

CS4442/9542b  
Artificial Intelligence II  
prof. Olga Veksler

*Lecture 9*  
*Computer Vision*  
**Edge Detection**

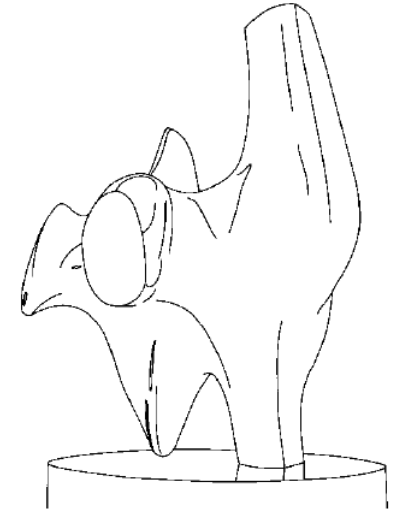
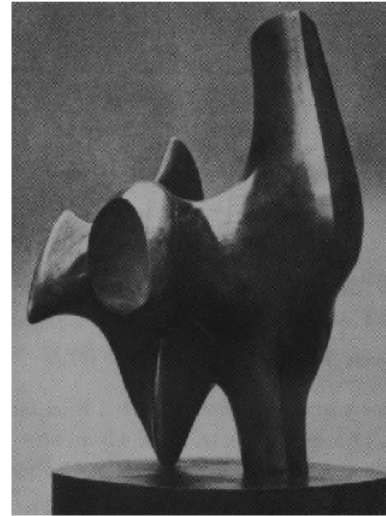
Some slides from: S.Seitz, D. Jacobs, D. Lowe, H.  
Man, K. Grauman, D. Hoiem, S. Lazebnik

# Outline

- Edge Detection
  - Edge types
  - Image Gradient
  - Canny Edge Detector
- Application
  - intelligent image resizing: Seam Carving

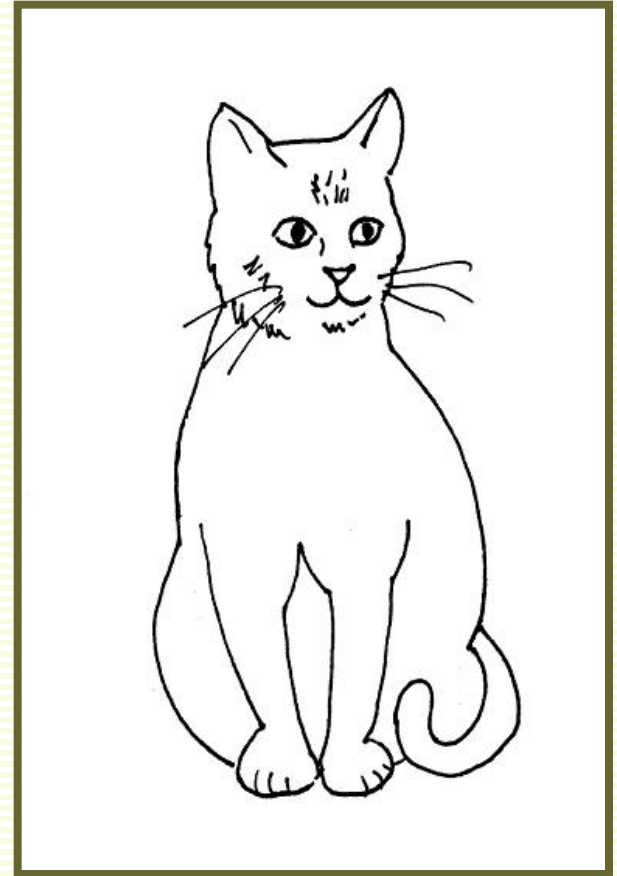
# Edge Detection

- Convert intensity image into binary (0 or 1) image that marks **prominent** curves
- What is a prominent curve?
  - no exact definition
  - intuitively, it is a place where abrupt changes occur
- Why perform edge detection?
  - most shape and semantic and information is encoded in edges
  - edges are stable to lighting and other changes, makes them good features for object recognition, etc.
  - more compact representation than intensity



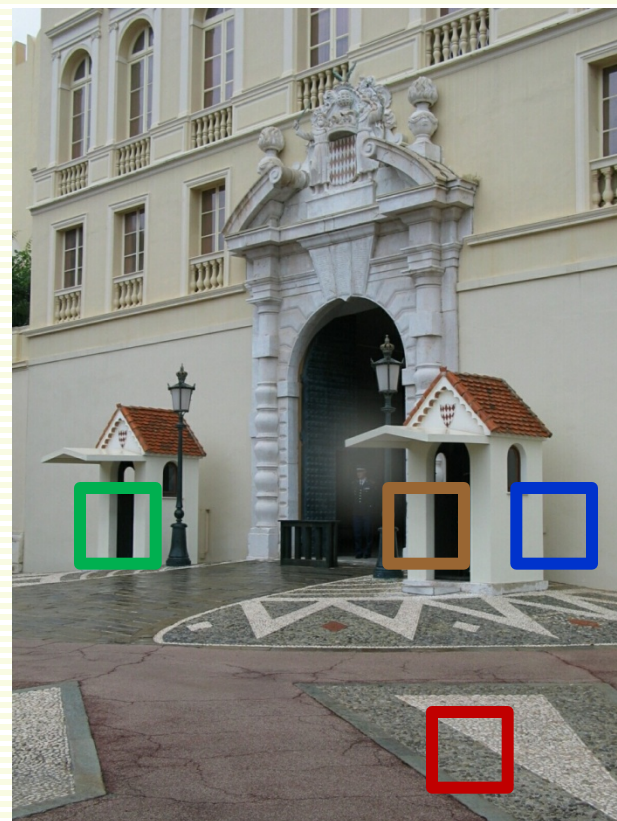
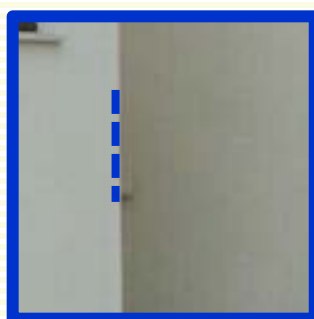
# Line Drawings

- Artists do it
  - and much better, as they use high level knowledge which edges are more perceptually important



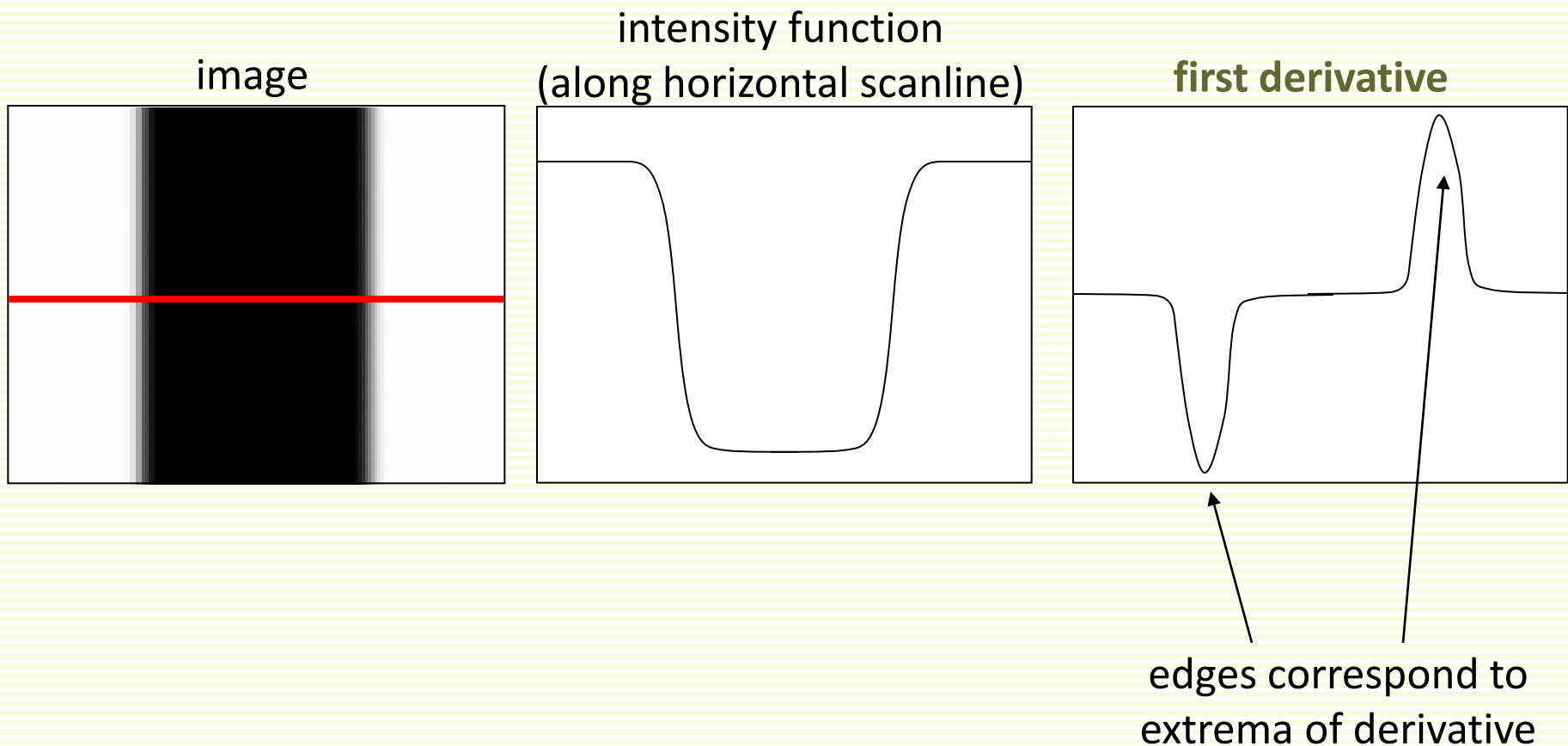
# Origin of Edges

- Many **discontinuity** causes:
  - surface color or texture discontinuity
  - depth discontinuity (object boundary)
  - surface normal discontinuity
  - illumination discontinuity (shadows)



# Derivatives and Edges

- An edge is a place of rapid change in intensity



# Derivatives with Convolution

- For 2D function  $f(x,y)$ , partial derivative in horizontal direction

$$\frac{\partial f(x, y)}{\partial x} = \lim_{\varepsilon \rightarrow 0} \frac{f(x + \varepsilon, y) - f(x, y)}{\varepsilon}$$

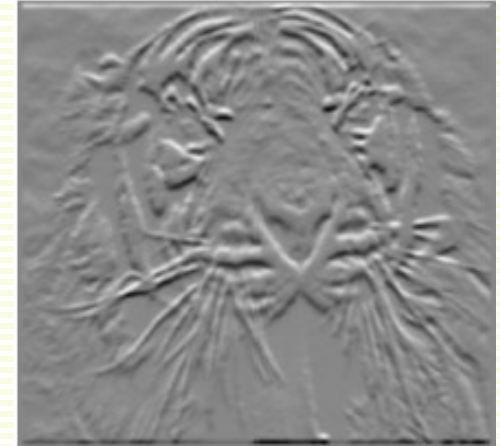
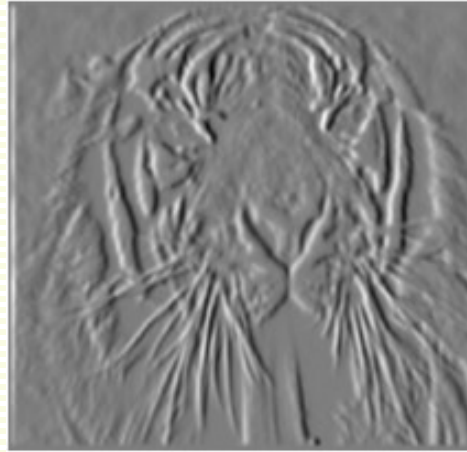
- For discrete data, approximate

$$\frac{\partial f(x, y)}{\partial x} \approx \frac{f(x + 1, y) - f(x, y)}{1}$$

- Similarly, approximate vertical partial derivative (wrt  $y$ )
- How to implement as a convolution?

