Absolute Factorization of Bivariate Polynomials with Floating Point Coefficients

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Given a polynomial $p(x, y)$ of degree d and complex floating point coefficients, we seek two polynomials f_1 and f_2 of degrees d_1 and d_2 such that $d_1 + d_2 = d$ and $f_1 \cdot f_2 = p + \Delta p$ for small Δp.

We view $p(x, y) = 0$ as defining $y(x)$, and develop y as a series in x about a generic point. Candidate factors are formed using truncations of $y(x)$ in bivariate polynomials of increasing degree.

The candidate factors are tested by approxiamate division with p, and the algorithm terminates when a pair f_1, f_2 is found which satisfy a given tolerance for Δp, or when the degree of the candidate factor exceeds $d/2$. The approximate division step requires the approximate solution of a linear system, for which we use the SVD to determine the numeric rank.

This a dense method, in the sense that Δp may introduce terms which are not in the support of p. To simplify treatment of leading terms we replace x, y with a randomized linear combination at the outset. We believe this first step is not essential, and could be replaced by a careful combinatoric reasoning.

This method can be applied to multivariate polynomials by considering generic bivariate linear restrictions of the given polynomial.