

# Algorithms for the Functional Decomposition of Symbolic Polynomials

Stephen M. Watt  
University of Western Ontario, Canada

Earlier work has presented algorithms to factor and compute GCD of symbolic Laurent polynomials, that is multivariate polynomials whose exponents are themselves integer-valued polynomials. More recently we have extended the notion of univariate polynomial decomposition to symbolic polynomials and have presented an algorithm to compute these decompositions. For example, the symbolic polynomial  $f(x) = 2x^{n^2+n} - 4x^{n^2} + 2x^{n^2-n} + 1$  can be decomposed as  $f = g \circ h$  where  $g(x) = 2x^2 + 1$  and  $h(x) = x^{n^2/2+n/2} - x^{n^2/2-n/2}$ . This decomposition of symbolic Laurent polynomials relies on a functional decomposition of the usual multivariate Laurent polynomials with specific integer exponents. This paper shows this reduction and shows that the required multivariate Laurent polynomial decomposition is about the same cost as the decomposition of multivariate polynomials.