# Image Partial Derivatives

Which is with respect to x?



$$\frac{\partial f(x, y)}{\partial x}$$

$$\frac{\partial f(x, y)}{\partial y}$$

|    |    |
|----|----|
| -1 | 1  |
| or |    |
| 1  | -1 |

|    |    |
|----|----|
| -1 | 1  |
| 1  | -1 |



# Finite Difference Filters

- Other filters for derivative approximation

Prewitt:  $H_x = \frac{1}{6}$

|    |   |   |
|----|---|---|
| -1 | 0 | 1 |
| -1 | 0 | 1 |
| -1 | 0 | 1 |

$$H_y = \frac{1}{6}$$

|    |    |    |
|----|----|----|
| 1  | 1  | 1  |
| 0  | 0  | 0  |
| -1 | -1 | -1 |

Sobel:  $H_x = \frac{1}{8}$

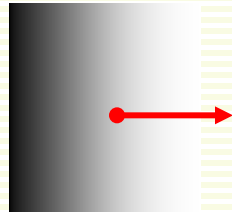
|    |   |   |
|----|---|---|
| -1 | 0 | 1 |
| -2 | 0 | 2 |
| -1 | 0 | 1 |

$$H_y = \frac{1}{8}$$

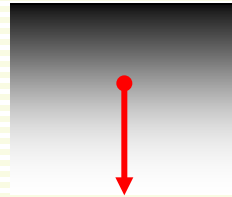
|    |    |    |
|----|----|----|
| 1  | 2  | 1  |
| 0  | 0  | 0  |
| -1 | -2 | -1 |

# Image Gradient

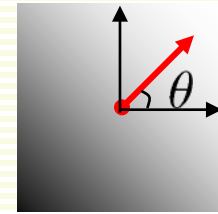
- Combine both partial derivatives into vector  $\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$   
image gradient
- Gradient points in the direction of most rapid increase in intensity



$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ 0 \end{bmatrix}$$



$$\nabla f = \begin{bmatrix} 0 \\ \frac{\partial f}{\partial y} \end{bmatrix}$$



$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

- **Direction** perpendicular to edge:

$$\theta = \tan^{-1} \left( \frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}} \right)$$

gradient orientation

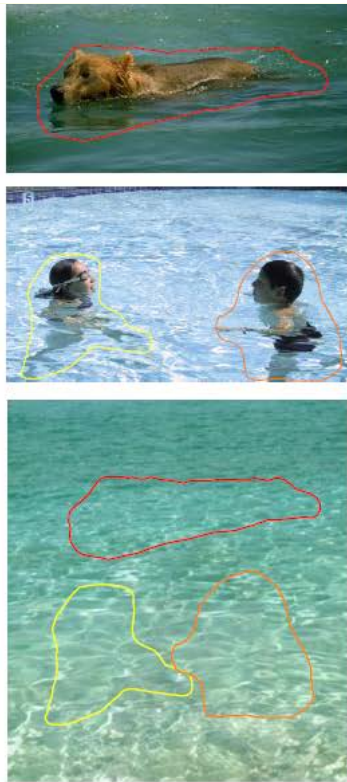
- Edge **strength**

$$\|\nabla f\| = \sqrt{\left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2}$$

gradient magnitude

# Application: Gradient-domain Image Editing

- Goal: solve for pixel values in the target region to match gradients of the source region while keeping background pixels the same



sources/destinations



cloning



seamless cloning

# Simplest Edge Detector

- Compute gradient magnitude at each pixel

$$g(x, y) = \|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

- Threshold gradient magnitude to get binary edge map  $e(x,y)$

$$e(x, y) = \begin{cases} 1 & \text{if } g(x,y) > T \\ 0 & \text{otherwise} \end{cases}$$

# Effects of Noise



original image



$$\frac{\partial f}{\partial y}$$



$$\frac{\partial f}{\partial x}$$

- Too many pixels with large gradient magnitude due to image noise

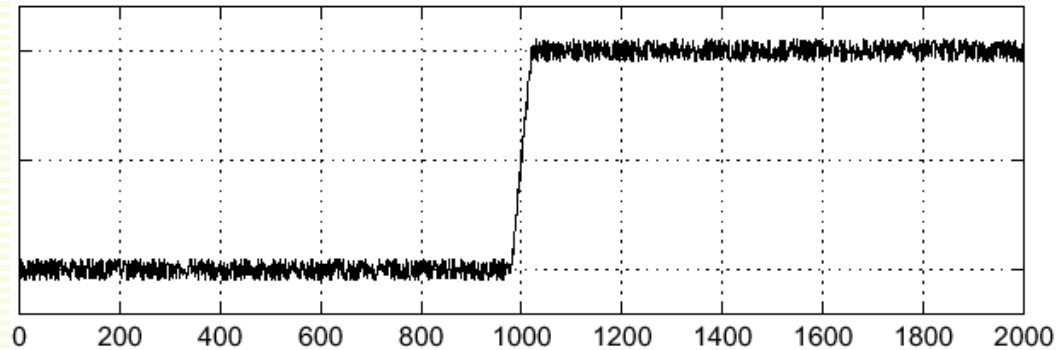


$$\|\nabla f\|$$

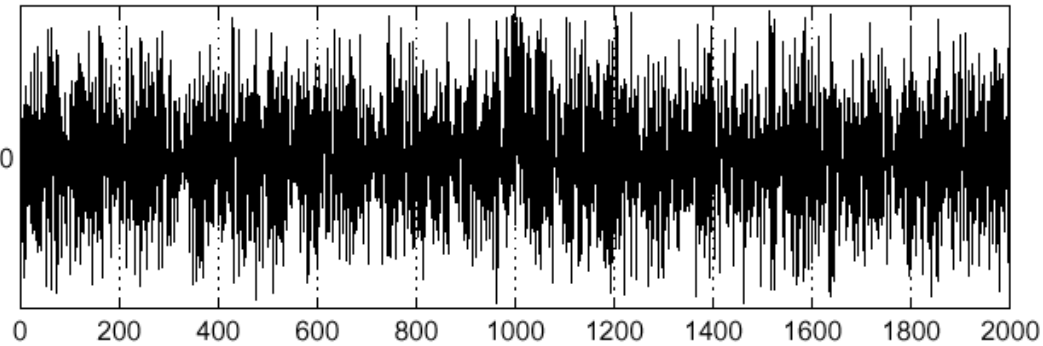
# Effects of noise

- Consider a single row of the image
- Plot intensity as a function of  $x$

$$f(x)$$



$$\frac{\partial}{\partial x} f(x)$$



- Where is the edge?

# Effects of Noise

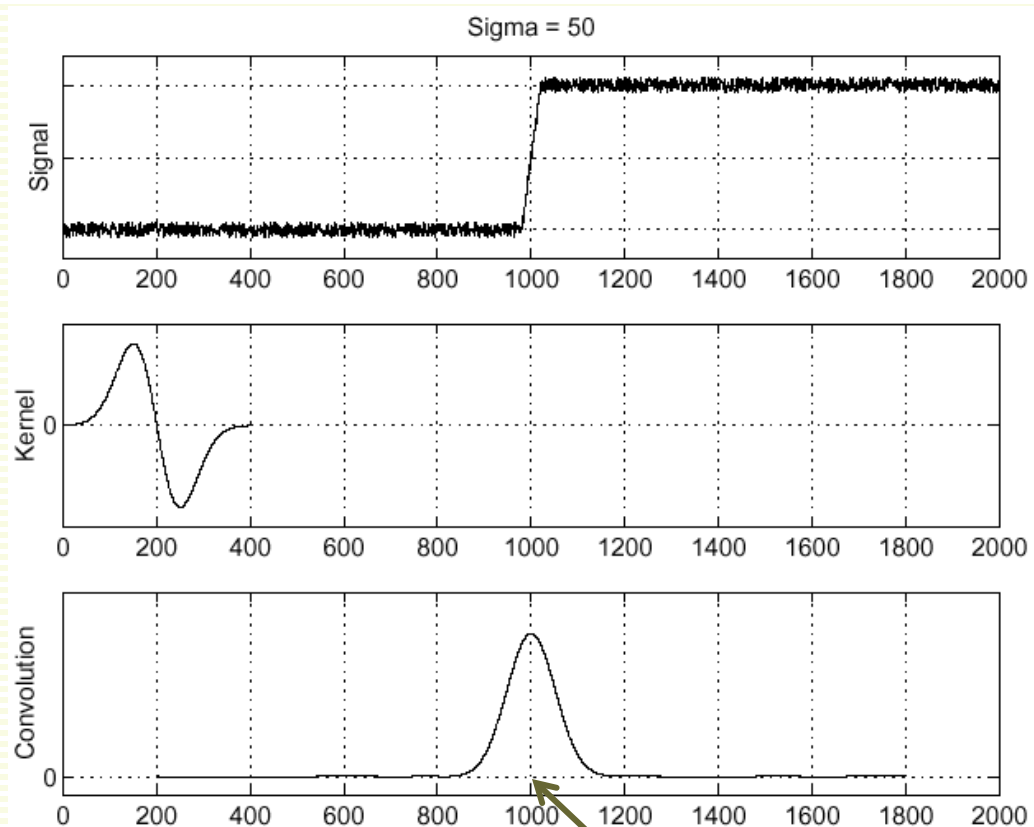
- How do we deal with noise?
- We already know, filter the noise out using Gaussian kernel
- First convolve image with a Gaussian filter
- Then take derivative

# Derivative Theorem of Convolution

$$\frac{\partial}{\partial x}(H * f) = \left(\frac{\partial}{\partial x} H\right) * f$$

- This saves us one step

$f$



$\frac{\partial}{\partial x} H$

$\left(\frac{\partial}{\partial x} H\right) * f$

edge

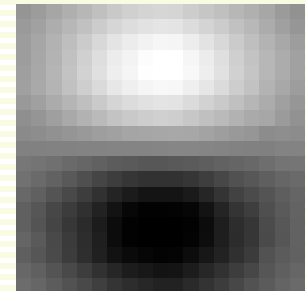
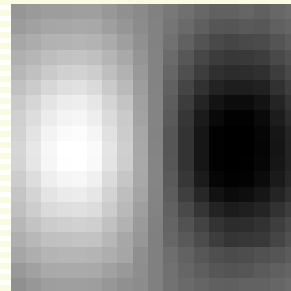
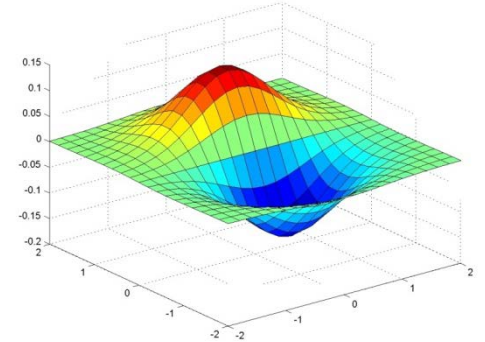
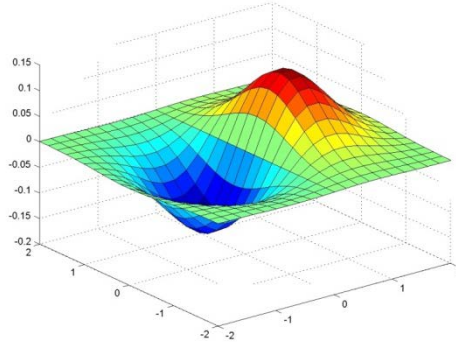
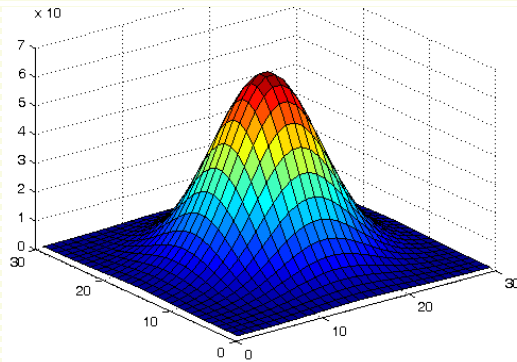


# Derivative of Gaussian

$$G_{\sigma}$$

$$\frac{\partial}{\partial x} G_{\sigma}$$

$$\frac{\partial}{\partial y} G_{\sigma}$$



white is positive values, dark negative, gray zero

- Which finds horizontal, which vertical edges?

# Derivative of Gaussian: Example

- Ignoring constant:

$$G_{\sigma}(x, y) = e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

- Differentiate:

$$\frac{\partial}{\partial x} G_{\sigma}(x, y) = -\frac{x}{\sigma^2} \cdot e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

$$\frac{\partial}{\partial y} G_{\sigma}(x, y) = -\frac{y}{\sigma^2} \cdot e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

- Plug in  $\sigma = 5$ , and take  $5 \times 5$  window

|         |         |        |        |        |
|---------|---------|--------|--------|--------|
| (-2,-2) | (-1,-2) | (0,-2) | (1,-2) | (2,-2) |
| (-2,-1) | (-1,-1) | (0,-1) | (1,-1) | (2,-1) |
| (-2,0)  | (-1,0)  | (0,0)  | (1,0)  | (2,0)  |
| (-2,1)  | (-1,1)  | (0,1)  | (1,1)  | (2,1)  |
| (-2,2)  | (-1,2)  | (0,2)  | (1,2)  | (2,2)  |

coordinates in window

|      |      |   |       |       |
|------|------|---|-------|-------|
| 0.04 | 0.08 | 0 | -0.08 | -0.04 |
| 0.16 | 0.37 | 0 | -0.37 | -0.16 |
| 0.27 | 0.61 | 0 | -0.61 | -0.27 |
| 0.16 | 0.37 | 0 | -0.37 | -0.16 |
| 0.04 | 0.08 | 0 | -0.08 | -0.04 |

$H_x$

|       |       |       |       |       |
|-------|-------|-------|-------|-------|
| -0.04 | -0.16 | -0.27 | -0.16 | -0.04 |
| -0.08 | -0.37 | -0.61 | -0.37 | -0.08 |
| 0     | 0     | 0     | 0     | 0     |
| 0.08  | 0.37  | 0.61  | 0.37  | 0.08  |
| 0.04  | 0.16  | 0.27  | 0.16  | 0.04  |

