Examples of MathML

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About one year ago, the World Wide Web Consortium issued a standard recommendation for MathML[1], a mark-up language intended to allow mathematical notations to be included in Web documents and to be transmitted between applications. Since then several software packages have implemented this specification.

This note gives a handful of examples illustrating MathML's so-called presentation tags. These examples are taken from an odd collection: In recent years the IBM T.J. Watson Research Center Mathematical Sciences Department has held a contest to design a T-shirt to be distributed to all summer students. In 1997, while the formulation of MathML was still underway, the summer T-shirt design indicated IBM's involvement in the Web Consortium Math Working Group, and tried to capture some of the notations and concepts that MathML would ultimately have to embrace. That summer IBM had more people in MathML[2] than any other organization.

Mathematical Sciences 1997

IBM T.J. Watson Research Center

`<math>`

$$\int_{\mathcal{C}} \mathrm{d}\,\omega \;=\; \int_{\partial\mathcal{C}} \omega$$

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) \;=\; (-1)^{\frac{p-1}{2}\cdot\frac{q-1}{2}}$$

$$G(E/F) \;=\; G(K/F)\;/\;G(K/E)$$

$$\nabla^{\mu}\nabla_{\mu}A^{\nu} - \nabla^{\nu}\nabla_{\mu}A^{\mu} = j^{\nu}$$

$$\partial_{n-1}\partial_{n}\,c \;=\; 0$$

`</math>`

1. Stokes' theorem

$$\int_{C} \mathrm{d}\,\omega \;=\; \int_{\partial C} \omega$$

This theorem is expressed most elegantly when cast in the language of differential forms, as above. The equation generalizes many integral identities, including the Ostrogradzky-Green and Riemann-Ampère-Stokes formulas. The fundamental theorem of the differential calculus, $\int_a^b \mathrm{d}f = f(b) - f(a)$, is a degenerate case.

```
<mrow>
   <mrow>
      <msub>
         <mo> &Integral;</mo>
         <mi> C </mi>
      </msub>
      <mrow>
         <mo> &DifferentialD; </mo>
         <mi> &omega; </mi>
      </mrow>
   </mrow>
   <mo> &equals; </mo>
   <mrow>
      <msub>
         <mo> &Integral; </mo>
         <mrow>
            <mo> &PartialD; </mo>
            <mi> C </mi>
         </mrow>
      </msub>
      <mi> &omega; </mi>
   </mrow>
</mrow>
```

2. The law of quadratic reciprocity

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) \;=\; (-1)^{\frac{p-1}{2}\times\frac{q-1}{2}}$$

This relation was discovered by Legendre, and proven by Gauss, who called it "the queen of number theory." Here p and q are odd primes and $\left(\frac{p}{q}\right)$ is the Legendre symbol, equal to ± 1 depending on whether or not p is a square mod q.

```
<mrow>
   <mrow>
      <mfenced open="(" close=")">
         <mfrac>
            <mi> p </mi>
            <mi> q </mi>
         </mfrac>
      </mfenced>
      <mo> &InvisibleTimes; </mo>
      <mfenced open="(" close=")">
         <mfrac>
            <mi> q </mi>
            <mi> p </mi>
         </mfrac>
      </mfenced>
   </mrow>
   <mo> &equals; </mo>
   <msup>
      <mfenced open="(" close=")">
         <mn> -1 </mn>
      </mfenced>
      <mrow>
         <mfrac>
            <mrow>
               <mi> p </mi>
               <mo> - </mo>
               <mn> 1 </mn>
            </mrow>
            <mn> 2 </mn>
         </mfrac>
         <mo> &times; </mo>
         <mfrac>
            <mrow>
               <mi> q </mi>
               <mo> - </mo>
               <mn> 1 </mn>
            </mrow>
            <mn> 2 </mn>
         </mfrac>
      </mrow>
   </msup>
</mrow>
```

3. Galois theory

$$G(E/F) \;=\; G(K/F)\;/\;G(K/E)$$

The main theorem of Galois theory states that for a finite normal extension $\boldsymbol{K}$ of a field $\boldsymbol{F}$, there is a one-to-one correspondence between the subgroups of $\boldsymbol{G(K/F)}$ and the intermediate fields $\boldsymbol{E}$. The notation $\boldsymbol{G(K/F)}$ denotes the group of all automorphisms of $\boldsymbol{K}$ leaving $\boldsymbol{F}$ fixed.

```
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     <mrow>
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       <mo> / </mo>
       <mi> F </mi>
      </mrow>
   </mfenced>
   <mo> &equals; </mo>
   <mi> G </mi>
   <mo> &ApplyFunction; </mo>
   <mfenced open="(" close=")">
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      <mi> K </mi>
      <mo> / </mo>
      <mi> F </mi>
     </mrow>
   </mfenced>
   <mi> / </mi>
   <mi> G </mi>
   <mo> &ApplyFunction; </mo>
   <mfenced open="(" close=")">
     <mrow>
      <mi> K </mi>
      <mo> / </mo>
      <mi> E </mi>
     </mrow>
   </mfenced>
</mrow>
```

