LOGISMOS – Cost Functions

Graph-Based Image Segmentation: LOGISMOS

Milan Sonka
&
The IIBI Team

Iowa Institute for Biomedical Imaging
The University of Iowa, Iowa City, IA, USA

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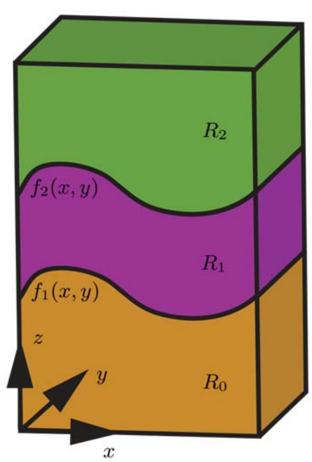
Surface set feasibility

A surface set { $f_1(x,y), \ldots, f_n(x,y)$ } is considered **feasible** if:

- Each surface in the set
  - satisfies surface smoothness constraints
- Each pair of surfaces
  - satisfies surface interaction constraints.



Cost Function – Surface Costs

- One-to-one correspondence between each feasible surface set and each closed set in the graph
  - *Graph arcs* reflect surface feasibility constraints
  - *Node costs* defined so that cost of each closed set corresponds to cost of a set of surfaces
- → Minimum-cost closed set corresponds to optimal set of surfaces

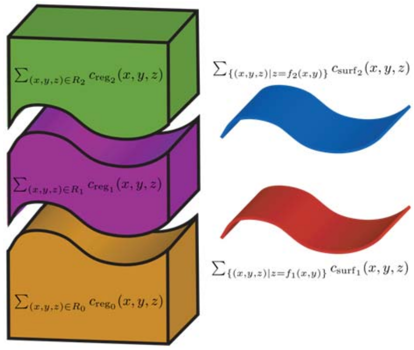
- Edge-based costs are a logical option
  - Strong edge → low node cost
  - Edge costs not always the best/only option → regional costs

Multi-Surface Regional Costs

- Edge based costs:
  - Each voxel → corresponding local edge/gradient cost
    - n "on-surface" costs – unlikeliness of belonging to each of the n surfaces

- Edge + region costs:
  - each voxel assigned $2n + 1$ cost values:
    - $n + 1$ "in-region" cost values, reflecting the unlikeliness of belonging to each of the $n + 1$ regions
    - n "on-surface" costs – unlikeliness of belonging to each of the n surfaces

Edge + Region Costs

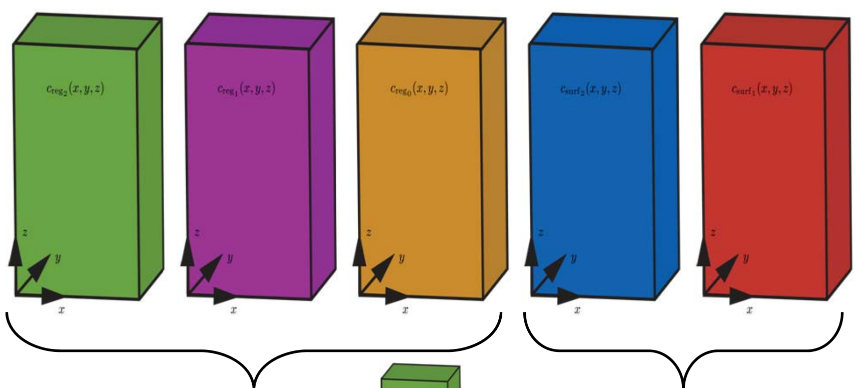


Summation of
- "on-surface" cost terms
- "in-region" cost terms

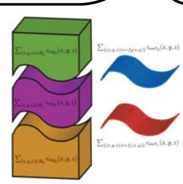
$$C_{\{f_1(x,y),f_2(x,y),\ldots,f_n(x,y)\}} = \sum_{i=1}^{n} C_{f_i(x,y)} + \sum_{i=0}^{n} C_{R_i}$$

$$\sum_{\{(x,y,z)|z=f_i(x,y)\}} c_{\mathrm{surf}_i}(x,y,z) \qquad \sum_{(x,y,z)\in R_i} c_{\mathrm{reg}_i}(x,y,z)$$

In-regions & on-surface costs



in-region cost functions



on-surface cost functions

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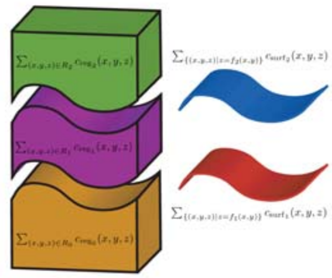
Extension 1: Including regional terms in the minimization problem