$H_y$

# Example Continued

$$H_x$$

|      |      |   |       |       |
|------|------|---|-------|-------|
| 0.04 | 0.08 | 0 | -0.08 | -0.04 |
| 0.16 | 0.37 | 0 | -0.37 | -0.16 |
| 0.27 | 0.61 | 0 | -0.61 | -0.27 |
| 0.16 | 0.37 | 0 | -0.37 | -0.16 |
| 0.04 | 0.08 | 0 | -0.08 | -0.04 |

$$H_y$$

|       |       |       |       |       |
|-------|-------|-------|-------|-------|
| -0.04 | -0.16 | -0.27 | -0.16 | -0.04 |
| -0.08 | -0.37 | -0.61 | -0.37 | -0.08 |
| 0     | 0     | 0     | 0     | 0     |
| 0.08  | 0.37  | 0.61  | 0.37  | 0.08  |
| 0.04  | 0.16  | 0.27  | 0.16  | 0.04  |

|     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|
| 121 | 121 | 122 | 123 | 122 | 123 |
| 121 | 121 | 122 | 123 | 122 | 123 |
| 122 | 123 | 124 | 123 | 124 | 123 |
| 120 | 122 | 122 | 123 | 122 | 123 |
| 121 | 121 | 124 | 123 | 124 | 123 |
| 125 | 120 | 124 | 123 | 124 | 123 |

|     |     |     |     |    |    |
|-----|-----|-----|-----|----|----|
| 121 | 121 | 122 | 123 | 20 | 20 |
| 121 | 121 | 122 | 123 | 22 | 22 |
| 122 | 123 | 124 | 123 | 24 | 21 |
| 120 | 122 | 122 | 123 | 22 | 22 |
| 121 | 121 | 124 | 123 | 24 | 23 |
| 125 | 120 | 124 | 123 | 24 | 24 |

apply  $H_x$  to pixel in red: -0.78

apply  $H_y$  to pixel in red: 0.46

apply  $H_x$  to pixel in red: **217**

apply  $H_y$  to pixel in red: 0.69

# Example Continued

$$H_x$$

|      |      |   |       |       |
|------|------|---|-------|-------|
| 0.04 | 0.08 | 0 | -0.08 | -0.04 |
| 0.16 | 0.37 | 0 | -0.37 | -0.16 |
| 0.27 | 0.61 | 0 | -0.61 | -0.27 |
| 0.16 | 0.37 | 0 | -0.37 | -0.16 |
| 0.04 | 0.08 | 0 | -0.08 | -0.04 |

$$H_y$$

|       |       |       |       |       |
|-------|-------|-------|-------|-------|
| -0.04 | -0.16 | -0.27 | -0.16 | -0.04 |
| -0.08 | -0.37 | -0.61 | -0.37 | -0.08 |
| 0     | 0     | 0     | 0     | 0     |
| 0.08  | 0.37  | 0.61  | 0.37  | 0.08  |
| 0.04  | 0.16  | 0.27  | 0.16  | 0.04  |

|     |     |     |     |    |    |
|-----|-----|-----|-----|----|----|
| 121 | 121 | 122 | 123 | 20 | 20 |
| 121 | 121 | 122 | 123 | 22 | 22 |
| 122 | 123 | 124 | 123 | 24 | 21 |
| 120 | 122 | 122 | 123 | 22 | 22 |
| 121 | 121 | 124 | 123 | 24 | 23 |
| 125 | 120 | 124 | 123 | 24 | 24 |

|     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|
| 121 | 121 | 122 | 120 | 121 | 125 |
| 121 | 121 | 123 | 122 | 121 | 120 |
| 122 | 122 | 124 | 122 | 124 | 124 |
| 123 | 123 | 123 | 123 | 123 | 123 |
| 20  | 22  | 24  | 22  | 24  | 24  |
| 20  | 22  | 21  | 22  | 23  | 24  |

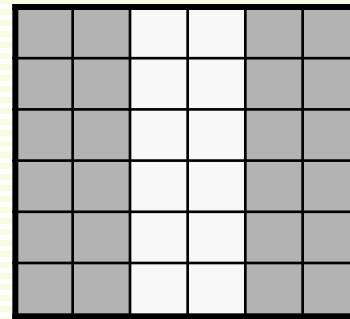
apply  $H_x$  to pixel in red: **217**  
 apply  $H_y$  to pixel in red: 0.69

apply  $H_x$  to pixel in red: -0.69  
 apply  $H_y$  to pixel in red: **-217**

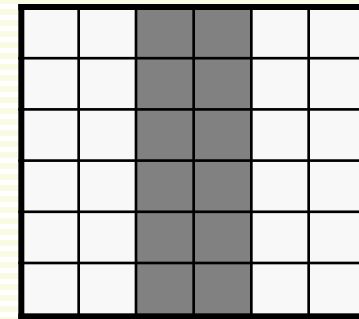
**Mask looks like the pattern it is trying to detect!**

# What does this Mask Detect?

|   |   |    |    |   |   |
|---|---|----|----|---|---|
| 2 | 2 | -4 | -4 | 2 | 2 |
| 2 | 2 | -4 | -4 | 2 | 2 |
| 2 | 2 | -4 | -4 | 2 | 2 |
| 2 | 2 | -4 | -4 | 2 | 2 |
| 2 | 2 | -4 | -4 | 2 | 2 |



strong negative response

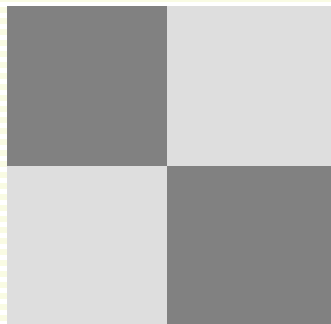


strong positive response

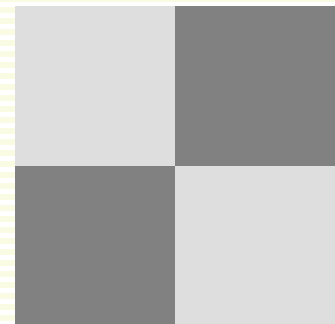
# What Does this Mask Detect?

|    |    |    |    |
|----|----|----|----|
| 2  | 2  | -2 | -2 |
| 2  | 2  | -2 | -2 |
| -2 | -2 | 2  | 2  |
| -2 | -2 | 2  | 2  |

**strong negative response**



**strong positive response**



# Canny Edge Detector



input image

# Canny Edge Detector



gradient magnitude



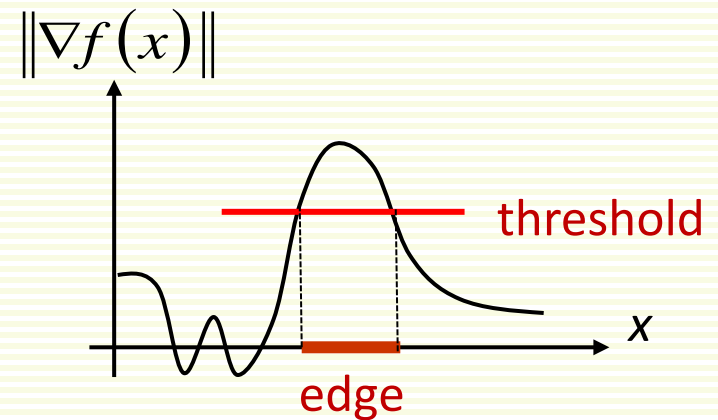
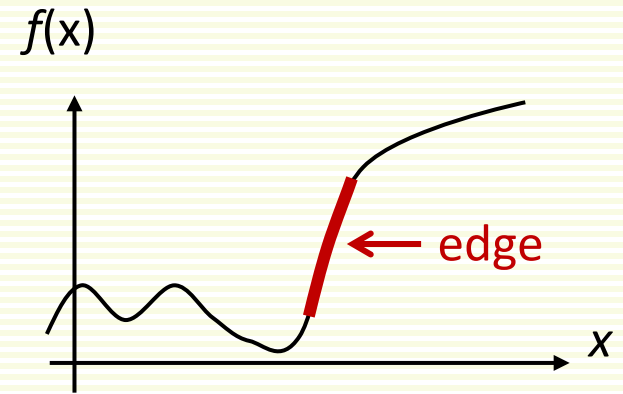
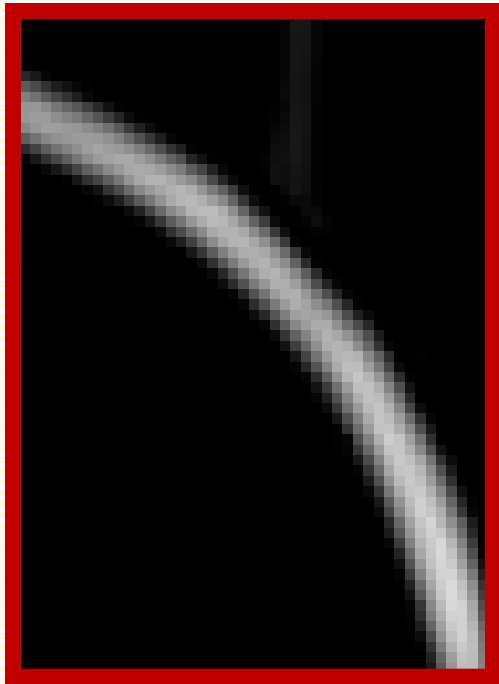
# Canny Edge Detector



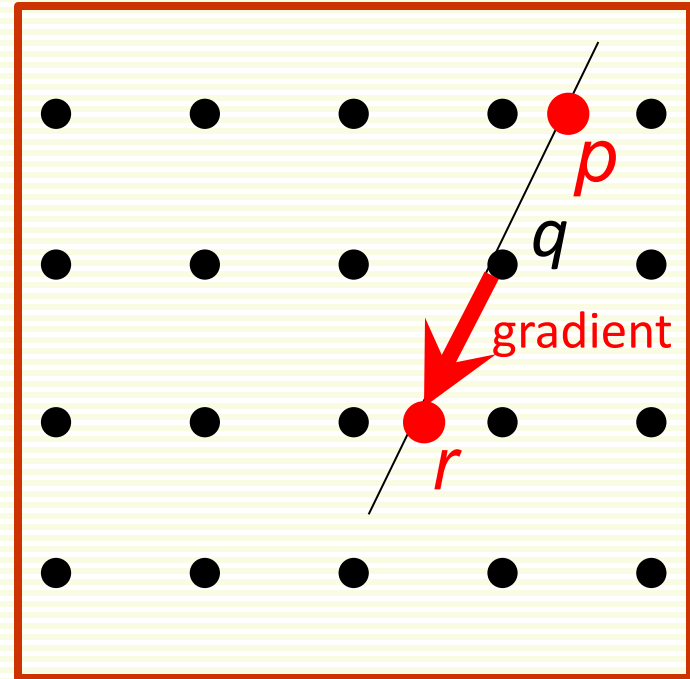
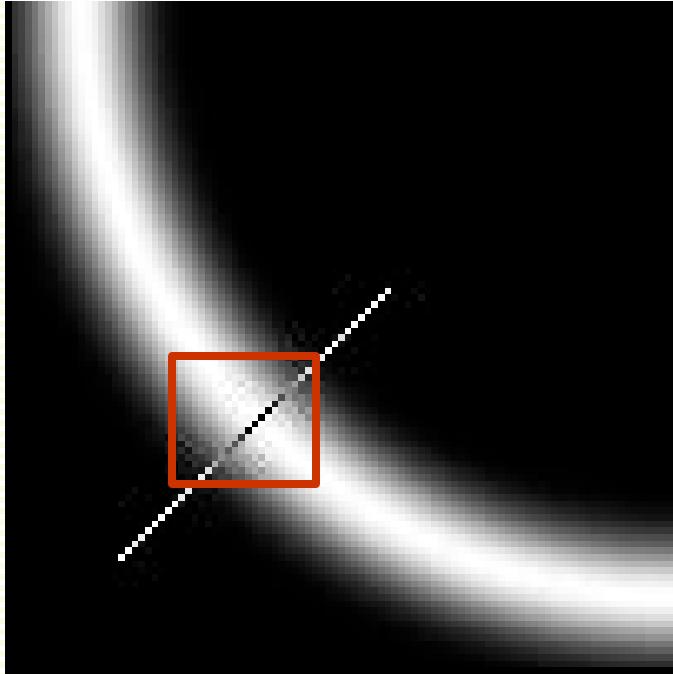
thresholding

# Canny Edge detector

- Why we get thick regions after thresholding?



# Edge Thinning: non-maximum suppression



- Check if pixel  $q$  is local maximum along gradient direction
  - take two neighbors in  $p$  and  $r$  in the gradient direction
    - requires checking interpolated pixels  $p$  and  $r$
  - turn off edge at pixel  $q$  if  $g(q) < g(p)$  or  $g(q) < g(r)$

# The Canny Edge Detector

- Another problem: some weak edge pixels do not survive thresholding



after thinning

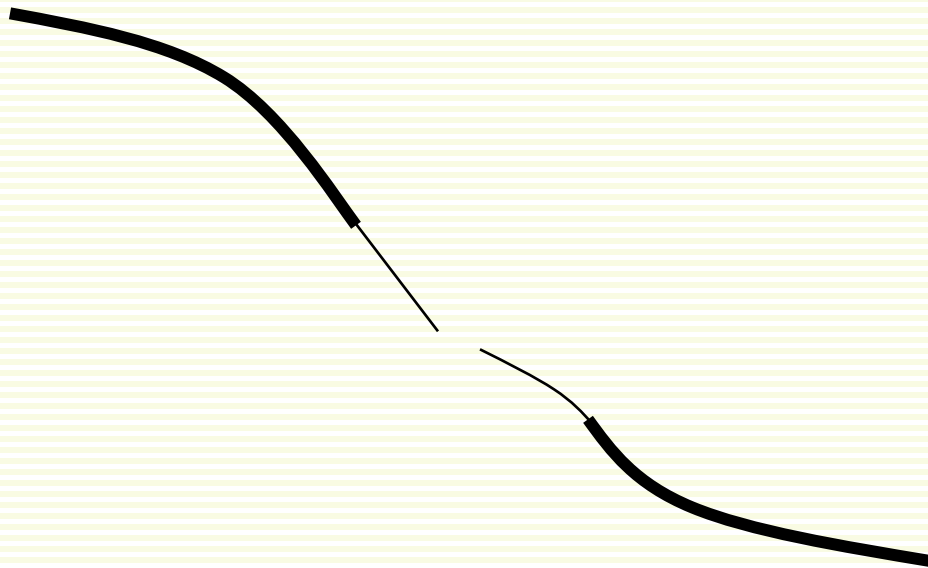
# The Canny Edge Detector

- Try a smaller threshold?
  - too many weak edges



# Hysteresis Thresholding

- Specify a **high** and **low** thresholds
- Use **high** threshold to start edge curves
  - Continue edge in the gradient direction
  - Use **low** threshold for continuation



