

Symbolic Symbolic Computation

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Abstract

Symbolic mathematical computation has become an important tool for modelling and analysis in most quantitative disciplines. It is now routine to represent, manipulate and reason about mathematical expressions involving unknowns. Efficient algorithms are known for a wide variety of commonly encountered linear, polynomial and differential problems. This has been achieved largely by phrasing symbolic mathematical computation in formal algebraic terms.

We argue that this highly developed state of the art has in some ways blinded us to the shortcomings of our approach. Most experienced users of symbolic computation systems know, for example, not to try to compute with polynomials of symbolic degree or matrices of symbolic size. For example, if n is an unspecified integer, how can one factor $X^{n^2/2-n/2} - 1$ or compute with a matrix of size $n \times n$? We show how such “symbolic” problems can be formulated precisely, show they lead to natural algebraic structures, and that a variety of algorithms may be applied to solve them.