Surface set cost function:

$$C_{\{f_1(x,y),f_2(x,y),\ldots,f_n(x,y)\}} = \sum_{i=1}^{n} C_{f_i(x,y)} + \sum_{i=0}^{n} C_{R_i}$$



On-surface costs:

$$C_{f_i(x,y)} = \sum_{\{(x,y,z)|z=f_i(x,y)\}} c_{\mathrm{surf}_i}(x,y,z)$$

In-region costs:

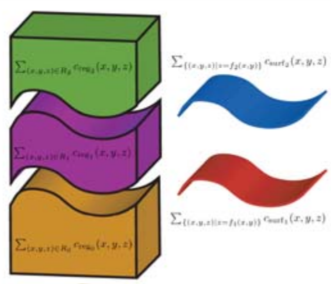
$$C_{R_i} = \sum_{(x,y,z)\in R_i} c_{\mathrm{reg}_i}(x,y,z)$$

Graph representation of in-region costs

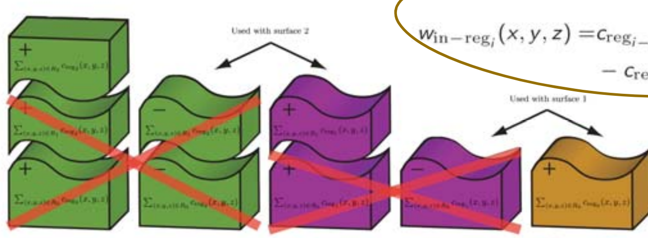
$$C_{\{f_1(x,y),f_2(x,y),\ldots,f_n(x,y)\}} = \sum_{i=1}^{n} C_{f_i(x,y)} + \sum_{i=0}^{n} C_{R_i}$$

$$C_{f_i(x,y)} = \sum_{\{(x,y,z)|z=f_i(x,y)\}} c_{\mathrm{surf}_i}(x,y,z)$$

$$C_{R_i} = \sum_{(x,y,z)\in R_i} c_{\mathrm{reg}_i}(x,y,z)$$



$$w_{\mathrm{in-reg}_i}(x,y,z) = c_{\mathrm{reg}_{i-1}}(x,y,z) - c_{\mathrm{reg}_i}(x,y,z)$$



Node-based Region Costs

$$w_{\text{in}-\text{reg}_i}(x,y,z) = c_{\text{reg}_{i-1}}(x,y,z) - c_{\text{reg}_i}(x,y,z)$$

Desired cost of closed set:

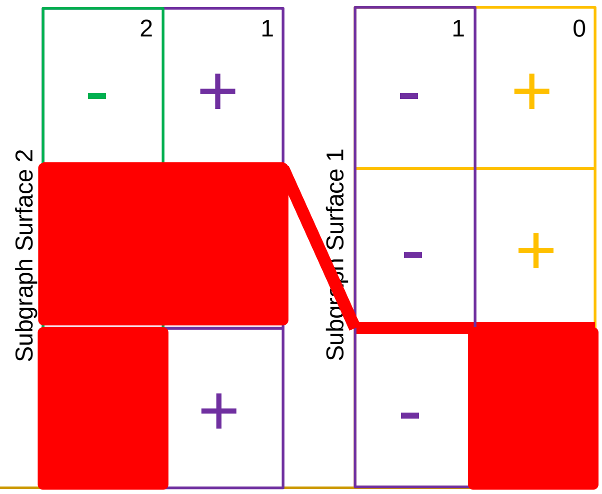
$R_{0cost}^{0} + R_{1cost}^{1} + R_{2cost}^{2}$

R_{2cost}^{2}
R_{1cost}^{1}
R_{0cost}^{0}

Cost of closed set ➔

$R_{0cost}^{0} - R_{1cost}^{0} + R_{1cost}^{1} + R_{1cost}^{0} - R_{2cost}^{0} - R_{2cost}^{1}$