# The Canny Edge Detector



low threshold



high threshold



hysteresis with low and high thresholds

# Effect of Kernel Size and Spread



original image



Canny with  $\sigma = 1$



Canny with  $\sigma = 2$

- Smaller  $\sigma$ /mask size detects fine scale edges
- Larger  $\sigma$ /mask detects large scale edges

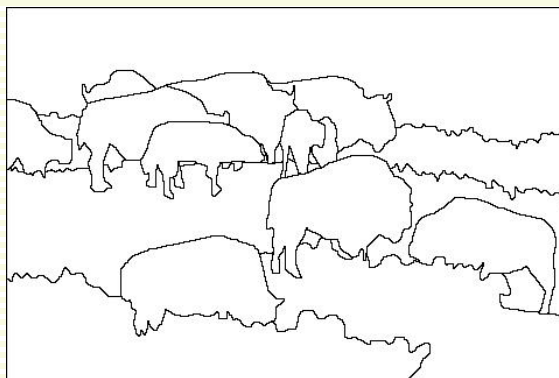


# Still Far From Human Vision

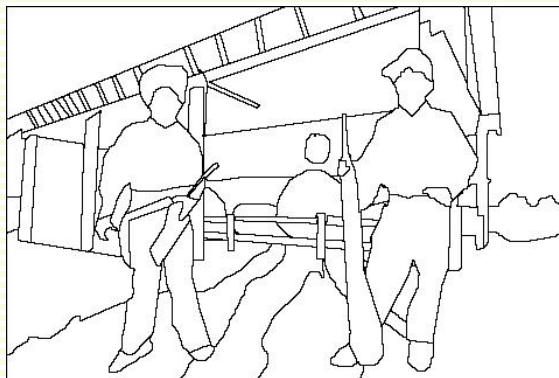
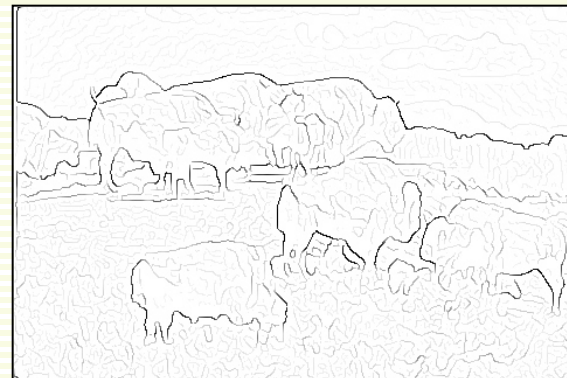
image



human segmentation



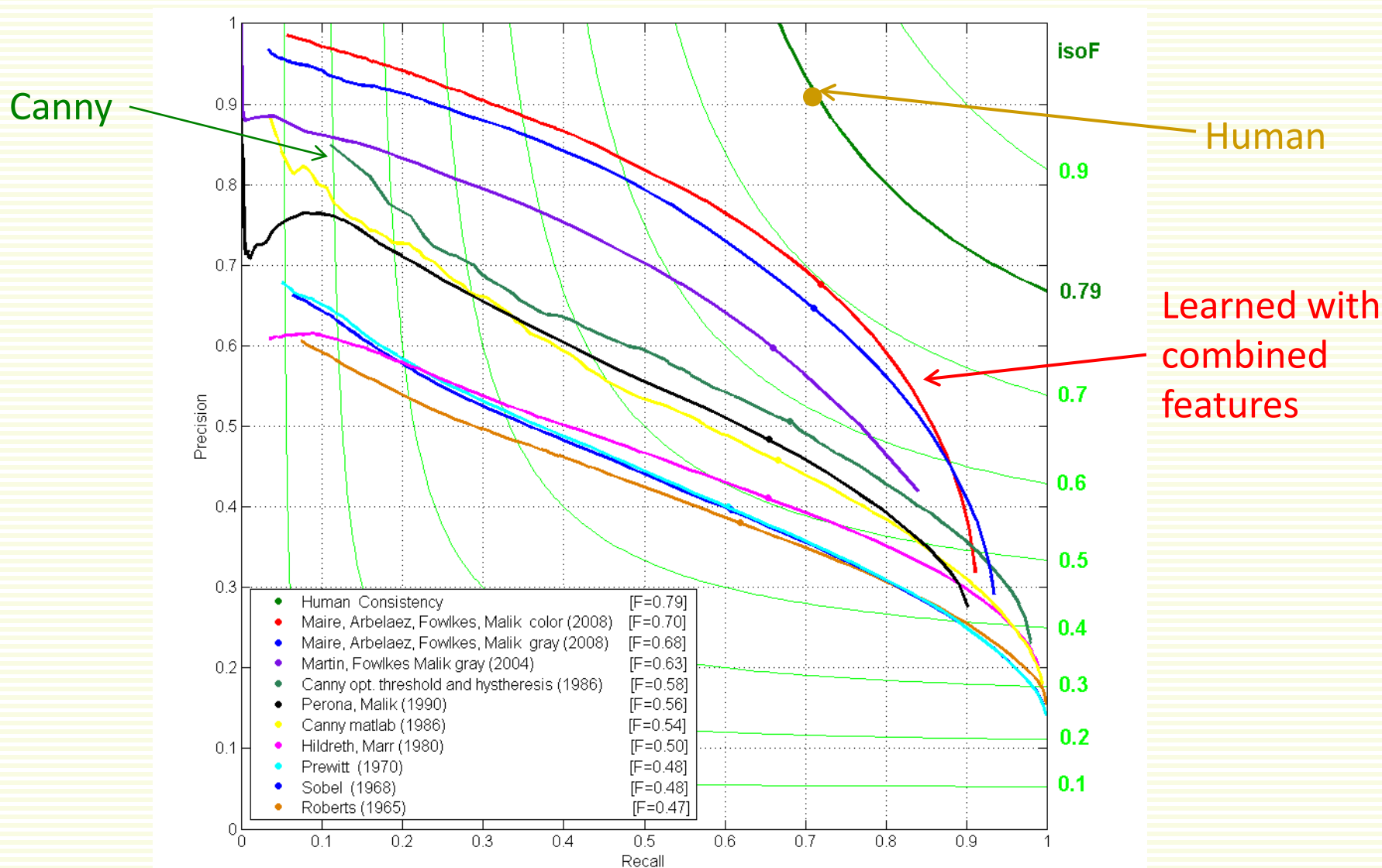
gradient magnitude



- Berkeley segmentation database:

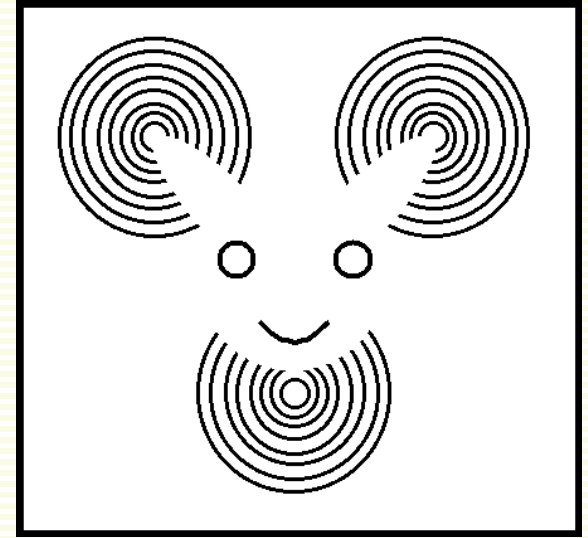
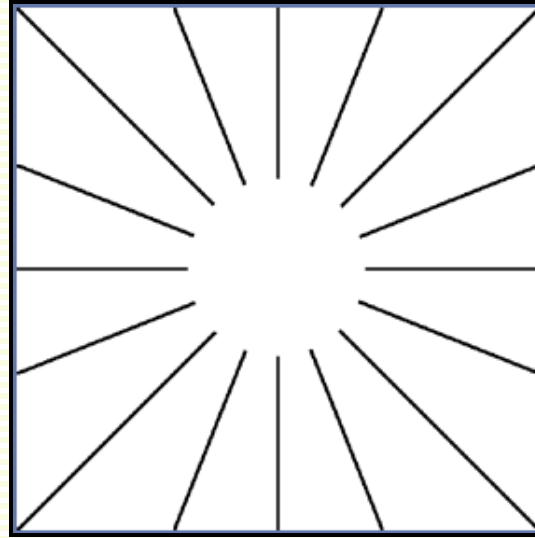
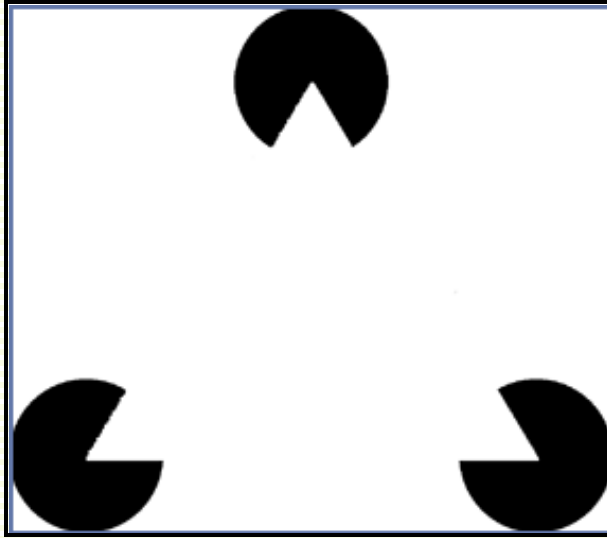
<http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/>

# State-of-the-Art in Contour Detection



Source: Jitendra Malik: <http://www.cs.berkeley.edu/~malik/malik-talks-ptns.html>

# Illusory Contours



- impossible detect the “illusory” contours using only local image gradients

# Application of Gradients: Intelligent Resizing

- In traditional image resizing, all dimensions change by the same ratio



input

75% smaller



75% larger

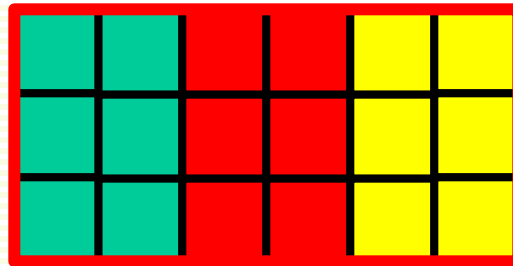
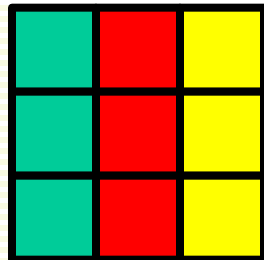


# Application of Gradients: Intelligent Resizing

- What if need to fit to a mobile device? Or resize to a web page?
  - often need different ratio in different dimensions
- Change width, height unchanged



- Object proportions are not preserved:





# Application of Gradients: Intelligent Resizing

- Intelligent resizing “seam carving”
  - Shai & Avidan, SIGGRAPH 2007



seam carving



naïve resizing

# Seam Carving: Main Idea

not  
interesting

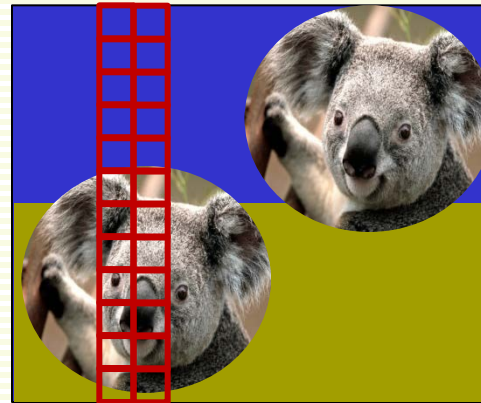


interesting

- Preserve the most “interesting” content
  - large gradient magnitude = interesting
  - small gradient magnitude = uninteresting
- **Prefer changes around low gradient magnitude pixels**

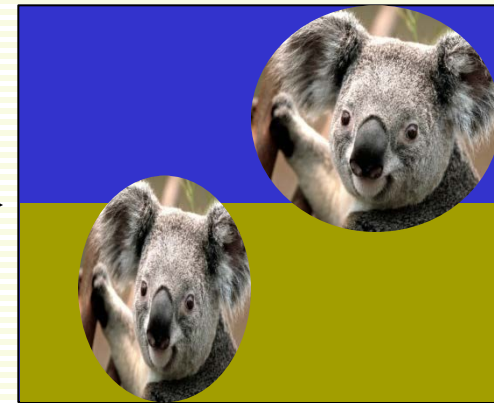
# Reducing Width by One Pixel

- Traditional resizing
  - works on regular seams
  - through random pixels
- Seam carving
  - find irregular seams
  - through low gradient (uninteresting) pixels

