Guest Editors' Foreword

The worlds of symbolic and numeric computation have had surprisingly little overlap since the invention of the digital computer. The problems addressed, the techniques, the literature — in short, the cultures — have had largely separate developments. This has been a reflection of the backgrounds of the mathematicians who were active in the two areas in the 1970's: The fundamental algorithms of symbolic computation, or computer *algebra*, were to a great extent developed by individuals with a background in modern algebra. Numerical computation or numerical *analysis* rather attracted mathematicians from applied mathematics, with hard core analysis at their fingertips. Since that time, computer algebra has largely viewed the world as consisting of *discrete* objects whereas numerical analysis sees a *continuous* world virtually everywhere.

This has been particularly striking in polynomial algebra, the problem area of this special issue, where a good number of natural problems have been addressed by both communities, with practically no intersection of results until fairly recently. The methods used in one setting are usually not directly transferable to the other: Algorithms designed for exact computation may be numerically unstable or break down in a numerical setting because tests like "$a \neq 0$?" become meaningless. Most numerical algorithms, on the other hand, cannot reasonably be applied to discrete problems or suffer problems of intermediate expression swell.

Thus, to conquer the no–man's–land between computer algebra and numerical analysis, the two cultures must move towards each other and acknowledge each other's existence and particular traits, and they must learn each other's language. Appropriate approaches must use ideas and algorithmic techniques from numeric computation together with standard and possibly extended procedures from classical computer algebra. Fundamental in this collaboration will be two things: the computer algebra community's realization that computational algebraic problems must be immersed in an analytic context if their numerical solution is to make sense, and the realization by the numerical analysis community that algebraic methods can reveal information about computational algebraic problems which facilitates its numerical solution decisively.

A similar collaboration has spawned the well-known area of "numerical linear algebra" which has become not only one of the fastest growing areas in mathematics but also the indispensable foundation for almost all of scientific computing. Thus, the emerging area of mathematics dealing with computational algebraic problems of an intrinsically nonlinear nature might appropriately be called "numerical nonlinear algebra". Within that area, the consideration of computational problems with *polynomials* will certainly be a core subject: Specific polynomials over a field of characteristic zero (complex numbers, reals, rationals) have numerical data, and many numerical results are associated with such polynomials in a natural way, e.g. zeros. And, most practically, polynomials serve as the basic nonlinear models in many areas of scientific computing! Thus, a fast further development of Symbolic-Numeric Algebra for Polynomials (SNAP), from its conceptual foundations to the design of reliable and efficient software, is overdue.

While there has been a steady trickle of interest and results in symbolic-numeric algorithms over the past decades, we have seen a vigourous growth in the area recently. A few papers appeared in 1995 introducing new approaches in the area. The European project FRISCO (FRamework for Integrated Symbolic-numeric COmputation) was launched in 1996, combining the efforts of several groups having backgrounds in symbolic and numerical computing. As one outgrowth, a SNAP workshop was organized at INRIA Sophia-Antipolis (near Nice, France) which brought together a group of some 40 researchers in the area from all over the world. The meeting became seminal for a further spread of activities: after only occasional contributions from SNAP in earlier ISSACs and still only 3 at ISSAC 96, the number has risen to about 8 in ISSAC 97 and about 12 plus a tutorial at ISSAC 98 (exact counts depending on personal views); this reflects not only the growing volume of research in SNAP but also the willingness of the program committees to consider contributions from SNAP as worthwhile for presentation. Numerical polynomial algebra has become part of the mainstream of symbolic and algebraic computation.

The idea of this special volume originated at the ISSAC 96 in Zürich: it was felt that a collection of papers as they had been presented at the SNAP workshop in Sophia-Antipolis should be collected into an archival publication. The JSC Special Issues Coordinator, Bruno Buchberger, reacted favorably to this idea and asked the two of us to serve as Guest Editors for a special issue of JSC on *Symbolic-Numeric Algebra for Polynomials*.

We were honoured to respond to this invitation. We had both become interested in the area of symbolic numeric algebra for polynomials in the early 90's, though from complementary points of departure: While one of us (SMW) came from a background of symbolic computing, the other (HJS) had been involved in various aspects of numerical computation throughout his career. In his work on computer algebra systems, SMW had been trying to develop a design which would allow both symbolic and floating point computation to be efficient. SNAP provided a rich test domain: his collaboration with Corless, Gianni and Trager adopted numerical linear algebra tools in solving algebraic problems, taking a geometric view of the problem space. Their ISSAC 95 paper introduced this new approach to the computer algebra community. HJS, on the other hand, had only felt challenged to develop a fuller understanding of the essentials of constructive polynomial algebra when he had become aware of the apparent discontinuities in many algebraic constructs and of the instabilities of many algebraic algorithms but also of their enormous potential for numerical computation.

