On the Approximate GCD in Initial Value Problems

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Abstract The computation of approximate greatest common divisors of polynomials has been considered by a number of authors under various different assumptions. Approximate GCDs have a number of applications, and have been used as an approach to ill-conditioned algebraic equations [Noda and Sasaki, J Comp and App Math 1991]. This paper examines the application of the approximate GCD to initial value problems. We consider initial value problems of the form $[\sum_{i=0}^{n} a_i D^i]\, y(t) = f(t)$ where a_i and $y^{(i)}(0)$ are given constants. Under appropriate conditions, equations such as this may be solved by integral transform methods. From the Laplace transform of the initial value problem, we see that $\mathcal{L}[y(t)](s)$ is a rational function whenever $\mathcal{L}[f(t)](s)$ is a rational function. Depending on a_i and $y^{(i)}(0)$, there may be a non-trivial approximate GCD between the numerator and denominator of $\mathcal{L}[y(t)](s)$. This paper examines the consequences of this fact and shows how the approximiate GCD may be used to remove spurious singularities.