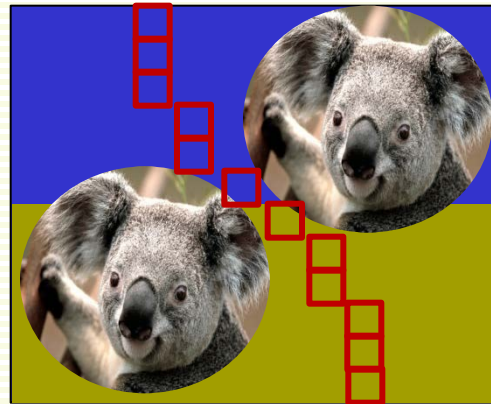


$w$

add two  
seams

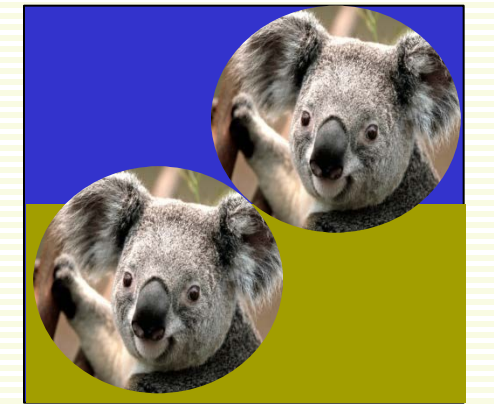


$w - 1$



$w$

remove  
seam



$w - 1$



# Seam Carving: Main Idea

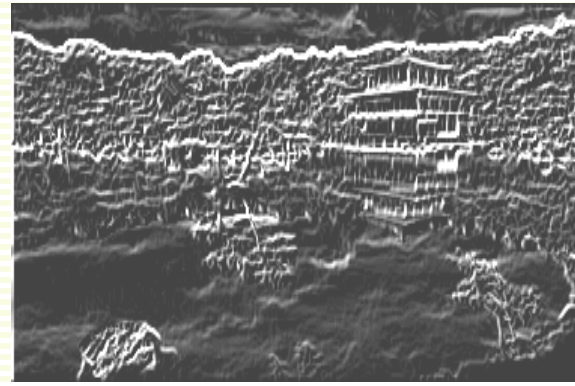


- **Prefer changes around low gradient magnitude pixels**
  - to reduce size in one dimension, **remove** irregular seams
  - to enlarge size in one dimension, **insert** irregular seams
- Many “uninteresting” seams
  - find the best (most boring) seam
  - with dynamic programming

# Seam Carving: Main Idea

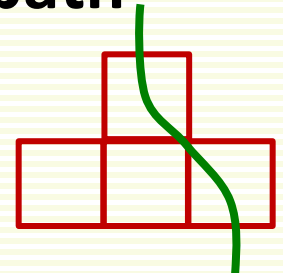
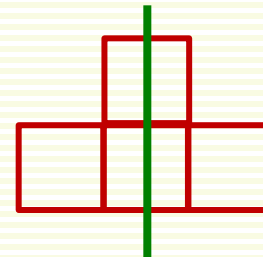
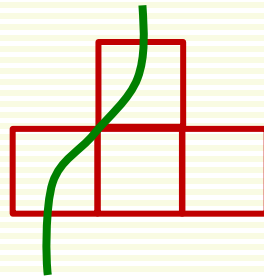


seams



$$Energy(f) = \|\nabla f\|$$

- Measure **energy** as gradient magnitude
- Removing low energy seam makes change less visible
- Choose seam based on **minimum total energy path**
- Path is **8-connected**