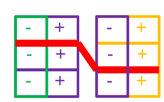
NOT quite correct



NOT quite correct ➔ Making it right

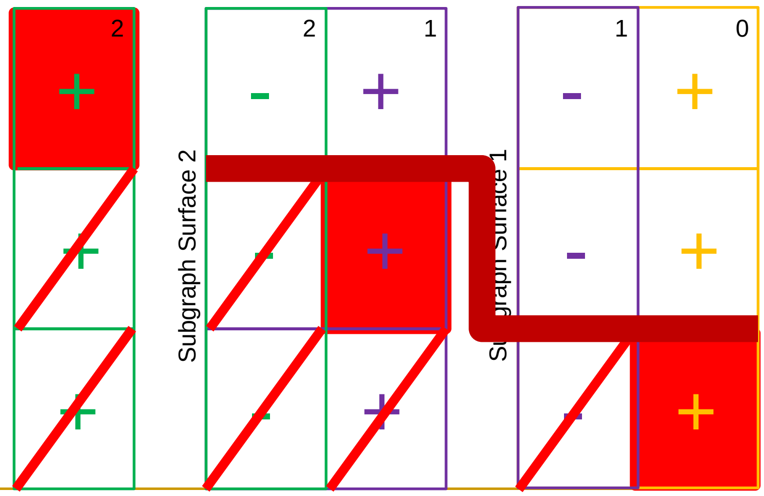
- Cost of closed set as defined so far:
  - $R_{0cost}^{0} + R_{1cost}^{1} + R_{1cost}^{0} - R_{1cost}^{0} - R_{2cost}^{0} - R_{2cost}^{1}$
- Correct cost to be minimized:
  - $\mathbf{R_{0cost}^{0} + R_{1cost}^{1} + R_{2cost}^{2}}$