A call for papers was issued in the late fall of 1996; it asked for papers contributing to the understanding of the interaction of symbolic and numeric computing and relevant for the design of algorithms for the solution of related problems. The response was quite encouraging: 27 manuscripts arrived at our desks for consideration. In the subsequent review process, it was our goal to delimit the scope of the special issue by a clear adherence to the four words represented in SNAP: Only papers which properly related to all four of them were seriously considered. We have to be grateful to a great number of referees who spent their valuable time in reading papers and providing us with their expert opinions. As it often happens, the review process took much longer than originally anticipated, which may have frustrated a good number of the authors; we wish to apologize to them for this delay.

Finally, nine papers were retained for publication in the special issue; they are of high scientific quality and also matched our image of the special issue; the given page limit was a further restricting factor. Thus many good manuscripts could not be considered; we trust that they will—perhaps in a modified form—find their ways to publication through different venues.

The problem where the desirability of a numerical embedding was first realized in the computer algebra community is the computation of the greatest common divisor of univariate polynomials. Here, the discontinuity of the algebraic concept is too apparent: Whenever p_1 and p_2 have a nontrivial g.c.d., it "disappears" upon almost all arbitrarily small changes in their coefficients. This has led to various definitions of a pseudo-g.c.d. and to various algorithms for its computation. The first four papers in this issue address the g.c.d. problem. The first paper (by Karmarkar and Lakshman) presents a comprehensive analysis, it is the most definitive paper on the subject to date; it is also prototypical for many current contributions to SNAP in formulating the embedded problem as a new "strict" problem (an optimization problem in this case) and solving it in the computer algebra spirit. The second paper (by Zhi and Wu) is an elaboration of the first one: it exposes some interesting details of a special situation. The other two papers (by Beckermann and Labahn) present a much more numerics-minded discussion of a more realistic formulation of the g.c.d. problem. The first one of the two companion papers presents the essentials of the authors' ideas while the second one contains a thorough analysis of the algorithmic aspects which displays the many issues which have to be addressed in a rigorous study of a symbolic-numeric algorithm.

A central problem area of SNAP is clearly the determination of zeros of multivariate polynomial systems. First of all, even in systems with integer coefficients, the zeros are generally not rational numbers so that their numerical values can only be determined approximately. Moreover, when such systems arise in scientific computing, some of their coefficients are usually known with limited accuracy only. Unfortunately, there are only two papers in this issue which treat aspects of this problem area directly. The first one (by Mourrain) addresses one of the central issues: the use of linear algebra techniques for polynomial systems solving. It has become well-known that 0-dimensional polynomial system are equivalent (for the purpose of root finding) to matrix eigenproblems; but there are various potential approaches to the computational generation of the associated matrices. Mourrain's paper explains and compares some of the latest techniques. The other paper (by Tran) addresses the iterative refinement of an approximate zero: polynomial systems are often not regular (their Jacobian is not square or it is singular along a manifold) so that Newton's method cannot be applied immediately. The paper uses the Wu-Ritt regularization technique to reformulate the system and pseudoinverses of the Jacobian for the iterative improvement.

Both papers assume exact input and strive to compute approximations of the exact zeros to a desired accuracy efficiently. The note by Shirayanagi and Sweedler on automatic algorithm stabilization suggests a general scheme for the use of approximate computation under these circumstances. Although the scope of this scheme has not yet been fully established, we have included its presentation as a stimulus for discussion.

The remaining two papers discuss special problems in the realm of SNAP: The one (by Huber, Sottile, and Sturmfels) develops numerical homotopy algorithms for solving the systems of polynomial equations which arise from the classical Schubert calculus. The other one (by Gianni, Seppälä, Silhol, and Trager) presents a theoretical basis and an algorithm for computing a representation of a compact Riemann surface as an algebraic plane curve and for finding a numerical approximation for its period matrix.

It must be emphasized that the content of this special issue does not give a balanced let alone a complete picture of the subject area; in view of the rapid development of SNAP this could not be expected. Also a call for a special issue of JSC had naturally more appeal to the computer algebra community than to the numerics community; the attention of the latter for the many challenging problems in SNAP has still to be aroused. With all its shortcomings, this special issue should help to make both established researchers and newcomers in algebraic computation more fully aware of the potentials of this area and thus stimulate its further growth.

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September, 1998