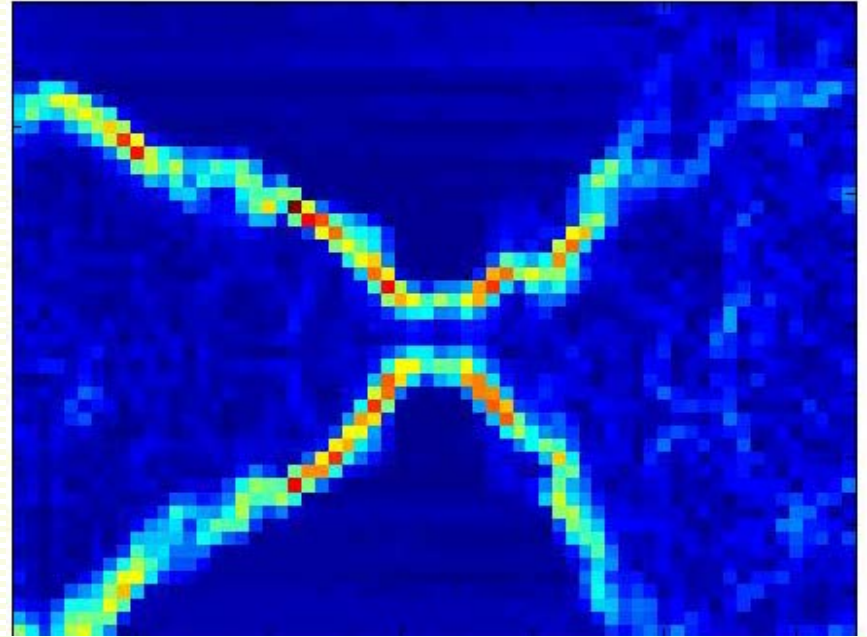


# Example

Original Image



Gradient Energy



blue = low energy

red = high energy

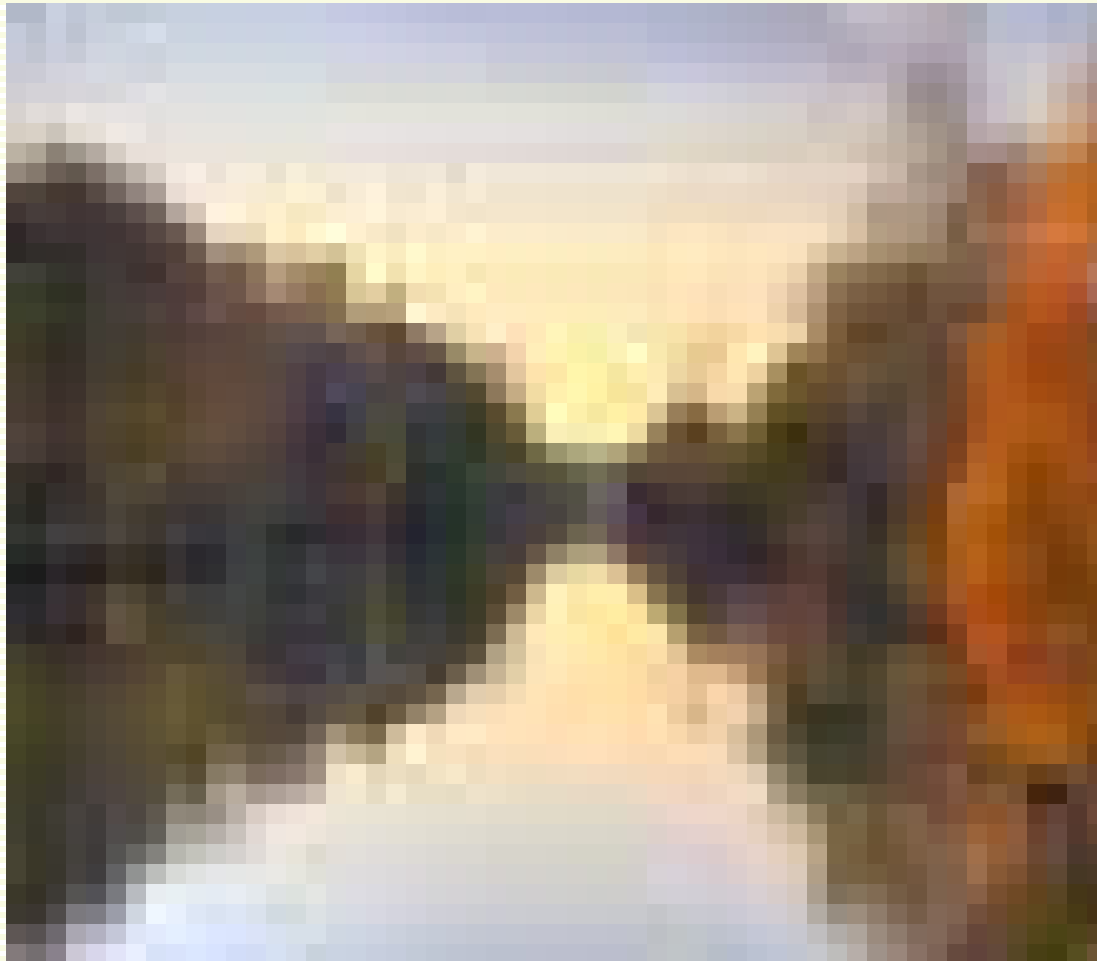
# Example



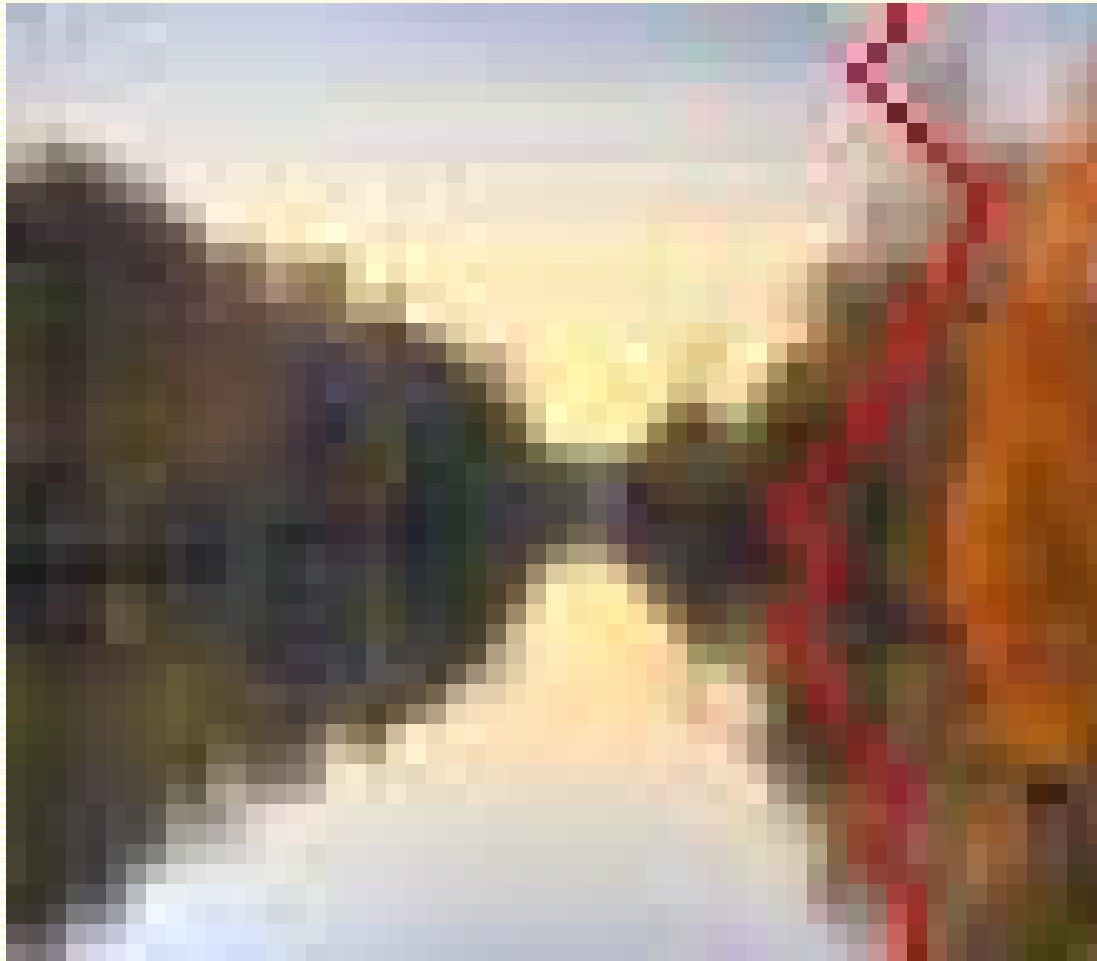
# Example



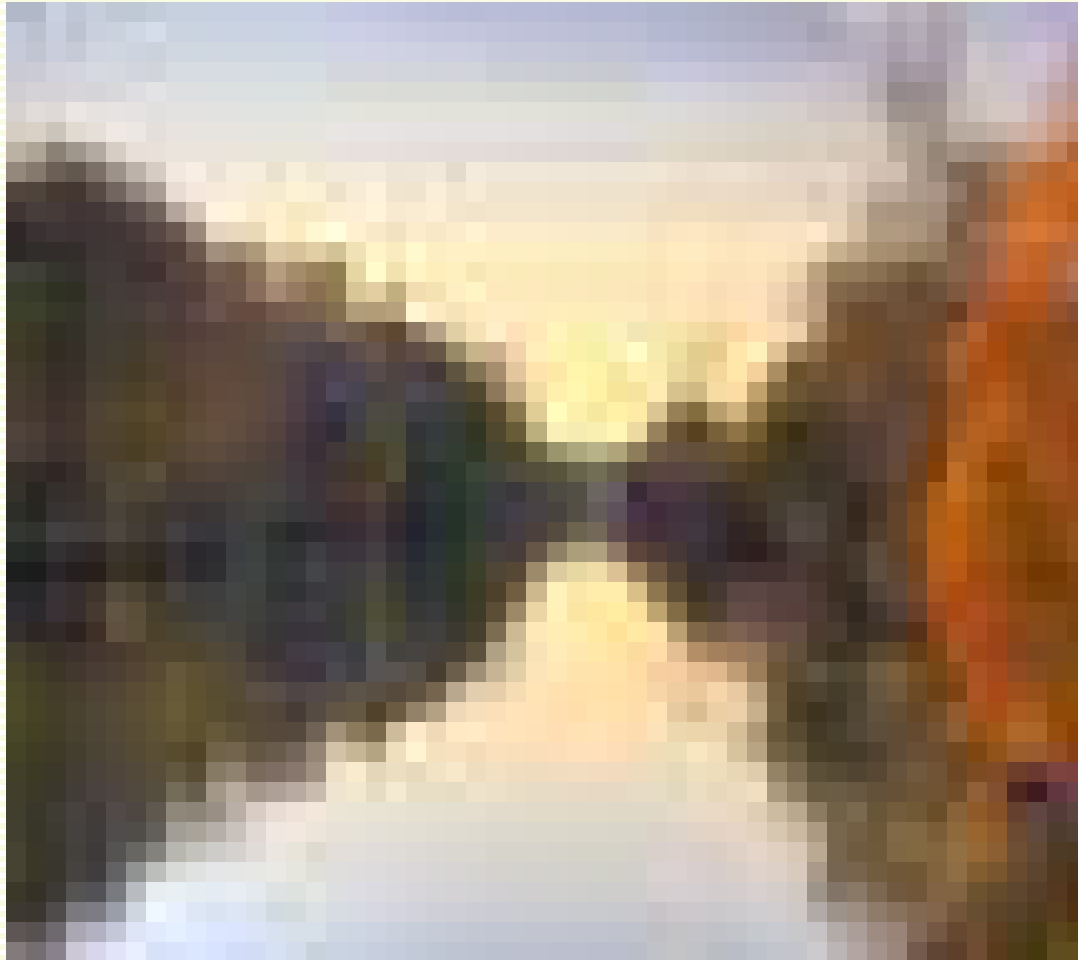
# Example



# Example

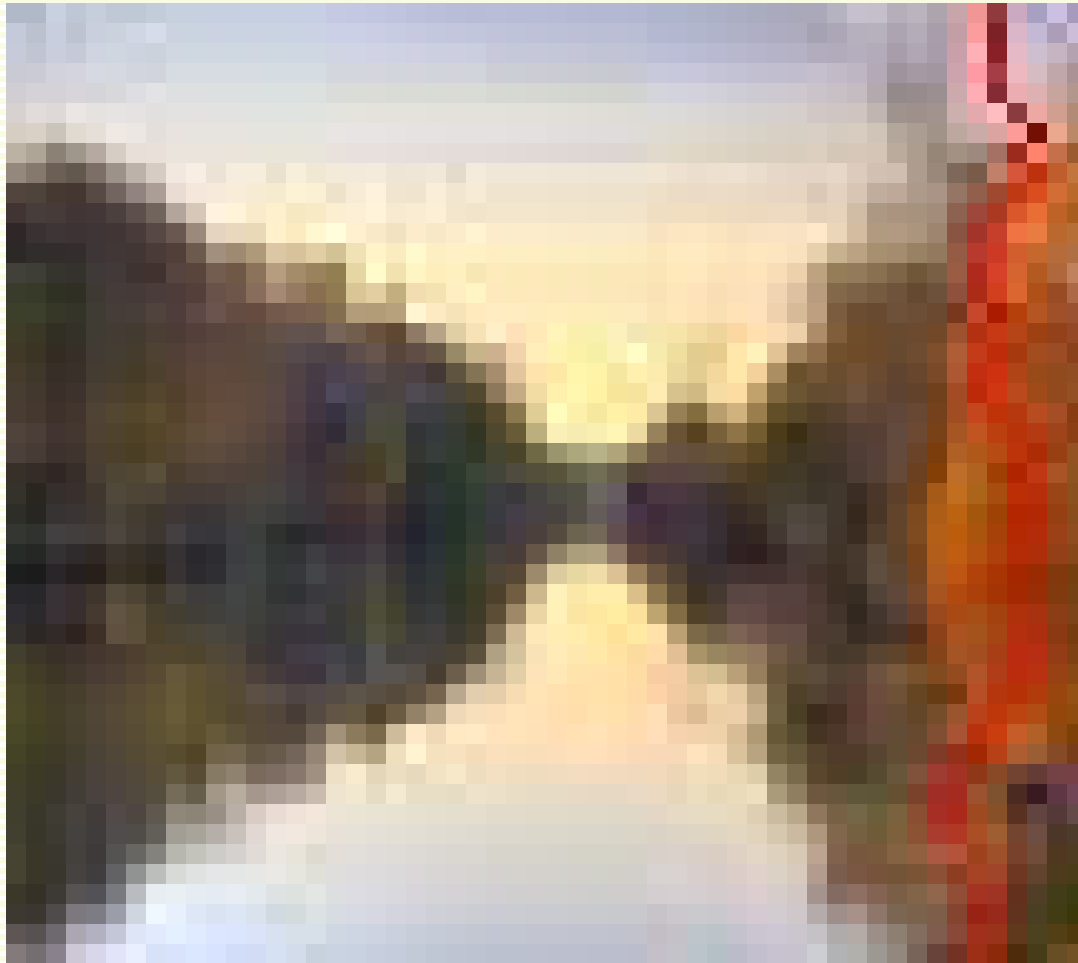


# Example





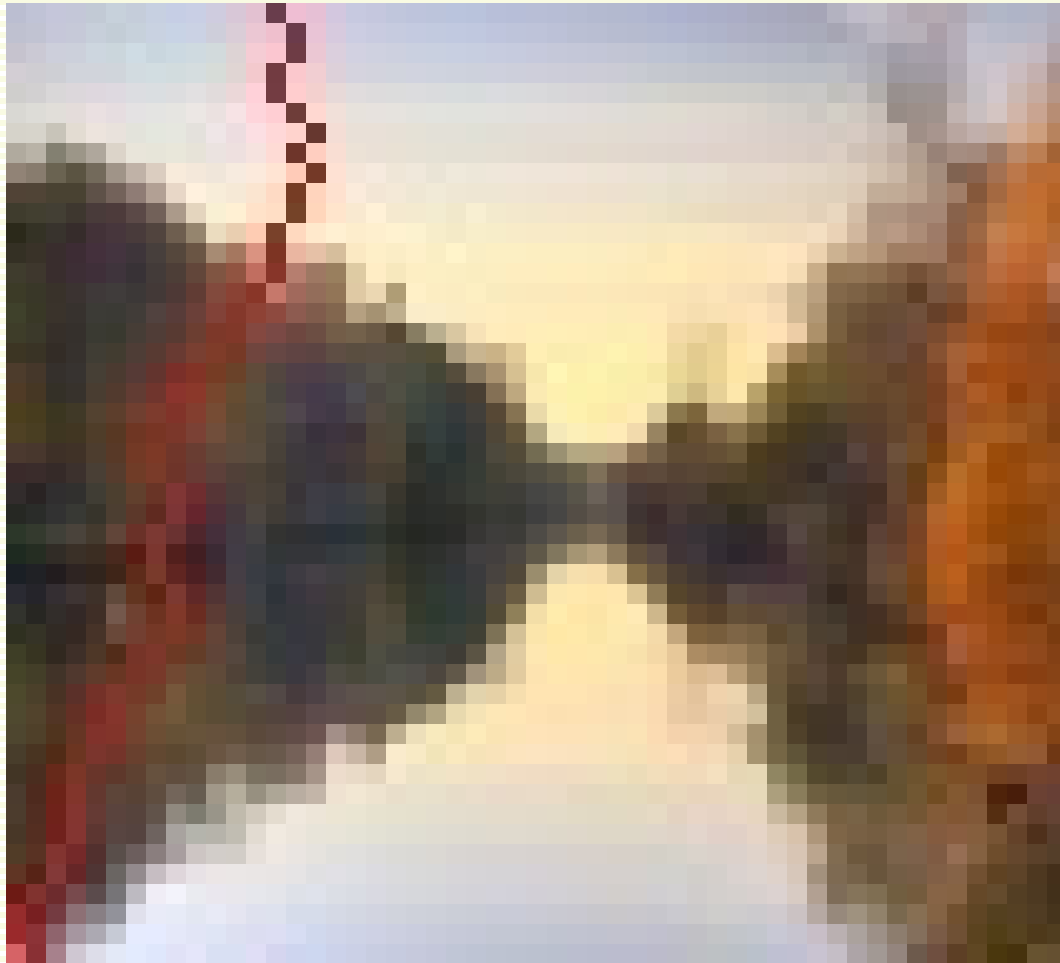
# Example



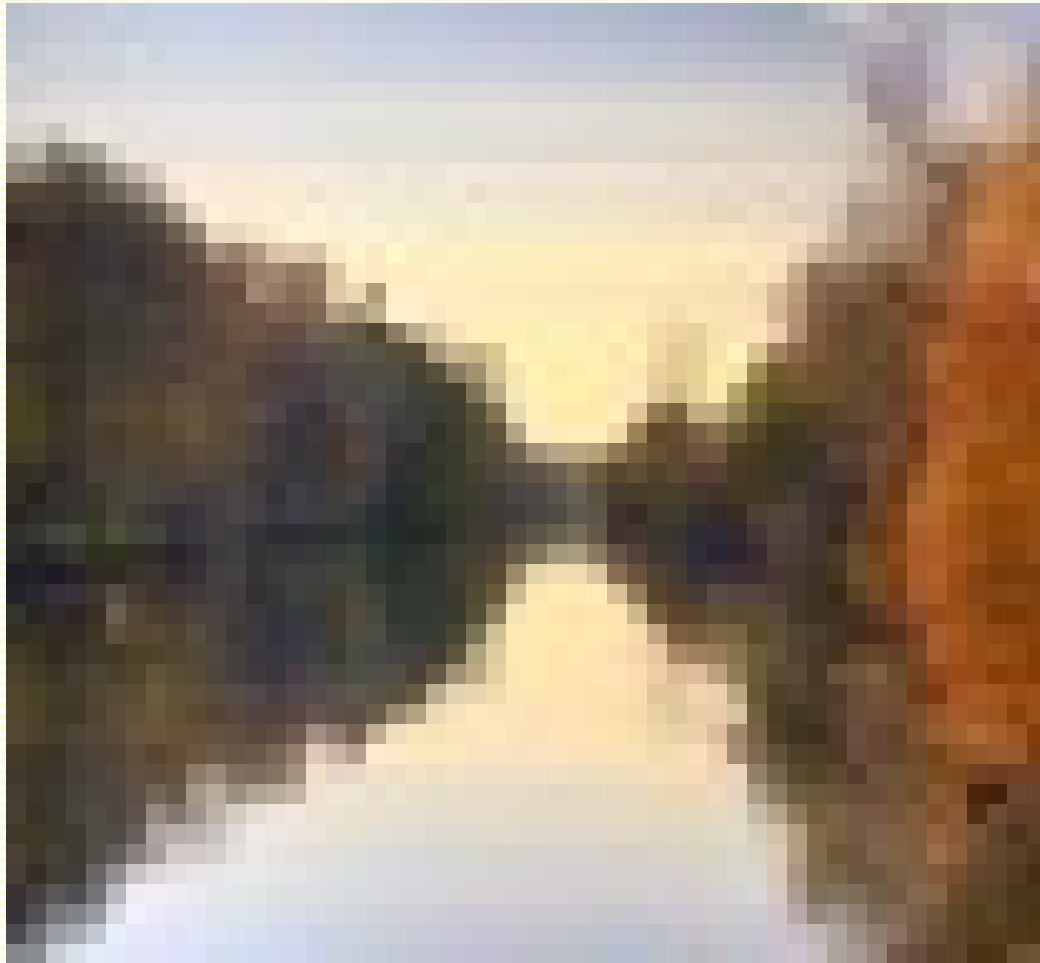
# Example



# Example



# Example



# Example



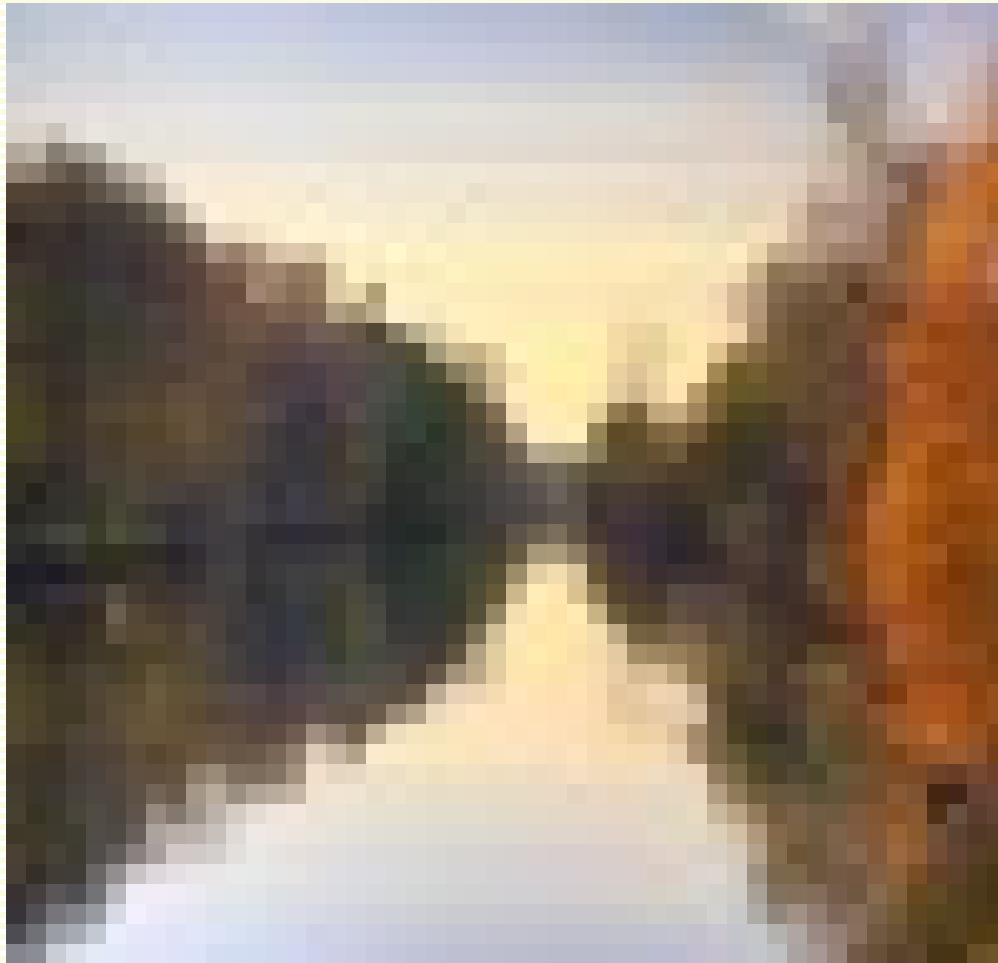
# Example



# Example



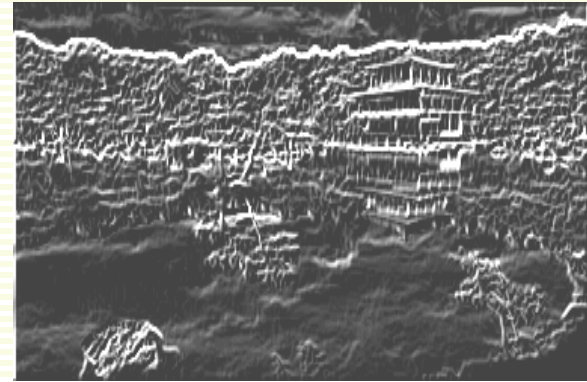
# Example





# Seam Carving: Algorithm

|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 0 | 6 | 4 | 2 |
| 1 | 3 | 6 | 6 | 4 |
| 4 | 3 | 4 | 6 | 2 |
| 4 | 6 | 0 | 0 | 4 |
| 1 | 6 | 5 | 6 | 6 |