4. Maxwell's equations

$$\nabla^{\mu}\nabla_{\mu}A^{\nu} - \nabla^{\nu}\nabla_{\mu}A^{\mu} = j^{\nu}$$

Maxwell's equations express the dynamics of electromagnetic and other fields. The equations are given here in a coordinate-independent form, where $\boldsymbol{A^{\mu}}$ is the gauge vector, $\boldsymbol{\nabla_{\mu}}$ is a covariant derivative operator and $\boldsymbol{j^{\mu}}$ is a current. With suitably generalized interpretation of the operators, these equations describe a broad class of phenomena, including non-abelian gauge fields and the electroweak interactions of elementary particles.

```
<mrow>
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      <mo> &nabla; </mo>
      <mi> &mu;    </mi>
   </msup>
   <msub>
      <mo> &nabla; </mo>
      <mi> &mu;    </mi>
   </msub>
   <msup>
      <mi> A    </mi>
      <mi> &nu; </mi>
   </msup>
   <mo> - </mo>
   <msup>
      <mo> &nabla; </mo>
      <mi> &nu;    </mi>
   </msup>
   <msub>
      <mo> &nabla; </mo>
      <mi> &mu;    </mi>
   </msub>
   <msup>
      <mi> A    </mi>
      <mi> &mu; </mi>
   </msup>
   <mo> &equals; </mo>
   <msup>
      <mi> j </mi>
      <mi> &nu; </mi>
   </msup>
</mrow>
```

5. The boundary of a boundary is empty

$$\partial_{n-1}\partial_n\, c \;=\; 0$$

This is one of the most important equations in homological algebra, if not all of mathematics. Here, $\partial_k : C_k(X) \;\rightarrow\; C_{k-1}(X)$ is the boundary operator on k-chains of a simplicial complex X. Stokes' theorem may be regarded as the de Rham cohomological version of this equation.

```
<mrow>
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      <mo> &PartialD; </mo>
      <mrow>
         <mi> n </mi>
         <mo> - </mo>
         <mn> 1 </mn>
      </mrow>
   </msub>
   <msub>
      <mo> &PartialD; </mo>
      <mi> n </mi>
   </msub>
   <mi> c </mi>
   <mo> &equals; </mo>
   <mn> 0 </mn>
</mrow>
```
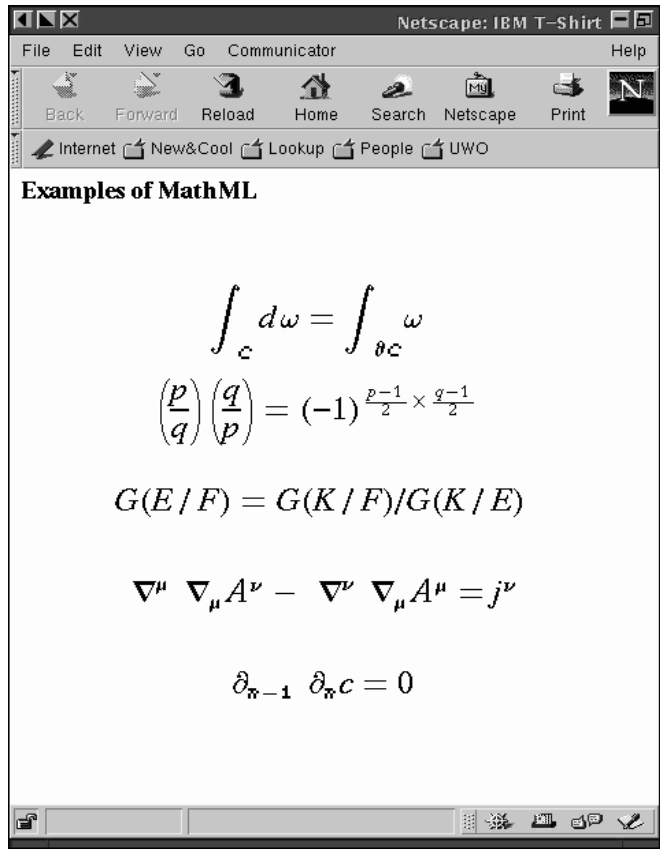

A Web Page

These examples can be downloaded from the web page `http://www.csd.uwo.ca/~watt/pub/1999/ibmtshirt.html`

At the time of writing, both Netscape and Internet Explorer require an applet to display MathML. After installing WebEQ[3] version 2.3, the page appears as below:



Notes

1. World Wide Web Consortium (W3C), *Mathematical Markup Language (MathML) 1.0 Specification*, `http://www.w3.org/TR/1998/REC-MathML-19980407`.

2. Apparel.

3. Geometry Technologies, Inc. `http://www.webeq.com`. [The authors thank Geometry Technologies for a demonstration copy of WebEQ.]