the curve fitting step
- system specs: LLVM 11, CUDA 11, CLAPACK, python 2.7.
Annotations and preprocessing the source code

```c
#pragma kernel_info_size_param_idx_Sample = 1;
#pragma kernel_info_dim_sample_kernel = 2;

__global__ void Sample (int *A, int N) {
    int tid_x = threadIdx.x + blockIdx.x*blockDim.x;
    int tid_y = threadIdx.y + blockIdx.y*blockDim.y;
    ...
}
```

- Annotating and preprocessing the source code makes it compatible with the KLARAPTOR tool, enabling the automatic determination of optimal kernel launch parameters.
- CUDA program should target at least CUDA Compute Capability 7.5, no CUDA runtime API calls, and block and grid dimensions must be declared as dim3 structs.
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Figure: Comparing kernel execution time (log-scaled) for the thread block configuration chosen by KLARAPTOR versus the minimum and maximum times as determined by an exhaustive search over all possible configurations. Kernels are part of the PolyBench/GPU benchmark suite and executed on a RTX 2070 SUPER with a data size of $N = 8192$ (except convolution3d with $N = 512$)
Table: KLARAPTOR Optimization Times on Polybench/GPU, RTX 2070 SUPER

Comparing times for (1) compile-time optimization steps of KLARAPTOR, (2) exhaustive search over all thread block configurations, the execution time for a kernel given (3) the best thread block configuration, and (4) the worst thread block configuration. Exhaustive search is given as a sum for values up to $N = 8192$ (except convolution3d with $N = 512$).

<table>
<thead>
<tr>
<th>Kernel</th>
<th>KLARAPTOR Time (s) $128 \leq N &lt; \infty$</th>
<th>Ex. Search Time (s) $128 \leq N \leq 8192$</th>
<th>Min Time (s) $N = 8192$</th>
<th>Max Time (s) $N = 8192$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2DCONV</td>
<td>210.29</td>
<td>82.78</td>
<td>0.002</td>
<td>0.023</td>
</tr>
<tr>
<td>ATAX</td>
<td>507.59</td>
<td>59.60</td>
<td>0.006</td>
<td>1.940</td>
</tr>
<tr>
<td>MVT</td>
<td>508.03</td>
<td>60.03</td>
<td>0.005</td>
<td>1.978</td>
</tr>
<tr>
<td>BICG</td>
<td>510.91</td>
<td>60.16</td>
<td>0.006</td>
<td>2.050</td>
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Outline

1. Parallel programming patterns
2. BPAS implementation of parallel patterns
   2.1 Multi-threading in C++
   2.2 Implementing a thread pool
   2.3 Parallel patterns with ExecutorThreadPool
3. Parallelization and automatic parallelization
4. Pipeline pattern detection
5. Automatic determination of optimal launch parameters for GPU kernels
   5.1 Overview
   5.2 GPUs and the CUDA programming model
   5.3 Klaraptor
   5.4 Experimentation
6. Automatic prefetching of data to GPU’s shared memory
7. Conclusions
This is a joint work with:
- Delaram Talaashrafi (NVIDIA)
- Johannes Doerfert (Lawrence Livermore National Laboratory)

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Conclusions

Pipelines:

- Pipelining is an underutilized pattern with lots of potential application in computer algebra
- We presented a mechanism for discovering and exploiting pipelines over a sequence of for-loop nests
- This is completely (no competitive methods) and implemented in LLVM/Polly
- Ask Delaram or Johannes if you are interested.

Klaraptor:

- Kernel launch parameters of GPU kernels are important-and-hard right
- We propose an **AI-free** approach which works successfully with data-oblivious algorithms
- All other approaches for that problem use AI . . .
- Ask me if you are interested.
Thank You!

http://www.bpaslib.org/
References