$$Energy(f) = \|\nabla f\|$$

- Vertical seam  $s$  consists of  $n$  positions that form a path
  - $s = (s_1, s_2, \dots, s_n)$ : one pixel in every row
- Seam cost  $Energy(s) = Energy(s_1) + Energy(s_2) + \dots + Energy(s_n)$ 
  - **red seam** has cost  $0 + 6 + 3 + 6 + 1 = 16$
  - **green seam** has cost  $4 + 4 + 2 + 4 + 6 = 20$
- **Optimal** seam minimizes this cost

$$s^* = \operatorname{argmin}_s Energy(s)$$

# How to Find the Minimum Cost Seam?

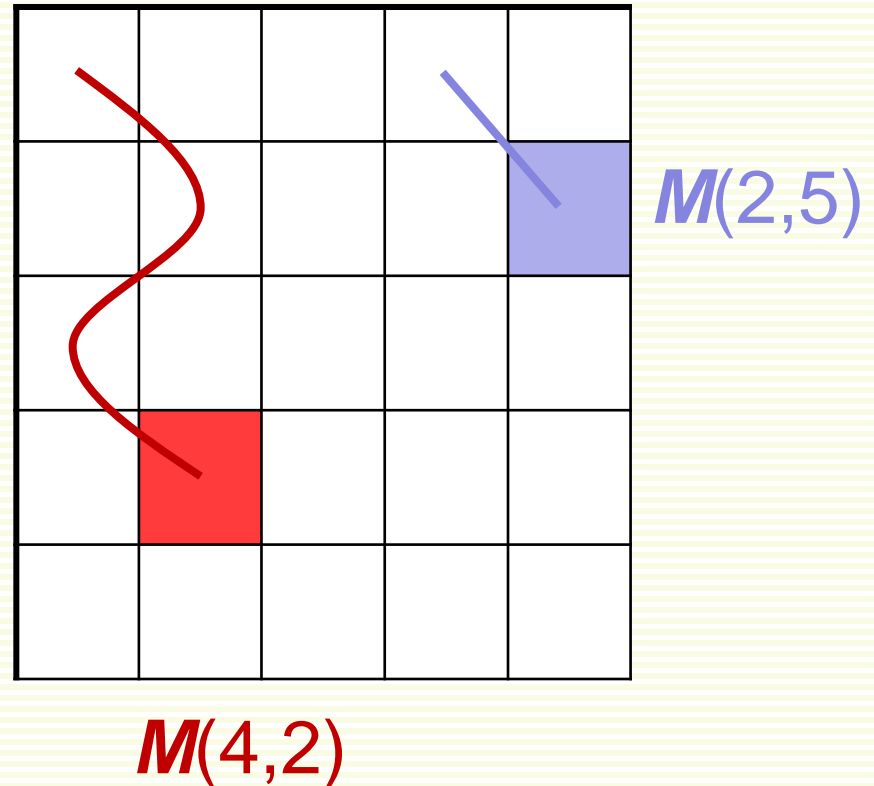
- First, consider a **greedy** approach on a small image
  - smaller number corresponds to smaller gradient

|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 0 | 6 | 4 | 2 |
| 1 | 3 | 6 | 6 | 4 |
| 4 | 3 | 4 | 6 | 2 |
| 4 | 6 | 0 | 0 | 4 |
| 0 | 6 | 5 | 6 | 6 |

- Greedy seam cost:  $0 + 1 + 3 + 0 + 5 = 9$
- Is this the best vertical seam?

# Optimal Seam Carving Algorithm

- Dynamic programming can find the best seam
- Work from the top row to the bottom row
- $M(r,c)$  is best seam cost that starts anywhere in row 0 and ends at position  $(r,c)$
- After  $M$  is computed, the best cost path is the smallest value of  $M$  in the last row
- Also keep track of the parent on the path,  $P(r,c)$



# Seam Carving Algorithm: Initialization Step

Compute Energy image  $E$

for  $c = 1$  to  $maxCol$

$$M(1, c) = E(1, c)$$

$$P(1, c) = \text{null}$$

|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 0 | 6 | 4 | 2 |
| 1 | 3 | 6 | 6 | 4 |
| 4 | 3 | 4 | 6 | 2 |
| 4 | 6 | 0 | 0 | 4 |
| 0 | 6 | 5 | 6 | 6 |

$E$

|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 0 | 6 | 4 | 2 |
|   |   |   |   |   |
|   |   |   |   |   |
|   |   |   |   |   |
|   |   |   |   |   |

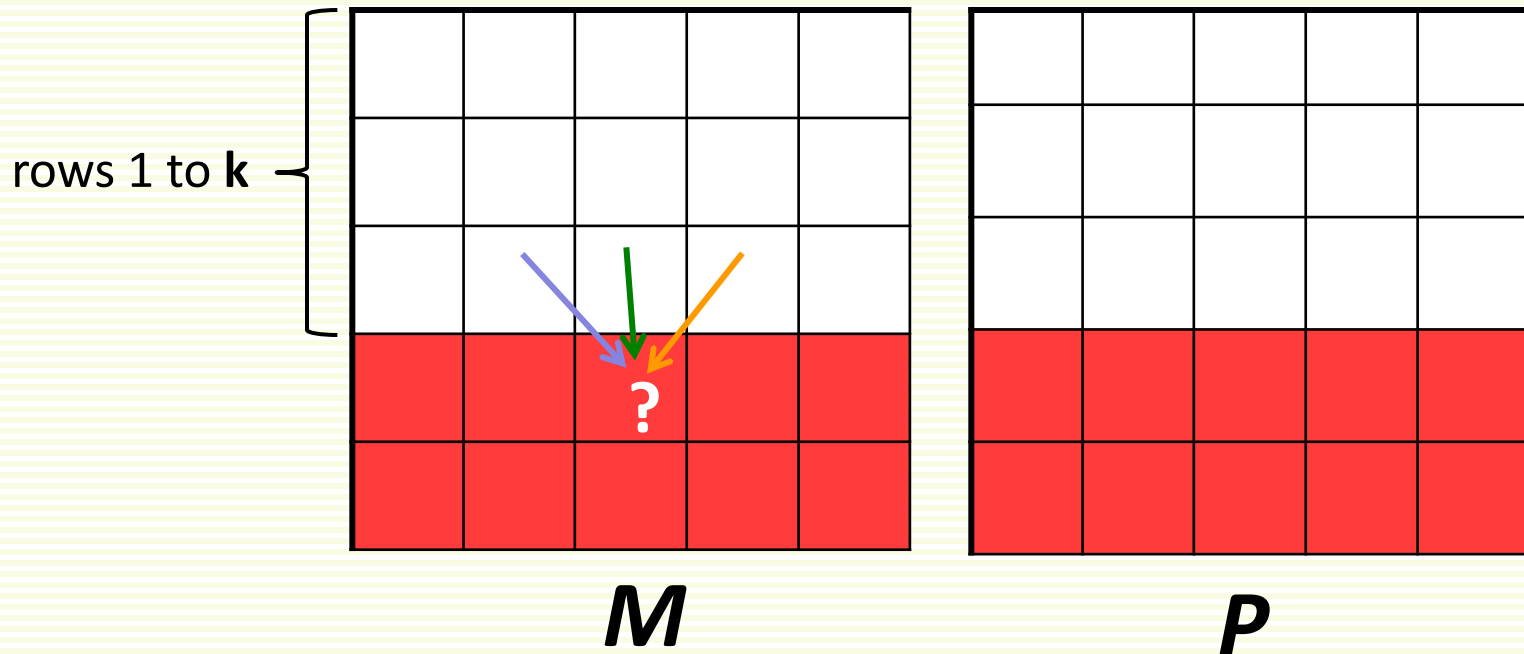
$M$

|      |      |      |      |      |
|------|------|------|------|------|
| null | null | null | null | null |
|      |      |      |      |      |
|      |      |      |      |      |
|      |      |      |      |      |
|      |      |      |      |      |

$P$

# Seam Carving Algorithm: Iteration Step

- Computed  $M, P$  for rows 0 to  $k$
- How to compute  $M, P$  for row  $k+1$ ?



- $M(r+1, c) = E(r+1, c) + \text{smallest in } \{M(r, c-1), M(r, c), M(r, c+1)\}$
- $P(r+1, c)$  stores corresponding column
  - either  $c-1$ , or  $c$ , or  $c+1$

# Optimal Seam Carving Algorithm: Iterations

```
for r = 1 to maxRow
  for c = 1 to maxCol
    option1 = M(r-1,c-1)
    option2 = M(r-1,c)
    option3 = M(r-1,c+1)
    if option1 ≤ option2 and option1 ≤ option3
      M(r,c) = E(r,c) + M(r-1,c-1)
      P(r,c) = c-1
    elseif option2 ≤ option1 and option2 ≤ option3
      M(r,c) = E(r,c) + M(r-1,c)
      P(r,c) = c
    else
      M(r,c) = E(r,c) + M(r-1,c+1)
      P(r,c) = c+1
```

!!!Note: have to implement  
matrix out of bounds check!!!

# Example: Initialization

|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 0 | 6 | 4 | 2 |
| 1 | 3 | 6 | 6 | 4 |
| 4 | 3 | 4 | 6 | 2 |
| 4 | 6 | 0 | 0 | 4 |
| 0 | 6 | 5 | 6 | 6 |

*E*

|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 0 | 6 | 4 | 2 |
|   |   |   |   |   |
|   |   |   |   |   |
|   |   |   |   |   |
|   |   |   |   |   |

*M*

|      |      |      |      |      |
|------|------|------|------|------|
| null | null | null | null | null |
|      |      |      |      |      |
|      |      |      |      |      |
|      |      |      |      |      |
|      |      |      |      |      |

*P*

# Example: Iteration 1

|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 0 | 6 | 4 | 2 |
| 1 | 3 | 6 | 6 | 4 |
| 4 | 3 | 4 | 6 | 2 |
| 4 | 6 | 0 | 0 | 4 |
| 0 | 6 | 5 | 6 | 6 |

*E*

|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 0 | 6 | 4 | 2 |
| 1 | 3 | 6 | 8 | 6 |
|   |   |   |   |   |
|   |   |   |   |   |
|   |   |   |   |   |

*M*

|      |      |      |      |      |
|------|------|------|------|------|
| null | null | null | null | null |
| 2    | 2    | 2    | 5    | 5    |
|      |      |      |      |      |
|      |      |      |      |      |
|      |      |      |      |      |

*P*



# Example: Iteration 2

|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 0 | 6 | 4 | 2 |
| 1 | 3 | 6 | 6 | 4 |
| 4 | 3 | 4 | 6 | 2 |
| 4 | 6 | 0 | 0 | 4 |
| 0 | 6 | 5 | 6 | 6 |

*E*

|   |   |   |    |   |
|---|---|---|----|---|
| 3 | 0 | 6 | 4  | 2 |
| 1 | 3 | 6 | 8  | 6 |
| 5 | 4 | 7 | 12 | 8 |
|   |   |   |    |   |
|   |   |   |    |   |

*M*

|      |      |      |      |      |
|------|------|------|------|------|
| null | null | null | null | null |
| 2    | 2    | 2    | 5    | 5    |
| 1    | 1    | 2    | 3    | 5    |
|      |      |      |      |      |
|      |      |      |      |      |

*P*

# Example: Iteration 3

|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 0 | 6 | 4 | 2 |
| 1 | 3 | 6 | 6 | 4 |
| 4 | 3 | 4 | 6 | 2 |
| 4 | 6 | 0 | 0 | 4 |
| 0 | 6 | 5 | 6 | 6 |

*E*

|   |    |   |    |    |
|---|----|---|----|----|
| 3 | 0  | 6 | 4  | 2  |
| 1 | 3  | 6 | 8  | 6  |
| 5 | 4  | 7 | 12 | 8  |
| 8 | 10 | 4 | 7  | 12 |
|   |    |   |    |    |

