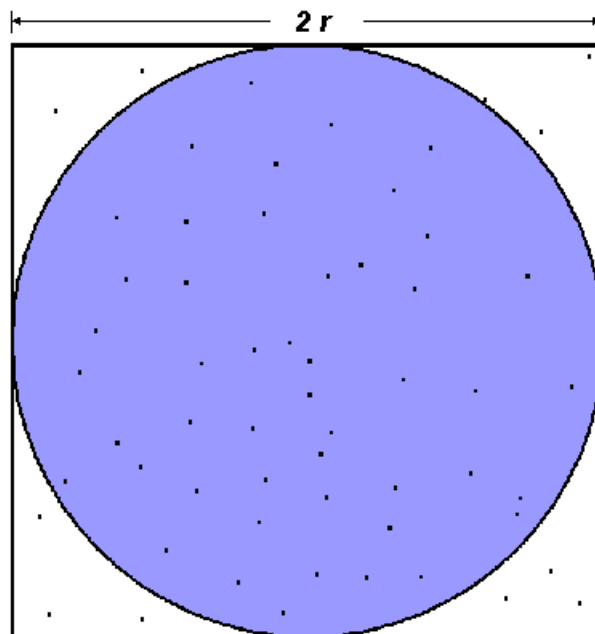


Problem Set 2

PROBLEM 1. [40 points]

The value of π can be calculated in a number of ways. Consider the following method of approximating π :

- (1) Inscribe a circle (of radius r) into a square of size $2r$.
- (2) Randomly generate N points in the square (for a large N , say $N = 1000000$).
- (3) Determine the number of points in the square that are also in the circle.
- (4) Let f be the number of points in the circle divided by the number of points in the square.
- (5) π can be estimated as $4f$.



$$A_S = (2r)^2 = 4r^2$$

$$A_C = \pi r^2$$

$$\pi = 4 \times \frac{A_C}{A_S}$$

Question 1. [10 points] Explain why π can be estimated by $4f$.

Question 2. [10 points] Write a serial Julia program implementing the above algorithm.

Question 3. [10 points] Write a parallel Julia program implementing the above algorithm.

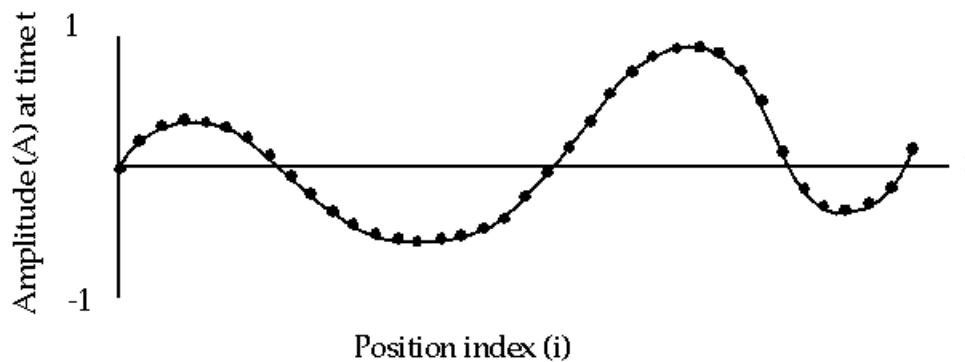
Question 4. [10 points] Compare the running times of the two programs for various values of N (say $N = i \times 20000$, for $i = 1 : 10$) and various values of the number p of workers ($p = 2, 3, 4$).

PROBLEM 2. [60 points]

In this exercise, we are interested in simulating a one-dimension wave. See for instance the first animation at the page:

http://en.wikipedia.org/wiki/Wave_equation

To be more precise, we are studying a vibrating string (after a specified amount of time has elapsed) and we are estimating the amplitude at various points of that string.



The computations are performed by the following formula:

$$A(i, t + 1) = (2.0 * A(i, t)) - A(i, t - 1) + (c(A(i - 1, t) - (2.0 * A(i, t)) + A(i + 1, t))) \quad (1)$$

where c is a constant and $A(i, t)$ is the amplitude of the point of coordinate i at time step t .

Note that the amplitude $A(i, t + 1)$ will depend on previous time-steps ($t, t - 1$) and neighboring points ($i - 1, i + 1$). Data dependence will mean that a parallel solution will involve communications.

We shall assume that c is given (say $c = 2$) and that $A(i, t)$ is known for $1 \leq i \leq 100000$ at $t = 0$ and $t = 1$. A natural choice is to assume that $A(i, 0)$ is a sine wave and $A(i, 1)$ a small translation (or shift) of $A(i, 0)$.

http://en.wikipedia.org/wiki/Sine_wave

For instance $A(i, 0) = \sin(\frac{\pi}{75}i)$ and $A(i, 1) = \sin(\frac{\pi}{75}(i - 1))$.

Question 1. [20 points] Write a serial `Julia` program simulating this vibrating string at time-steps $2 \leq t \leq 100000$.

Question 2. [30 points] Write a parallel `Julia` program simulating this vibrating string at time-steps $2 \leq t \leq 100000$. Consider using a distributed array for encoding the amplitude A . Note that, at each time step, the different workers will need to exchange data since for updating the amplitude at each point in the string needs to know the amplitude at the neighboring points.

Question 3. [10 points] Compare the running times of the two programs for various values of the number p of workers ($p = 2, 3, 4$).

Submission instructions.

Format: Problems 1 and 2 involve programming with `Julia`: they must be submitted as two input files to be called `Pb1.jl` and `Pb2.jl`, respectively. Each of these two files must be a valid input file for `Julia`. In addition, each user defined function must be documented and the measured the running times should be presented in a separate PDF file called `Pb1+2.pdf`.

To summarize, each assignment submission consists of three files: `Pb1.jl`, `Pb2.jl` and `Pb1+2.pdf`.

Submission: The assignment should be returned to the instructor **and** the TA by email.

Collaboration. You are expected to do this assignment *on your own* without assistance from anyone else in the class. However, you can use literature and if you do so, briefly list your references in the assignment. Be careful! You might find on the web solutions to our problems that are not appropriate. For instance, because the cache memory model is different. So please, avoid those traps and work out the solutions by yourself. You should not hesitate to contact the instructor or the TA if you have any question regarding this assignment. We will be more than happy to help.

Marking. This assignment will be marked out of 100. A 10 % bonus will be given if your answers are clearly organized, precise and concise. Messy assignments (unclear statements, lack of correctness in the reasoning, many typographical or language mistakes) may give rise to a 10 % malus.