R_{2cost}^{2}
R_{1cost}^{1}
R_{0cost}^{0}

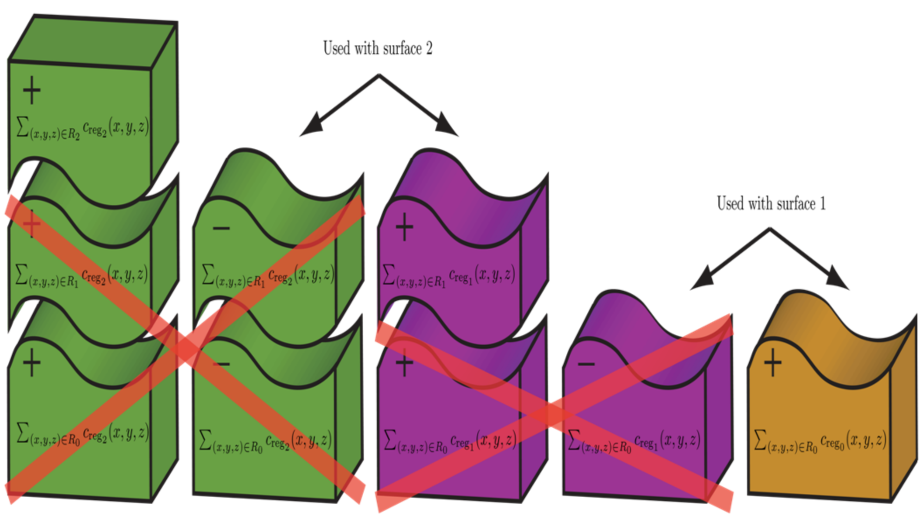


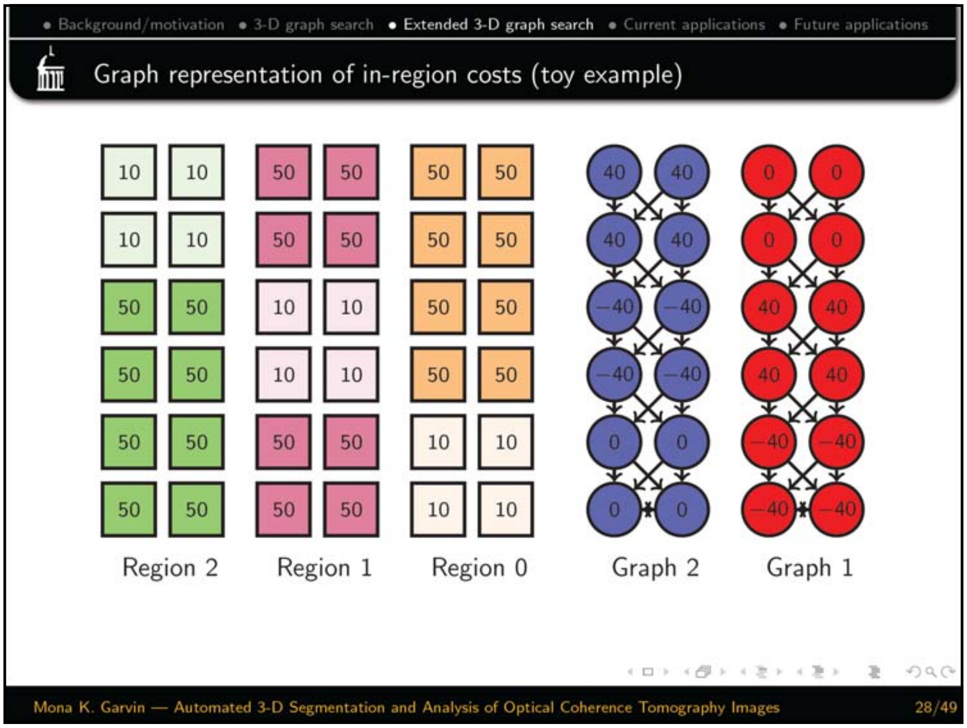
- $R_{0cost}^{0} - \cancel{R_{1cost}^{0}} + \cancel{R_{1cost}^{0}} + R_{1cost}^{1} - \cancel{R_{2cost}^{0}} - \cancel{R_{2cost}^{1}} +$ $+ \cancel{R_{2cost}^{0}} + \cancel{R_{2cost}^{1}} + R_{2cost}^{2}$
- **Adding** $K = R_{2cost}^{0} + R_{2cost}^{1} + R_{2cost}^{2}$ **solves the problem**
- **K is a constant ➔ OK to add a constant**

$$w_{\text{in-reg}_i}(x,y,z) = c_{\text{reg}_{i-1}}(x,y,z) - c_{\text{reg}_i}(x,y,z)$$