*M*

|      |      |      |      |      |
|------|------|------|------|------|
| null | null | null | null | null |
| 2    | 2    | 2    | 5    | 5    |
| 1    | 1    | 2    | 3    | 5    |
| 2    | 2    | 2    | 3    | 5    |
|      |      |      |      |      |

*P*

# Example: Iteration 4

- Best seam has cost 8, better than what greedy algorithm finds
  - end of the best seam is in column 0

|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 0 | 6 | 4 | 2 |
| 1 | 3 | 6 | 6 | 4 |
| 4 | 3 | 4 | 6 | 2 |
| 4 | 6 | 0 | 0 | 4 |
| 0 | 6 | 5 | 6 | 6 |

*E*

|   |    |   |    |    |
|---|----|---|----|----|
| 3 | 0  | 6 | 4  | 2  |
| 1 | 3  | 6 | 8  | 6  |
| 5 | 4  | 7 | 12 | 8  |
| 8 | 10 | 4 | 7  | 12 |
| 8 | 10 | 9 | 10 | 13 |

*M*

|      |      |      |      |      |
|------|------|------|------|------|
| null | null | null | null | null |
| 2    | 2    | 2    | 5    | 5    |
| 1    | 1    | 2    | 3    | 5    |
| 2    | 2    | 2    | 3    | 5    |
| 1    | 3    | 3    | 3    | 4    |

*P*

# Example: Finishing Up

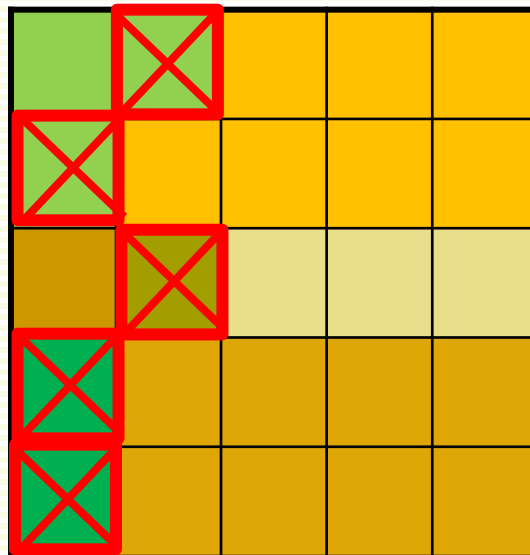
*M*

|   |    |   |    |    |
|---|----|---|----|----|
| 3 | 0  | 6 | 4  | 2  |
| 1 | 3  | 6 | 8  | 6  |
| 5 | 4  | 7 | 12 | 8  |
| 8 | 10 | 4 | 7  | 12 |
| 8 | 10 | 9 | 10 | 13 |

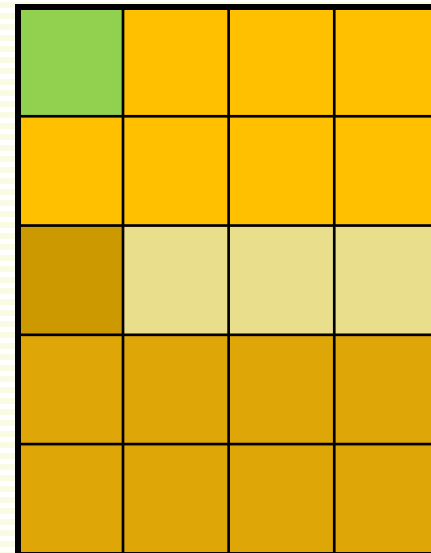
*P*

|      |      |      |      |      |
|------|------|------|------|------|
| null | null | null | null | null |
| 2    | 2    | 2    | 5    | 5    |
| 1    | 1    | 2    | 3    | 5    |
| 2    | 2    | 2    | 3    | 5    |
| 1    | 3    | 3    | 3    | 4    |

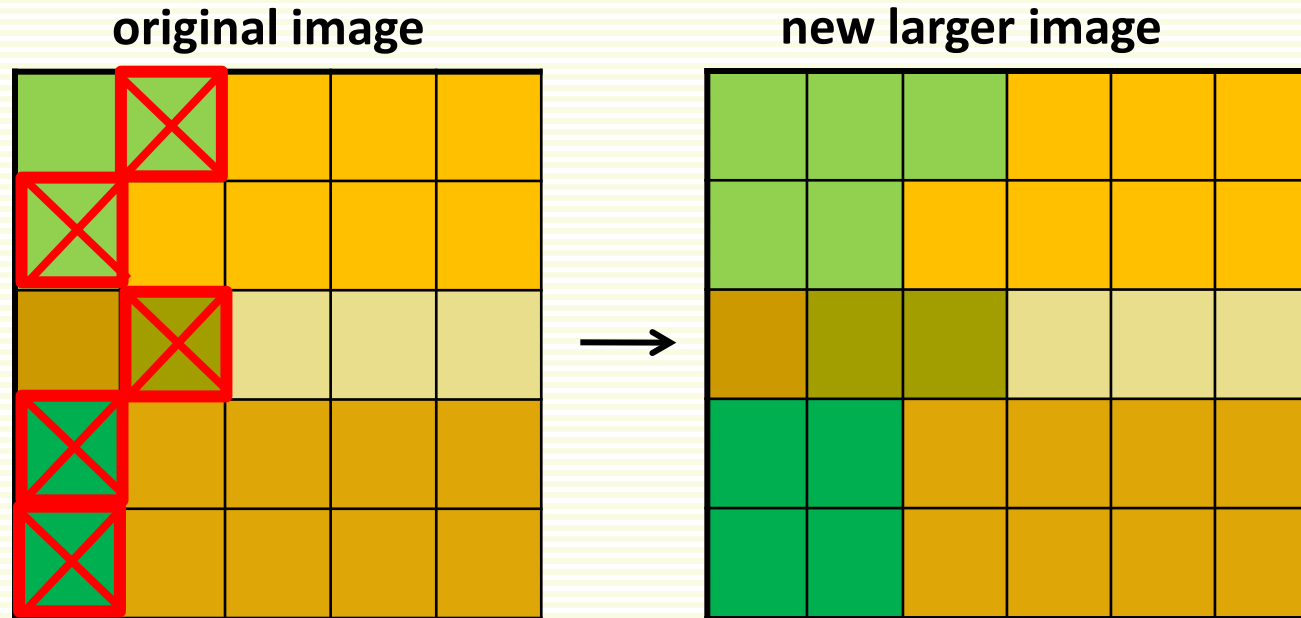
original image



new smaller image



# Other notes on seam carving



- Can also insert seams to *increase* size of image
  - duplicate optimal seam, averaged with neighbors
- Analogous procedure for horizontal seams
- Other energy functions may be plugged in
  - e.g., color-based
- Can remove (or keep, or enlarge) marked objects

# Some Results



**brings friends closer**



**or draws them apart**

# Include Color in Energy

- Want to remove objects of red color
  - $R_f, G_f, B_f$  are red, green blue color channels of image  $f$



$\|\nabla f\|$

+



$-2 \cdot R_f + G_f + B_f$

=



$energy(f)$



input image  $f$



carving out red



# Include Color in Energy

- That hat is too big - get rid of some green

$$energy(f) = \|\nabla f\| - 2G_f + R_f + B_f$$



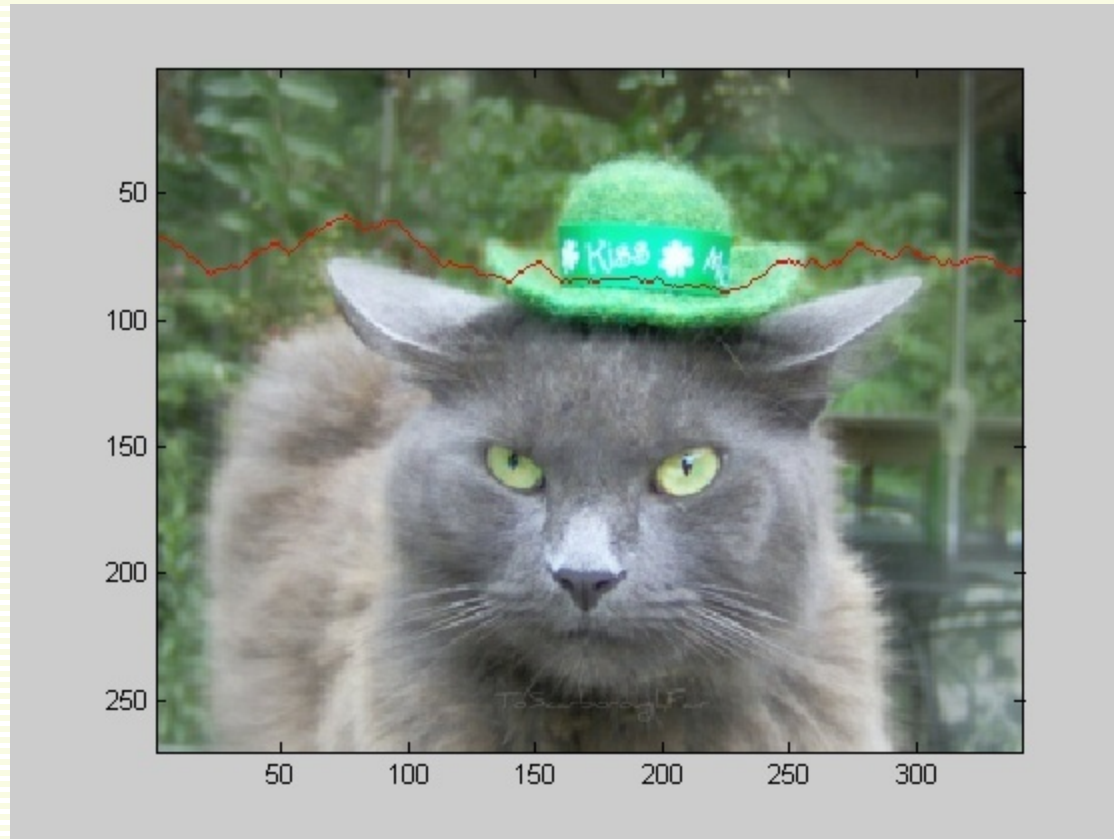
input image  $f$



carved



# Some Results



# Removal of a Marked Object

- Mask image  $M$  is 1 for object, 0 otherwise
  - remove vertical and horizontal seams



$\|\nabla f\|$

- 1000 ·



$M$

=



$energy(f)$



# Insert More Marked Object

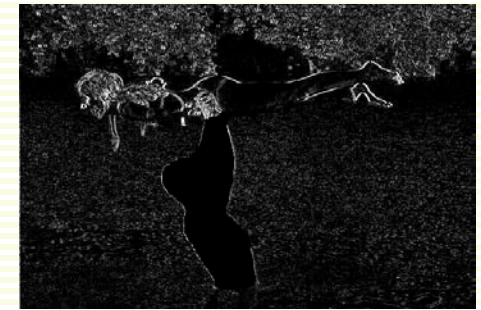
- Same energy, now insert vertical seams



- 1000·



=



$\|\nabla f\|$

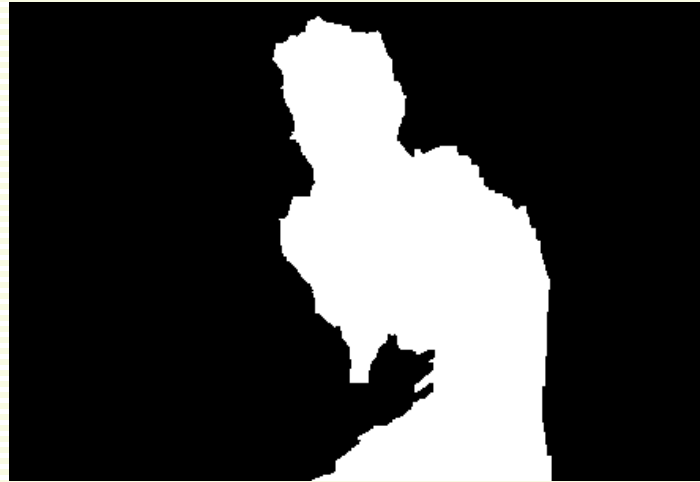
$M$

$energy(f)$





# Sometimes Remove Mask not Enough



# Remove and Preserve Mask

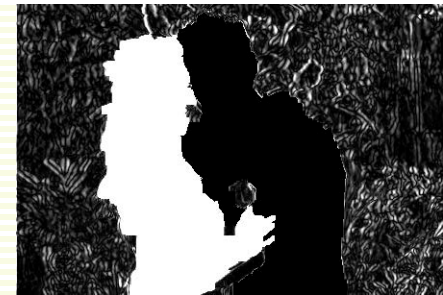
- $M$  is 1 for pixels to remove, -1 for pixels to keep, 0 for neutral



- 1000 ·



=



$\|\nabla f\|$

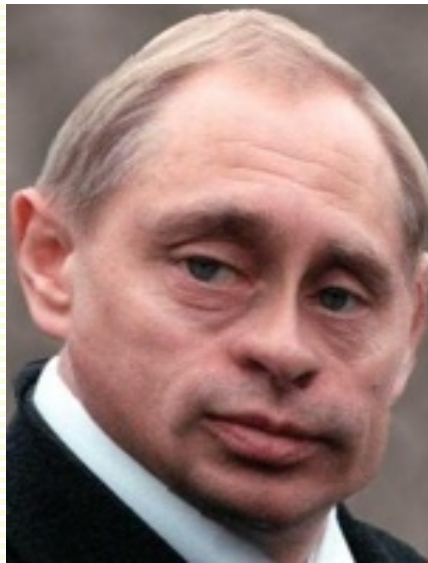
$M$

$energy(f)$





# Carving a Caricature



# Sometimes it Fails