In-region Costs → Vertex Costs




• Background/motivation • 3-D graph search • Extended 3-D graph search • Current applications • Future applications
Graph representation of in-region costs (toy example)
Region 2
Region 1
Region 0
Graph 2
Graph 1
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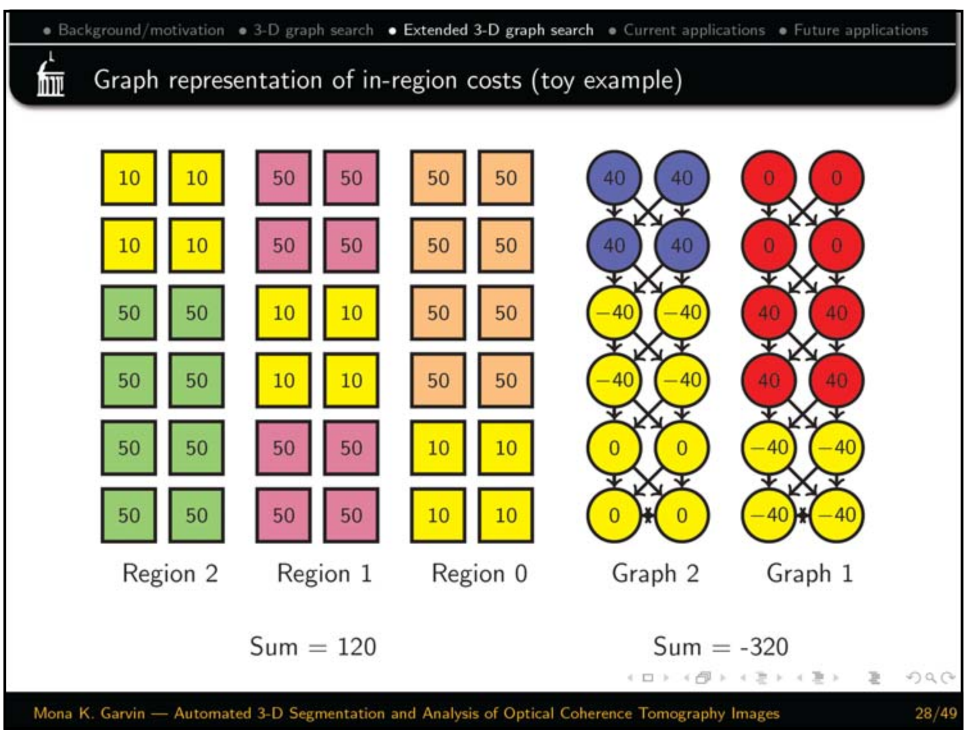
• Background/motivation • 3-D graph search • Extended 3-D graph search • Current applications • Future applications
Graph representation of in-region costs (toy example)
Region 2
Region 1
Region 0
Graph 2
Graph 1
Sum = 120
Sum = -320
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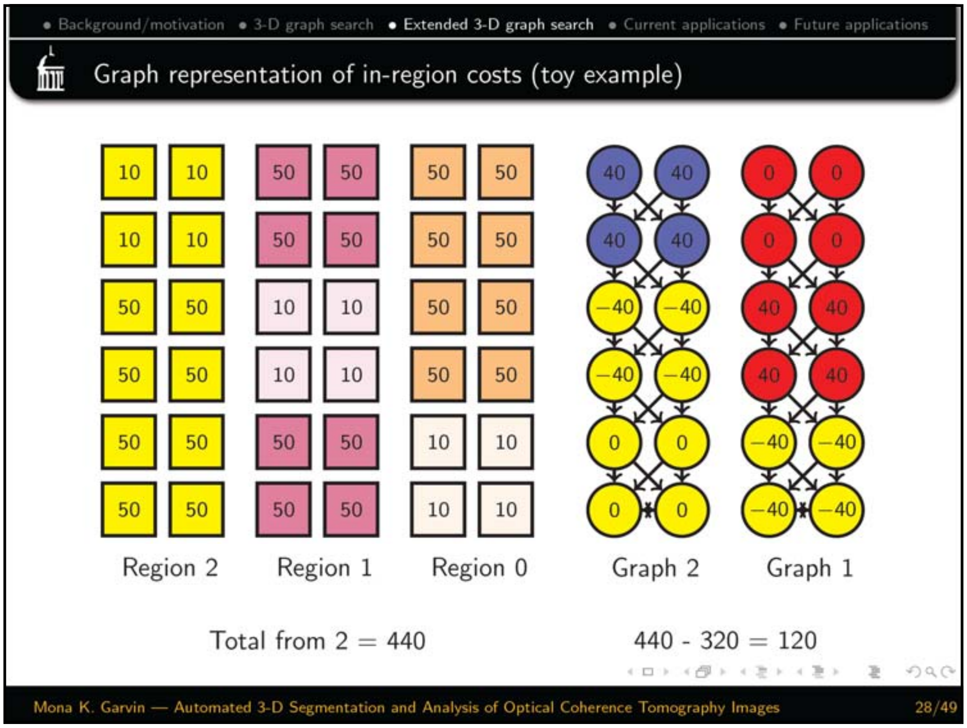
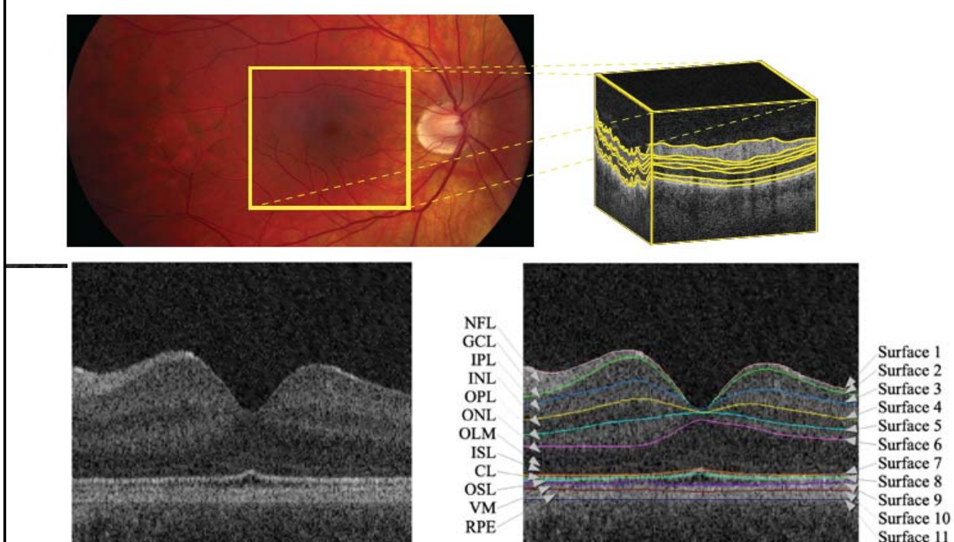


Retinal OCT



nerve fiber layer - NFL, ganglion cell layer -GCL, inner plexiform layer IPL, inner nuclear layer - INL, outer plexiform layer - OPL, outer nuclear layer - ONL, outer limiting membrane - OLM, inner segment layer - ISL, connecting cilia - CL, outer segment layer - OSL, Verhoeff's membrane - VM, and retinal pigment epithelium - RPE